FARI QUÆ SENTIA
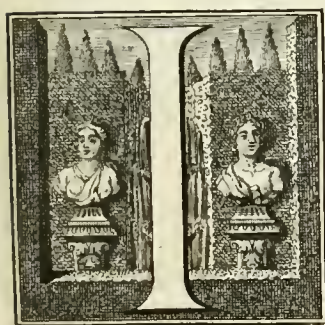
To the Noble and Right Honourable

SIR *ROBERT WALPOLE.*

SIR,

I Take the liberty to send you this view of Sir I s a a c N e w-t o n's philofophy, which, if it were performed fuitable to the dignity of the fubject, might not be a prefent unworthy the acceptance of the greatest perfon. For his philofophy affords us the only true account of the

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DEDICATION.

operations of nature, which for so many ages
had imployed the curiosity of mankind; though
no one before him was furnished with the
strength of mind necessary to go any depth in
this difficult search. However, I am encouraged
to hope, that this attempt, imperfect as it is, to
give our countrymen in general some concep-
tion of the labours of a person, who shall al-
ways be the boast of this nation, may be re-
ceived with indulgence by one, under whose
influence these kingdoms enjoy so much hap-
piness. Indeed my admiration at the surprizing
inventions of this great man, carries me to con-
ceive of him as a person, who not only must
raise the glory of the country, which gave him
birth; but that he has even done honour to hu-
man nature, by having extended the greatest
and most noble of our faculties, reason, to sub-
jects, which, till he attempted them, appeared
to be wholly beyond the reach of our limi-
ted capacities. And what can give us a
<div align="right">more</div>

DEDICATION.

more pleafing profpect of our own condition, than to fee fo exalted a proof of the ftrength of that faculty, whereon the conduct of our lives, and our happinefs depends; our paffions and all our motives to action being in fuch manner guided by our opinions, that where thefe are juft, our whole behaviour will be praife-worthy? But why do I prefume to detain you, SIR, with fuch reflections as thefe, who muft have the fulleft experience within your own mind, of the effects of right reafon? For to what other fource can be afcribed that amiable franknefs and unreferved condefcenfion among your friends, or that mafculine perfpicuity and ftrength of argument, whereby you draw the admiration, of the publick, while you are engaged in the moft important of all caufes, the liberties of mankind?

I humbly crave leave to make the only acknowledgement within my power, for the benefits, which.

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which I receive in common with the rest of my countrymen from these high talents, by sub-scribing my self

SIR,

Your most faithful,

and

Most humble Servant,

HENRY PEMBERTON.

PREFACE.

I Drew up the following papers many years ago at the defire of fome friends, who, upon my taking care of the late edition of Sir Isaac Newton's Principia, perfwaded me to make them publick. I laid hold of that opportunity, when my thoughts were afrefh employed on this fubject, to revife what I had formerly written. And I now fend it abroad not without fome hopes of anfwering thefe two ends. My firft intention was to convey to fuch, as are not ufed to mathematical reafoning, fome idea of the philofophy of a perfon, who has acquired an univerfal reputation, and rendered our nation famous for thefe fpeculations in the learned world. To which purpofe I have avoided ufing terms of art as much as poffible, and taken care to define fuch as I was obliged to ufe. Though this caution was the lefs neceffary at prefent, fince many of them are become familiar words to our language, from the great number of books wrote in it upon philofophical fubjects, and the courfes of experiments, that have of late years been given by feveral ingenious men. The other view I had, was to encourage fuch young gentlemen as have a turn for the mathematical fciences, to purfue thofe ftudies the more chearfully, in order to underftand in our author himfelf the demonftrations of the things I here declare. And to facilitate their progrefs herein, I intend to proceed ftill farther in the explanation of Sir Isaac Newton's philofophy. For as I have received very much pleafure from perufing his writings, I hope it is no illaudable ambition to endeavour the rendering them more eafily underftood, that greater numbers may enjoy the fame fatisfaction.

It will perhaps be expected, that I fhould fay fomething particular of a perfon, to whom I muft always acknowledge my felf to be much obliged. What I have to declare on this head will be but fhort; for it was in the very laft years of Sir Isaac's life, that I had the ho-

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P R E F A C E.

nour of his acquaintance. This happened on the following occasion. Mr. Polenus, *a Professor in the University of* Padua, *from a new experiment of his, thought the common opinion about the force of moving bodies was overturned, and the truth of* Mr. Libnitz's *notion in that matter fully proved. The contrary of what* Polenus *had asserted I demonstrated in a paper, which* Dr. MEAD, *who takes all opportunities of obliging his friends, was pleased to shew Sir* ISAAC NEWTON. *This was so well approved of by him, that he did me the honour to become a fellow-writer with me, by annexing to what I had written, a demonstration of his own drawn from another consideration. When I printed my discourse in the philosophical transactions, I put what Sir* ISAAC *had written in a scholium by it self, that I might not seem to usurp what did not belong to me. But I concealed his name, not being then sufficiently acquainted with him to ask whether he was willing I might make use of it or not. In a little time after he engaged me to take care of the new edition he was about making of his* Principia. *This obliged me to be very frequently with him, and as he lived at some distance from me, a great number of letters passed between us on this account. When I had the honour of his conversation, I endeavoured to learn his thoughts upon mathematical subjects, and something historical concerning his inventions, that I had not been before acquainted with. I found, he had read fewer of the modern mathematicians, than one could have expected ; but his own prodigious invention readily supplied him with what he might have an occasion for in the pursuit of any subject he undertook. I have often heard him censure the handling geometrical subjects by algebraic calculations; and his book of Algebra he called by the name of Universal Arithmetic, in opposition to the injudicious title of Geometry, which* Des Cartes *had given to the treatise, wherein he shews, how the geometer may assist his invention by such kind of computations. He frequently praised* Slusius, Barrow *and* Huygens *for not being influenced by the false taste, which then began to prevail. He used to commend the laudable attempt of* Hugo de Omerique *to restore the ancient analysis, and very much esteemed* Apolonius's *book* De sectione rationis *for giving us a clearer notion of that analysis than we had before.* Dr. Barrow *may be esteemed as having*

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ing shewn a compass of invention equal, if not superior to any of the moderns, our author only excepted; but Sir Isaac Newton has several times particularly recommended to me Huygens's stile and manner. He thought him the most elegant of any mathematical writer of modern times, and the most just imitator of the antients. Of their taste, and form of demonstration Sir Isaac always professed himself a great admirer: I have heard him even censure himself for not following them yet more closely than he did; and speak with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of Des Cartes and other algebraic writers, before he had considered the elements of Euclide with that attention, which so excellent a writer deserves. As to the history of his inventions, what relates to his discoveries of the methods of series and fluxions, and of his theory of light and colours, the world has been sufficiently informed of already. The first thoughts, which gave rise to his Principia, he had, when he retired from Cambridge in 1666 on account of the plague. As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not found sensibly diminished at the remotest distance from the center of the earth, to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains; it appeared to him reasonable to conclude, that this power must extend much farther than was usually thought; why not as high as the moon, said he to himself? and if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby. However, though the power of gravity is not sensibly weakened in the little change of distance, at which we can place our selves from the center of the earth; yet it is very possible, that so high as the moon this power may differ much in strength from what it is here. To make an estimate, what might be the degree of this diminution, he considered with himself, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried round the sun by the like power. And by comparing the periods of the several planets with their distances from the sun, he found, that if any power like gravity held them in their courses, its strength must decrease in the duplicate proportion of the increase of distance. This

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he concluded by supposing them to move in perfect circles concentrical to the sun, from which the orbits of the greatest part of them do not much differ. Supposing therefore the power of gravity, when extended to the moon, to decrease in the same manner, he computed whether that force would be sufficient to keep the moon in her orbit. In this computation, being absent from books, he took the common estimate in use among geographers and our seamen, before Norwood had measured the earth, that 60 English miles were contained in one degree of latitude on the surface of the earth. But as this is a very faulty supposition, each degree containing about 69½ of our miles, his computation did not answer expectation; whence he concluded, that some other cause must at least join with the action of the power of gravity on the moon. On this account he laid aside for that time any farther thoughts upon this matter. But some years after, a letter which he received from Dr. Hook, put him on inquiring what was the real figure, in which a body let fall from any high place descends, taking the motion of the earth round its axis into consideration. Such a body, having the same motion, which by the revolution of the earth the place has whence it falls, is to be considered as projected forward and at the same time drawn down to the center of the earth. This gave occasion to his resuming his former thoughts concerning the moon; and Picart in France having lately measured the earth, by using his measures the moon appeared to be kept in her orbit purely by the power of gravity; and consequently, that this power decreases as you recede from the center of the earth in the manner our author had formerly conjectured. Upon this principle he found the line described by a falling body to be an ellipsis, the center of the earth being one focus. And the primary planets moving in such orbits round the sun, he had the satisfaction to see, that this inquiry, which he had undertaken merely out of curiosity, could be applied to the greatest purposes. Hereupon he composed near a dozen propositions relating to the motion of the primary planets about the sun. Several years after this, some discourse he had with Dr. Halley, who at Cambridge made him a visit, engaged Sir I s a a c N e w t o n to resume again the consideration of this subject; and gave occasion

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to his writing the treatise which he published under the title of *mathematical principles of natural philosophy*. This treatise, full of such a variety of profound inventions, was composed by him from scarce any other materials than the few propositions before mentioned, in the space of one year and an half.

Though his memory was much decayed, I found he perfectly understood his own writings, contrary to what I had frequently heard in discourse from many persons. This opinion of theirs might arise perhaps from his not being always ready at speaking on these subjects, when it might be expected he should. But as to this, it may be observed, that great genius's are frequently liable to be absent, not only in relation to common life, but with regard to some of the parts of science they are the best informed of. Inventors seem to treasure up in their minds, what they have found out, after another manner than those do the same things, who have not this inventive faculty. The former, when they have occasion to produce their knowledge, are in some measure obliged immediately to investigate part of what they want. For this they are not equally fit at all times: so it has often happened, that such as retain things chiefly by means of a very strong memory, have appeared off hand more expert than the discoverers themselves.

As to the moral endowments of his mind, they were as much to be admired as his other talents. But this is a field I leave others to exspatiate in. I only touch upon what I experienced my self during the few years I was happy in his friendship. But this I immediately discovered in him, which at once both surprized and charmed me: Neither his extreme great age, nor his universal reputation had rendred him stiff in opinion, or in any degree elated. Of this I had occasion to have almost daily experience. The Remarks I continually sent him by letters on his *Principia* were received with the utmost goodness. These were so far from being any ways displeasing to him, that on the contrary it occasioned him to speak many kind things of me to my friends, and to honour me with a publick testimony of his good opinion. He also approved of the following treatise, a great part of which we read together. As many alterations were

made

PREFACE.

made in the late edition of his *Principia*, so there would have been many more if there had been a sufficient time. But whatever of this kind may be thought wanting, I shall endeavour to supply in my comment on that book. I had reason to believe he expected such a thing from me, and I intended to have published it in his life time, after I had printed the following discourse, and a mathematical treatise Sir ISAAC NEWTON had written a long while ago, containing the first principles of fluxions, for I had prevailed on him to let that piece go abroad. I had examined all the calculations, and prepared part of the figures; but as the latter part of the treatise had never been finished, he was about letting me have other papers, in order to supply what was wanting. But his death put a stop to that design. As to my comment on the *Principia*, I intend there to demonstrate whatever Sir ISAAC NEWTON has set down without express proof, and to explain all such expressions in his book, as I shall judge necessary. This comment I shall forthwith put to the press, joined to an english translation of his *Principia*, which I have had some time by me. A more particular account of my whole design has already been published in the new memoirs of literature for the month of march 1727.

I have presented my readers with a copy of verses on Sir ISAAC NEWTON, which I have just received from a young Gentleman, whom I am proud to reckon among the number of my dearest friends. If I had any apprehension that this piece of poetry stood in need of an apology, I should be desirous the reader might know, that the author is but sixteen years old, and was obliged to finish his composition in a very short space of time. But I shall only take the liberty to observe, that the boldness of the digressions will be best judged of by those who are acquainted with PINDAR.

A POEM

A

P O E M

O N

Sir *I S A A C   N E W T O N.*

TO Newton's genius, and immortal fame
 Th' advent'rous muſe with trembling pinion ſoars.
 Thou, heav'nly truth, from thy ſeraphick throne
 Look favourable down, do thou aſſiſt
My lab'ring thought, do thou inſpire my ſong.
Newton, who firſt th' almighty's works diſplay'd,
And ſmooth'd that mirror, in whoſe poliſh'd face
The great creator now conſpicuous ſhines;
Who open'd nature's adamantine gates,
And to our minds her ſecret powers expos'd;
Newton demands the muſe; his ſacred hand
Shall guide her infant ſteps; his ſacred hand
Shall raiſe her to the Heliconian height,
Where, on its lofty top inthron'd, her head
Shall mingle with the Stars. Hail nature, hail,
O Goddeſs, handmaid of th' ethereal power,
Now lift thy head, and to th' admiring world
Shew thy long hidden beauty. Thee the wiſe
Of ancient fame, immortal Plato's ſelf,
The Stagyrite, and Syracuſian ſage,

From

A Poem on Sir Isaac Newton.

From black obfcurity's abyfs to raife,
(Drooping and mourning o'er thy wondrous works)
With vain inquiry fought. Like meteors thefe
In their dark age bright fons of wifdom fhone :
But at thy N e w t o n all their laurels fade,
They fhrink from all the honours of their names.
So glimm'ring ftars contract their feeble rays,
When the fwift luftre of A u r o r a's face
Flows o'er the skies, and wraps the heav'ns in light.

T h e Deity's omnipotence, the caufe,
Th' original of things long lay unknown.
Alone the beauties prominent to fight
(Of the celeftial power the outward form)
Drew praife and wonder from the gazing world.
As when the deluge overfpread the earth,
Whilft yet the mountains only rear'd their heads
Above the furface of the wild expanfe,
Whelm'd deep below the great foundations lay,
Till fome kind angel at heav'n's high command
Roul'd back the rifing tides, and haughty floods,
And to the ocean thunder'd out his voice :
Quick all the fwelling and imperious waves,
The foaming billows and obfcuring furge,
Back to their channels and their ancient feats
Recoil affrighted : from the darkfome main
Earth raifes fmiling, as new-born, her head,
And with frefh charms her lovely face arrays.
So his extenfive thought accomplifh'd firft
The mighty task to drive th' obftructing mifts
Of ignorance away, beneath whofe gloom
Th' infhrouded majefty of Nature lay.
He drew the veil and fwell'd the fpreading fcene.
How had the moon around th' ethereal void

Rang'd,

A Poem on Sir Isaac Newton.

Rang'd, and eluded lab'ring mortals care,
Till his invention trac'd her fecret fteps,
While fhe inconftant with unfteady rein
Through endlefs mazes and meanders guides
In its unequal courfe her changing carr :
Whether behind the fun's fuperior light
She hides the beauties of her radiant face,
Or, when confpicuous, fmiles upon mankind,
Unveiling all her night-rejoicing charms.
When thus the filver-treffed moon difpels
The frowning horrors from the brow of night,
And with her fplendors chears the fullen gloom,
While fable-mantled darknefs with his veil
The vifage of the fair horizon fhades,
And over nature fpreads his raven wings ;
Let me upon fome unfrequented green
While fleep fits heavy on the drowfy world,
Seek out fome folitary peaceful cell,
Where darkfome woods around their gloomy brows
Bow low, and ev'ry hill's protended fhade
Obfcures the dusky vale, there filent dwell,
Where contemplation holds its ftill abode,
There trace the wide and pathlefs void of heav'n,
And count the ftars that fparkle on its robe.
Or elfe in fancy's wild'ring mazes loft
Upon the verdure fee the fairy elves
Dance o'er their magick circles, or behold,
In thought enraptur'd with the ancient bards,
Medea's baleful incantations draw
Down from her orb the paly queen of night.
But chiefly NEWTON let me foar with thee,
And while furveying all yon ftarry vault
With admiration I attentive gaze,
Thou fhalt defcend from thy celeftial feat,

And

And waft aloft my high-afpiring mind,
Shalt fhew me there how nature has ordain'd
Her fundamental laws, fhalt lead my thought
'Through all the wand'rings of th' uncertain moon,
And teach me all her operating powers.
She and the fun with influence conjoint
Wield the huge axle of the whirling earth,
And from their juft direction turn the poles,
Slow urging on the progrefs of the years.
The conftellations feem to leave their feats,
And o'er the skies with folemn pace to move.
You, fplendid rulers of the day and night,
The feas obey, at your refiftlefs fway
Now they contract their waters, and expofe
The dreary defart of old ocean's reign.
The craggy rocks their horrid fides difclofe ;
Trembling the failor views the dreadful fcene,
And cautioufly the threat'ning ruin fhuns.
But where the fhallow waters hide the fands,
There ravenous deftruction lurks conceal'd,
There the ill-guided veffel falls a prey,
And all her numbers gorge his greedy jaws.
But quick returning fee th' impetuous tides
Back to th' abandon'd fhores impell the main.
Again the foaming feas extend their waves,
Again the rouling floods embrace the fhoars,
And veil the horrours of the empty deep.
Thus the obfequious feas your power confefs,
While from the furface healthful vapours rife
Plenteous throughout the atmofphere diffus'd,
Or to fupply the mountain's heads with fprings,
Or fill the hanging clouds with needful rains,
That friendly ftreams, and kind refrefhing fhow'rs
May gently lave the fun-burnt thirfty plains,

Or

A Poem on Sir Isaac Newton.

Or to replenish all the empty air
With wholsome moisture to increase the fruits
Of earth, and bless the labours of mankind.
O Newton, whether flies thy mighty soul,
How shall the feeble muse pursue through all
The vast extent of thy unbounded thought,
That even seeks th' unseen recesses dark
To penetrate of providence immense.
And thou the great dispenser of the world
Propitious, who with inspiration taught'st
Our greatest bard to send thy praises forth ;
Thou, who gav'st Newton thought ; who smil'dst serene,
When to its bounds he stretch'd his swelling soul ;
Who still benignant ever blest his toil,
And deign'd to his enlight'ned mind t' appear
Confess'd around th' interminated world :
To me O thy divine infusion grant
(O thou in all so infinitely good)
That I may sing thy everlasting works,
Thy inexhausted store of providence,
In thought effulgent and resounding verse.
O could I spread the wond'rous theme around,
Where the wind cools the oriental world,
To the calm breezes of the Zephir's breath,
To where the frozen hyperborean blasts,
To where the boist'rous tempest-leading south
From their deep hollow caves send forth their storms.
Thou still indulgent parent of mankind,
Left humid emanations should no more
Flow from the ocean, but dissolve away
Through the long series of revolving time ;
And left the vital principle decay,
By which the air supplies the springs of life ;
Thou hast the fiery visag'd comets form'd

[b]

With

A Poem on Sir Isaac Newton.

With vivifying fpirits all replete,
Which they abundant breathe about the void,
Renewing the prolifick foul of things.
No longer now on thee amaz'd we call,
No longer tremble at imagin'd ills,
When comets blaze tremendous from on high,
Or when extending wide their flaming trains
With hideous grafp the skies engirdle round,
And fpread the terrors of their burning locks.
For thefe through orbits in the length'ning fpace
Of many tedious rouling years compleat
Around the fun move regularly on ;
And with the planets in harmonious orbs,
And myftick periods their obeyfance pay
To him majeftick ruler of the skies
Upon his throne of circled glory fixt.
He or fome god confpicuous to the view,
Or elfe the fubftitute of nature feems,
Guiding the courfes of revolving worlds.
He taught great NEWTON the all-potent laws
Of gravitation, by whofe fimple power
The univerfe exifts.   Nor here the fage
Big with invention ftill renewing ftaid.
But O bright angel of the lamp of day,
How fhall the mufe difplay his greateft toil?
Let her plunge deep in Aganippe's waves,
Or in Caftalia's ever-flowing ftream,
That re-infpired fhe may fing to thee,
How NEWTON dar'd advent'rous to unbraid
The yellow treffes of thy fhining hair.
Or didft thou gracious leave thy radiant fphere,
And to his hand thy lucid fplendours give,
T' unweave the light-diffufing wreath, and part

A Poem on Sir Isaac Newton.

The blended glories of thy golden plumes?
He with laborious, and unerring care,
How diff'rent and imbodied colours form
Thy piercing light, with juft diftinction found.
He with quick fight purfu'd thy darting rays,
When penetrating to th' obfcure recefs
Of folid matter, there perfpicuous faw,
How in the texture of each body lay
The power that feparates the diff'rent beams.
Hence over nature's unadorned face
Thy bright diverfifying rays dilate
Their various hues: and hence when vernal rains
Defcending fwift have burft the low'ring clouds,
Thy fplendors through the diffipating mifts
In its fair vefture of unnumber'd hues
Array the fhow'ry bow.   At thy approach
The morning rifen from her pearly couch
With rofy blufhes decks her virgin cheek;
The ev'ning on the frontifpiece of heav'n
His mantle fpreads with many colours gay;
The mid-day skies in radiant azure clad,
The fhining clouds, and filver vapours rob'd
In white tranfparent intermixt with gold,
With bright variety of fplendor cloath
All the illuminated face above.
When hoary-headed winter back retires
To the chill'd pole, there folitary fits
Encompafs'd round with winds and tempefts bleak
In caverns of impenetrable ice,
And from behind the diffipated gloom
Like a new Venus from the parting furge
The gay-apparell'd fpring advances on;
When thou in thy meridian brightnefs fitt'ft,
And from thy throne pure emanations flow

A Poem on Sir Isaac Newton.

Of glory burſting o'er the radiant skies:
Then let the muſe Olympus' top aſcend,
And o'er 'Theſſalia's plain extend her view,
And count, O Tempe, all thy beauties o'er.
Mountains, whoſe ſummits graſp the pendant clouds,
Between their wood-invelop'd ſlopes embrace
The green-attired vallies.   Every flow'r
Here in the pride of bounteous nature clad
Smiles on the boſom of th' enamell'd meads.
Over the ſmiling lawn the ſilver floods
Of fair Peneus gently roul along,
While the reflected colours from the flow'rs,
And verdant borders pierce the lympid waves,
And paint with all their variegated hue
The yellow ſands beneath.   Smooth gliding on
The waters haſten to the neighbouring ſea.
Still the pleas'd eye the floating plain purſues;
At length, in Neptune's wide dominion loſt,
Surveys the ſhining billows, that ariſe
Apparell'd each in Phœbus' bright attire:
Or from a far ſome tall majeſtick ſhip,
Or the long hoſtile lines of threat'ning fleets,
Which o'er the bright uneven mirror ſweep,
In dazling gold and waving purple deckt;
Such as of old, when haughty Athens power
Their hideous front, and terrible array
Againſt Pallene's coaſt extended wide,
And with tremendous war and battel ſtern
The trembling walls of Potidæa ſhook.
Creſted with pendants curling with the breeze
The upright maſts high briſtle in the air,
Aloft exalting proud their gilded heads.
The ſilver waves againſt the painted prows
Raiſe their reſplendent boſoms, and impearl

The

A Poem on Sir Isaac Newton.

The fair vermillion with their glist'ring drops:
And from on board the iron-cloathed hoft
Around the main a gleaming horrour cafts;
Each flaming buckler like the mid-day fun,
Each plumed helmet like the filver moon,
Each moving gauntlet like the light'ning's blaze,
And like a ftar each brazen pointed fpear.
But lo the facred high-erected fanes,
Fair citadels, and marble-crowned towers,
And fumptuous palaces of ftately towns
Magnificent arife, upon their heads
Bearing on high a wreath of filver light.
But fee my mufe the high Pierian hill,
Behold its fhaggy locks and airy top,
Up to the skies th' imperious mountain heaves
The fhining verdure of the nodding woods.
See where the filver Hippocrene flows,
Behold each glitt'ring rivulet, and rill
Through mazes wander down the green defcent,
And fparkle through the interwoven trees.
Here reft a while and humble homage pay,
Here, where the facred genius, that infpir'd
Sublime MÆONIDES and PINDAR's breaft,
His habitation once was fam'd to hold.
Here thou, O HOMER, offer'dft up thy vows;
Thee, the kind mufe CALLIOPÆA heard,
And led thee to the empyrean feats,
There manifefted to thy hallow'd eyes
The deeds of gods; thee wife MINERVA taught
The wondrous art of knowing human kind;
Harmonious PHOEBUS tun'd thy heav'nly mind,
And fwell'd to rapture each exalted fenfe;
Even MARS the dreadful battle-ruling god,
MARS taught thee war, and with his bloody hand

<div align="right">Inftructed</div>

A Poem on Sir Isaac Newton.

Inſtructed thine, when in thy ſounding lines
We hear the rattling of Bellona's carr,
The yell of diſcord, and the din of arms.
Pindar, when mounted on his fiery ſteed,
Soars to the ſun, oppoſing eagle like
His eyes undazled to the fierceſt rays.
He firmly ſeated, not like Glaucus' ſon,
Strides his ſwift-winged and fire-breathing horſe,
And born aloft ſtrikes with his ringing hoofs
The brazen vault of heav'n, ſuperior there
Looks down upon the ſtars, whoſe radiant light
Illuminates innumerable worlds,
That through eternal orbits roul beneath.
But thou all hail immortalized ſon
Of harmony, all hail thou Thracian bard,
To whom Apollo gave his tuneful lyre.
O might'ſt thou, Orpheus, now again revive,
And Newton ſhould inform thy liſt'ning ear
How the ſoft notes, and ſoul-inchanting ſtrains
Of thy own lyre were on the wind convey'd.
He taught the muſe, how ſound progreſſive floats
Upon the waving particles of air,
When harmony in ever-pleaſing ſtrains,
Melodious melting at each lulling fall,
With ſoft alluring penetration ſteals
Through the enraptur'd ear to inmoſt thought,
And folds the ſenſes in its ſilken bands.
So the ſweet muſick, which from Orpheus' touch
And fam'd Amphion's, on the ſounding ſtring
Aroſe harmonious, gliding on the air,
Pierc'd the tough-bark'd and knotty-ribbed woods,
Into their ſaps ſoft inſpiration breath'd
And taught attention to the ſtubborn oak.
Thus when great Henry, and brave Marlb'rough led

Th'

A Poem on Sir Isaac Newton.

Th' imbattled numbers of Britannia's fons,
The trump, that fwells th' expanded cheek of fame,
That adds new vigour to the gen'rous youth,
And rouzes fluggifh cowardize it felf,
The trumpet with its Mars-inciting voice,
The winds broad breaft impetuous fweeping o'er
Fill'd the big note of war.   Th' infpired hoft
With new-born ardor prefs the trembling Gaul ;
Nor greater throngs had reach'd eternal night,
Not if the fields of Agencourt had yawn'd
Expofing horrible the gulf of fate ;
Or roaring Danube fpread his arms abroad,
And overwhelm'd their legions with his floods.
But let the wand'ring mufe at length return ;
Nor yet, angelick genius of the fun,
In worthy lays her high-attempting fong
Has blazon'd forth thy venerated name.
Then let her fweep the loud-refounding lyre
Again, again o'er each melodious ftring
Teach harmony to tremble with thy praife.
And ftill thine ear O favourable grant,
And fhe fhall tell thee, that whatever charms,
Whatever beauties bloom on nature's face,
Proceed from thy all-influencing light.
That when arifing with tempeftuous rage,
The North impetuous rides upon the clouds
Difperfing round the heav'ns obftructive gloom,
And with his dreaded prohibition ftays
The kind effufion of thy genial beams ;
Pale are the rubies on Aurora's lips,
No more the rofes blufh upon her cheeks,
Black are Peneus' ftreams and golden fands
In Tempe's vale dull melancholy fits,
And every flower reclines its languid head.

By

A Poem on Sir Isaac Newton.

By what high name ſhall I invoke thee, ſay,
Thou life-infuſing deity, on thee
I call, and look propitious from on high,
While now to thee I offer up my prayer.
O had great NEWTON, as he found the cauſe,
By which ſound rouls thro' th' undulating air,
O had he, baffling times reſiſtleſs power,
Diſcover'd what that ſubtle ſpirit is,
Or whatſoe'er diffuſive elſe is ſpread
Over the wide-extended univerſe,
Which cauſes bodies to reflect the light,
And from their ſtraight direction to divert
The rapid beams, that through their ſurface pierce.
But ſince embrac'd by th' icy arms of age,
And his quick thought by times cold hand congeal'd,
Ev'n NEWTON left unknown this hidden power;
Thou from the race of human kind ſelect
Some other worthy of an angel's care,
With inſpiration animate his breaſt,
And him inſtruct in theſe thy ſecret laws.
O let not NEWTON, to whoſe ſpacious view,
Now unobſtructed, all th' extenſive ſcenes
Of the ethereal ruler's works ariſe;
When he beholds this earth he late adorn'd,
Let him not ſee philoſophy in tears,
Like a fond mother ſolitary ſit,
Lamenting him her dear, and only child.
But as the wiſe PYTHAGORAS, and he,
Whoſe birth with pride the fam'd Abdera boaſts,
With expectation having long ſurvey'd
This ſpot their antient ſeat, with joy beheld
Divine philoſophy at length appear
In all her charms majeſtically fair,
Conducted by immortal NEWTON's hand:

So

A Poem on Sir Isaac Newton.

So may he fee another fage arife,
That fhall maintain her empire : then no more
Imperious ignorance with haughty fway
Shall ftalk rapacious o'er the ravag'd globe :
Then thou, O Newton, fhalt protect thefe lines,
The humble tribute of the grateful mufe ;
Ne'er fhall the facrilegious hand defpoil
Her laurel'd temples, whom his name preferves :
And were fhe equal to the mighty theme,
Futurity fhould wonder at her fong ;
Time fhould receive her with extended arms,
Seat her confpicuous in his rouling carr,
And bear her down to his extreameft bound.

Fables with wonder tell how Terra's fons
With iron force unloos'd the ftubborn nerves
Of hills, and on the cloud-infhrouded top
Of Pelion Offa pil'd.   But if the vaft
Gigantick deeds of favage ftrength demand
Aftonifhment from men, what then fhalt thou,
O what expreffive rapture of the foul,
When thou before us, Newton, doft difplay
The labours of thy great excelling mind ;
When thou unveileft all the wondrous fcene,
The vaft idea of th' eternal king,
Not dreadful bearing in his angry arm
The thunder hanging o'er our trembling heads ;
But with th' effulgency of love replete,
And clad with power, which form'd th' extenfive heavens.
O happy he, whofe enterprizing hand
Unbars the golden and relucid gates
Of th' empyrean dome, where thou enthron'd
Philofophy art feated.   Thou fuftain'd
By the firm hand of everlafting truth

Defpifeft

Defpifeft all the injuries of time :
Thou never know'ft decay when all around,
Antiquity obfcures her head.   Behold
Th' Egyptian towers, the Babylonian walls,
And Thebes with all her hundred gates of brafs,
Behold them fcatter'd like the duft abroad.
Whatever now is flourifhing and proud,
Whatever fhall, muft know devouring age.
Euphrates' ftream, and feven-mouthed Nile,
And Danube, thou that from Germania's foil
To the black Euxine's far remoted fhore,
O'er the wide bounds of mighty nations fweep'ft
In thunder loud thy rapid floods along.
Ev'n you fhall feel inexorable time ;
To you the fatal day fhall come ; no more
Your torrents then fhall fhake the trembling ground,
No longer then to inundations fwol'n
Th' imperious waves the fertile paftures drench,
But fhrunk within a narrow channel glide ;
Or through the year's reiterated courfe
When time himfelf grows old, your wond'rous ftreams
Loft ev'n to memory fhall lie unknown
Beneath obfcurity, and Chaos whelm'd.
But ftill thou fun illuminateft all
The azure regions round, thou guideft ftill
The orbits of the planetary fpheres ;
The moon ftill wanders o'er her changing courfe,
And ftill, O Newton, fhall thy name furvive :
As long as nature's hand directs the world,
When ev'ry dark obftruction fhall retire,
And ev'ry fecret yield its hidden ftore,
Which thee dim-fighted age forbad to fee
Age that alone could ftay thy rifing foul.
And could mankind among the fixed ftars,

E'en

A Poem on Sir Isaac Newton.

E'en to th' extremeſt bounds of knowledge reach,
To thoſe unknown innumerable ſuns,
Whoſe light but glimmers from thoſe diſtant worlds,
Ev'n to thoſe utmoſt boundaries, thoſe bars
That ſhut the entrance of th' illumin'd ſpace
Where angels only tread the vaſt unknown,
Thou ever ſhould'ſt be ſeen immortal there:
In each new ſphere, each new-appearing ſun,
In fartheſt regions at the very verge
Of the wide univerſe ſhould'ſt thou be ſeen.
And lo, th' all-potent goddeſs N a t u r e takes
With her own hand thy great, thy juſt reward
Of immortality ; aloft in air
See ſhe diſplays, and with eternal graſp
Uprears the trophies of great N e w t o n's fame.

R. Glover.

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J. Pine Sculp:

INTRODUCTION.

THE manner, in which Sir Isaac Newton has publiſhed his philoſophical diſcoveries, occaſions them to lie very much concealed from all, who have not made the mathematics particularly their ſtudy. He once, indeed, intended to deliver, in a more familiar way, that part of his inventions, which relates to the ſyſtem of the world ; but upon farther conſideration he altered his deſign. For as the nature of thoſe diſcoveries made it impoſſible to prove them upon any other than geometrical principles ; he apprehended, that thoſe, who ſhould not fully perceive the force of his arguments, would hardly be prevailed on to exchange their former ſentiments for new opinions, ſo very different from

B   what

what were commonly received [a].   He therefore chofe rather
to explain himfelf only to mathematical readers; and declined
the attempting to inftruct fuch in any of his principles, who,
by not comprehending his method of reafoning, could not, at
the firft appearance of his difcoveries, have been perfuaded of
their truth.   But now, fince Sir I s a a c N e w t o n's doctrine
has been fully eftablifhed by the unanimous approbation of all,
who are qualified to underftand the fame; it is without doubt
to be wifhed, that the whole of his improvements in philofo-
phy might be univerfally known.   For this purpofe therefore
I drew up the following papers, to give a general notion of our
great philofopher's inventions to fuch, as are not prepared to
read his own works, and yet might defire to be informed of
the progrefs, he has made in natural knowledge; not doubting
but there were many, befides thofe, whofe turn of mind had
led them into a courfe of mathematical ftudies, that would take
great pleafure in tafting of this delightful fountain of fcience.

2. I t is a juft remark, which has been made upon the hu-
man mind, that nothing is more fuitable to it, than the con-
templation of truth; and that all men are moved with a ftrong
defire after knowledge ; efteeming it honourable to excel
therein ; and holding it, on the contrary, difgraceful to mi-
ftake, err, or be in any way deceived.   And this fentiment
is by nothing more fully illuftrated, than by the inclination
of men to gain an acquaintance with the operations of na-
ture; which difpofition to enquire after the caufes of things is

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[a] Philofoph. Nat. princ. math. L. iii. introduct.

fo general, that all men of letters, I believe, find themfelves influenced by it. Nor is it difficult to affign a reafon for this, if we confider only, that our defire after knowledge is an effect of that tafte for the fublime and the beautiful in things, which chiefly conftitutes the difference between the human life, and the life of brutes. Thefe inferior animals partake with us of the pleafures, that immediately flow from the bodily fenfes and appetites; but our minds are furnifhed with a fuperior fenfe, by which we are capable of receiving various degrees of delight, where the creatures below us perceive no difference. Hence arifes that purfuit of grace and elegance in our thoughts and actions, and in all things belonging to us, which principally creates imployment for the active mind of man. The thoughts of the human mind are too extenfive to be confined only to the providing and enjoying of what is neceffary for the fupport of our being. It is this tafte, which has given rife to poetry, oratory, and every branch of literature and fcience. From hence we feel great pleafure in conceiving ftrongly, and in apprehending clearly, even where the paffions are not concerned. Perfpicuous reafoning appears not only beautiful; but, when fet forth in its full ftrength and dignity, it partakes of the fublime, and not only pleafes, but warms and elevates the foul. This is the fource of our ftrong defire of knowledge; and the fame tafte for the fublime and the beautiful directs us to chufe particularly the productions of nature for the fubject of our contemplation: our creator having fo adapted our minds to the condition, wherein he has placed us, that all his vifible

works,

works, before we inquire into their make, ftrike us with the moft lively ideas of beauty and magnificence.

3. B U T if there be fo ftrong a paffion in contemplative minds for natural philofophy; all fuch muft certainly receive a particular pleafure in being informed of Sir Isaac Newton's difcoveries, who alone has been able to make any great advancements in the true courfe leading to natural knowledge : whereas this important fubject had before been u-fually attempted with that negligence, as cannot be re-flected on without furprize. Excepting a very few, who, by purfuing a more rational method, had gained a little true knowledge in fome particular parts of nature; the writers in this fcience had generally treated of it after fuch a manner, as if they thought, that no degree of certainty was ever to be ho-ped for. The cuftom was to frame conjectures; and if upon comparing them with things, there appeared fome kind of a-greement, though very imperfect, it was held fufficient. Yet at the fame time nothing lefs was undertaken than intire fy-ftems, and fathoming at once the greateft depths of nature ; as if the fecret caufes of natural effects, contrived and framed by infinite wifdom, could be fearched out by the flighteft endeavours of our weak underftandings. Whereas the only method, that can afford us any profpect of fuccefs in this difficult work, is to make our enquiries with the utmoft caution, and by very flow degrees. And after our moft dili-gent labour, the greateft part of nature will, no doubt, for e-ver remain beyond our reach.

<div align="right">4. THIS</div>

4. THIS neglect of the proper means to enlarge our knowledge, joined with the presumption to attempt, what was quite out of the power of our limited faculties, the Lord BACON judiciously observes to be the great obstruction to the progress of science [a]. Indeed that excellent person was the first, who expresly writ against this way of philosophizing; and he has laid open at large the absurdity of it in his admirable treatise, intitled NOVUM ORGANON SCIENTIARUM; and has there likewise described the true method, which ought to be followed.

5. THERE are, saith he, but two methods, that can be taken in the pursuit of natural knowledge. One is to make a hasty transition from our first and slight observations on things to general axioms, and then to proceed upon those axioms, as certain and uncontestable principles, without farther examination. The other method; (which he observes to be the only true one, but to his time unattempted;) is to proceed cautiously, to advance step by step, reserving the most general principles for the last result of our inquiries [b]. Concerning the first of these two methods; where objections, which happen to appear against any such axioms taken up in haste, are evaded by some frivolous distinction, when the axiom it self ought rather to be corrected [c]; he affirms, that the united endeavours of all ages cannot make it successful; because this original error in the first digestion of the mind (as he expresses himself) cannot afterwards be remedied [d]: whereby he would signify to us, that if we set out in a

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[a] Nov. Org. Scient. L. i. Aphorism. 9.
[b] Nov. Org. L. 1. Aph. 19.
[c] Ibid. Aph. 25.

[d] Aph. 30. Errores radicales & in prima digestione mentis ab excellentia functionum & remediorum sequentium non curantur.

wrong

wrong way ; no diligence or art, we can use, while we follow so erroneous a course, will ever bring us to our defigned end.   And doubtless it cannot prove otherwise; for in this spacious field of nature, if once we forsake the true path, we shall immediately lose our selves, and must for ever wander with uncertainty.

6. The impossibility of succeeding in so faulty a method of philosophizing his Lordship endeavours to prove from the many false notions and prejudices, to which the mind of man is exposed [a].   And since this judicious writer apprehends, that men are so exceeding liable to fall into these wrong tracts of thinking, as to incur great danger of being misled by them, even while they enter on the true course in pursuit of nature [b] ; I trust, I shall be excused, if, by insisting a little particularly upon this argument, I endeavour to remove whatever prejudice of this kind, might possibly entangle the mind of any of my readers.

7. His Lordship has reduced these prejudices and false modes of conception under four distinct heads [c].

8. The first head contains such, as we are subject to from the very condition of humanity, through the weakness both of our senses, and of the faculties of the mind [d]; seeing, as this author well observes, the subtilty of nature far exceeds the greatest subtilty of our senses or acutest reasonings [e].   One

[a] Aph. 38.
[b] Ibid.
[c] Aph. 39.

[d] Aph. 41.
[e] Aph. 10, 24.

of

of the falſe modes of conception, which he mentions un-
der this head, is the forming to our ſelves a fanciful ſim-
plicity and regularity in natural things. This he illuſtrates
by the following inſtances; the conceiving the planets to
move in perfect circles; the adding an orb of fire to the o-
ther three elements, and the ſuppoſing each of theſe to ex-
ceed the other in rarity, juſt in a decuple proportion [a].
And of the ſame nature is the aſſertion of DES CARTES,
without any proof, that all things are made up of three
kinds of matter only [b]. As alſo this opinion of another
philoſopher; that light, in paſſing through different me-
diums, was refracted, ſo as to proceed by that way, through
which it would move more ſpeedily, than through any o-
ther [c]. The ſecond erroneous turn of mind, taken notice of
by his Lordſhip under this head, is, that all men are in ſome
degree prone to a fondneſs for any notions, which they have
once imbibed; whereby they often wreſt things to reconcile
them to thoſe notions, and neglect the conſideration of what-
ever will not be brought to an agreement with them; juſt as
thoſe do, who are addicted to judicial aſtrology, to the obſer-
vation of dreams, and to ſuch-like ſuperſtitions; who care-
fully preſerve the memory of every incident, which ſerves to
confirm their prejudices, and let ſlip out of their minds all in-
ſtances, that make againſt them [d]. There is alſo a farther impe-
diment to true knowledge, mentioned under the ſame head by
this noble writer, which is; that whereas, through the weak-
neſs and imperfection of our ſenſes, many things are concealed:

[a] Aph. 45.
[b] Des Cartes Princ. Phil. Part. 3. §. 52.
[c] Fermat, in Oper. pag. 156, &c.
[d] Nov. Org. Aph. 46.

from

from us, which have the greateft effect in producing natural appearances ; our minds are ordinarily moft affected by that, which makes the ftrongeft impreffion on our organs of fenfe; whereby we are apt to judge of the real importance of things in nature by a wrong meafure [a]. So, becaufe the figuration and the motion of bodies ftrike our fenfes more immediately than moft of their other properties, Des Cartes and his followers will not allow any other explication of natural appearances, than from the figure and motion of the parts of matter. By which example we fee how juftly his Lordfhip obferves this caufe of error to be the greateft of any [b] ; fince it has given rife to a fundamental principle in a fyftem of philofophy, that not long ago obtained almoft an univerfal reputation.

9. These are the chief branches of thofe obftructions to knowledge, which this author has reduced under his firft head of falfe conceptions. The fecond head contains the errors, to which particular perfons are more efpecially obnoxious [c]. One of thefe is the confequence of a preceding obfervation: that as we are expofed to be captivated by any opinions, which have once taken poffeffion of our minds; fo in particular, natural knowledge has been much corrupted by the ftrong attachment of men to fome one part of fcience, of which they reputed themfelves the inventers, or about which they have fpent much of their time; and hence have been apt to conceive it to be of greater ufe in the ftudy of na-

[a] Aph. 50.
[b] Ibid.

[c] Aph. 53.

tural

tural philofophy than it was : like ARISTOTLE, who redu-
ced his phyfics to logical difputations; and the chymifts, who
thought, that nature could be laid open only by the force
of their fires [a].   Some again are wholly carried away by an
exceffive veneration for antiquity; others, by too great fond-
nefs for the moderns; few having their minds fo well balanced,
as neither to depreciate the merit of the ancients, nor yet to
defpife the real improvements of later times [b].   To this is
added by his Lordfhip a difference in the genius of men,
that fome are moft fitted to obferve the fimilitude, there is in
things, while others are more qualified to difcern the par-
ticulars, wherein they difagree; both which difpofitions of
mind are ufeful: but to the prejudice of philofophy men are
apt to run into excefs in each; while one fort of genius dwells
too much upon the grofs and fum of things, and the other
upon trifling minuteneffes and fhadowy diftinctions [c].

10. UNDER the third head of prejudices and falfe notions
this writer confiders fuch, as follow from the lax and indefi-
nite ufe of words in ordinary difcourfe; which occafions great
ambiguities and uncertainties in philofophical debates (as ano-
ther eminent philofopher has fince fhewn more at large [d];) in-
fomuch that this our author thinks a ftrict defining of terms to
be fcarce an infallible remedy againft this inconvenience [e]. And
perhaps he has no fmall reafon on his fide: for the common
inaccurate fenfe of words, notwithftanding the limitations
given them by definitions, will offer it felf fo conftantly to

[a] Aph. 54.
[b] Aph. 56.
[c] Aph. 55.

[d] Locke. On human underftanding. B. iii.
[e] Nov. Org. Aph. 59.

C

the mind, as to require great caution and circumſpection for us not to be deceived thereby. Of this we have a very eminent inſtance in the great diſputes, that have been raiſed about the uſe of the word attraction in philoſophy; of which we ſhall be obliged hereafter to make particular mention [a]. Words thus to be guarded againſt are of two kinds. Some are names of things, that are only imaginary [b]; ſuch words are wholly to be rejected. But there are other terms, that allude to what is real, though their ſignification is confuſed [c]. And theſe latter muſt of neceſſity be continued in uſe; but their ſenſe cleared up, and freed, as much as poſſible, from obſcurity.

11. The laſt general head of theſe errors comprehends ſuch, as follow from the various ſects of falſe philoſophies; which this author divides into three ſorts, the ſophiſtical, empirical, and ſuperſtitious [d]. By the firſt of theſe he means a philoſophy built upon ſpeculations only without experiments [e]; by the ſecond, where experiments are blindly adhered to, without proper reaſoning upon them [f]; and by the third, wrong opinions of nature fixed in mens minds either through falſe religions, or from miſunderſtanding the declarations of the true [g].

12. Theſe are the four principal canals, by which this judicious author thinks, that philoſophical errors have flowed in upon us. And he rightly obſerves, that the faulty method of

[a] In the concluſion.
[b] Nov. Org. L. i. Aph. 59.
[c] Ibid. Aph. 60.
[d] Ibid. Aph. 62.
[e] Aph. 63.
[f] Aph. 64.
[g] Aph. 65.

proceeding

proceeding in philofophy, againft which he writes [a], is fo far from affifting us towards overcoming thefe prejudices ; that he apprehends it rather fuited to rivet them more firmly to the mind [b]. How great reafon then has his Lordfhip to call this way of philofophizing the parent of error, and the bane of all knowledge [c]? For, indeed, what elfe but miftakes can fo bold and prefumptuous a treatment of nature produce? have we the wifdom neceffary to frame a world, that we fhould think fo eafily, and with fo flight a fearch to enter into the moft fecret fprings of nature, and difcover the original caufes of things ? what chimeras, what monfters has not this prepofte-rous method brought forth? what fchemes, or what hypothe-fis's of the fubtileft wits has not a ftricter enquiry into nature not only overthrown, but manifefted to be ridiculous and abfurd? Every new improvement, which we make in this fcience, lets us fee more and more the weaknefs of our gueffes. Dr. HARVEY, by that one difcovery of the circulation of the blood, has diffipated all the fpeculations and reafonings of many ages up-on the animal oeconomy. ASELLIUS, by detecting the la-cteal veins, fhewed how little ground all phyficians and phi-lofophers had in conjecturing, that the nutritive part of the aliment was abforbed by the mouths of the veins fpread upon the bowels : and then PECQUET, by finding out the thora-cic duct, as evidently proved the vanity of the opinion, which was perfifted in after the lacteal veffels were known, that the alimental juice was conveyed immediately to the liver, and there converted into blood.

[a] See above, § 4, 5.
[b] Nov. Org. L. i. Aph. 69.

[c] Ibid.

13. As

13. As these things set forth the great absurdity of proceeding in philosophy on conjectures, by informing us how far the operations of nature are above our low conceptions; so on the other hand, such instances of success from a more judicious method shew us, that our bountiful maker has not left us wholly without means of delighting our selves in the contemplation of his wisdom. That by a just way of inquiry into nature, we could not fail of arriving at discoveries very remote from our apprehensions; the Lord Bacon himself argues from the experience of mankind. If, says he, the force of guns should be described to any one ignorant of them, by their effects only; he might reasonably suppose, that those engines of destruction were only a more artificial composition, than he knew, of wheels and other mechanical powers: but it could never enter his thoughts, that their immense force should be owing to a peculiar substance, which would enkindle into so violent an explosion, as we experience in gunpowder: since he would no where see the least example of any such operation; except perhaps in earthquakes and thunder, which he would doubtless look upon as exalted powers of nature, greatly surpassing any art of man to imitate. In the same manner, if a stranger to the original of silk were shewn a garment made of it, he would be very far from imagining so strong a substance to be spun out of the bowels of a small worm; but must certainly believe it either a vegetable substance, like flax or cotton; or the natural covering of some animal, as wool is of sheep. Or had we been told, before the invention of the magnetic needle among us, that another people was in possession of a certain

<div align="right">contrivance</div>

contrivance, by which they were inabled to difcover the pofition of the heavens, with vaftly more eafe, than we could do; what could have been imagined more, than that they were provided with fome fitter aftronomical inftrument for this purpofe than we? That any ftone fhould have fo amazing a property, as we find in the magnet, muft have been. the remoteft from our thoughts [a].

14. But what furprizing advancements in the knowledge of nature may be made by purfuing the true courfe in philofophical inquiries; when thofe fearches are conducted by a genius equal to fo divine a work, will be beft underftood by confidering Sir Isaac Newton's difcoveries. That my reader may apprehend as juft a notion of thefe, as can be conveyed to him, by the brief account, which I intend to lay before him; I have fet apart this introduction for explaining, in the fulleft manner I am able, the principles, whereon Sir Isaac Newton proceeds. For without a clear conception of thefe, it is impoffible to form any true idea of the fingular excellence of the inventions of this great philofopher.

15. The principles then of this philofophy are; upon no confideration to indulge conjectures concerning the powers and laws of nature, but to make it our endeavour with all diligence to fearch out the real and true laws, by which the conftitution of things is regulated. The philofopher's firft care muft be to diftinguifh, what he fees to be within his power, from what

[a] Ibid. Aph. 109.

is beyond his reach ; to affume no greater degree of knowledge, than what he finds himfelf poffeffed of ; but to advance by flow and cautious fteps ; to fearch gradually into natural caufes ; to fecure to himfelf the knowledge of the moft immediate caufe of each appearance, before he extends his views farther to caufes more remote. This is the method, in which philofophy ought to be cultivated ; which does not pretend to fo great things, as the more airy fpeculations ; but will perform abundantly more : we fhall not perhaps feem to the unskilful to know fo much, but our real knowledge will be greater. And certainly it is no objection againft this method, that fome others promife, what is nearer to the extent of our wifhes : fince this, if it will not teach us all we could defire to be informed of, will however give us fome true light into nature ; which no other can do. Nor has the philofopher any reafon to think his labour loft, when he finds himfelf ftopt at the caufe firft difcovered by him, or at any other more remote caufe, fhort of the original : for if he has but fufficiently proved any one caufe, he has entered fo far into the real conftitution of things, has laid a fafe foundation for others to work upon, and has facilitated their endeavours in the fearch after yet more diftant caufes ; and befides, in the mean time he may apply the knowledge of thefe intermediate caufes to many ufeful purpofes. Indeed the being able to make practical deductions from natural caufes, conftitutes the great diftinction between the true philofophy and the falfe. Caufes affumed upon conjecture, muft be fo loofe and undefined, that nothing particular can be collected from them. But thofe caufes, which are brought to light by a ftrict examination

of

of things, will be more diftinct. Hence it appears to have been no unufeful difcovery, that the afcent of water in pumps is owing to the preffure of the air by its weight or fpring; though the caufes, which make the air gravitate, and render it elaftic, be unknown : for notwithftanding we are ignorant of the original, whence thefe powers of the air are derived ; yet we may receive much advantage from the bare knowledge of thefe powers. If we are but certain of the degree of force, wherewith they act, we fhall know the extent of what is to be expected from them; we fhall know the greateft height, to which it is poffible by pumps to raife water; and fhall thereby be prevented from making any ufelefs efforts towards improving thefe inftruments beyond the limits prefcribed to them by nature ; whereas without fo much knowledge as this, we might probably have wafted in attempts of this kind much time and labour. How long did philofophers bufy themfelves to no purpofe in endeavouring to perfect telefcopes, by forming the glaffes into fome new figure ; till Sir Isaac Newton demonftrated, that the effects of telefcopes were limited from another caufe, than was fuppofed; which no alteration in the figure of the glaffes could remedy ? What method Sir Isaac Newton himfelf has found for the improvement of telefcopes fhall be explained hereafter [a]. But at prefent I fhall proceed to illuftrate, by fome farther inftances, this diftinguifhing character of the true philofophy, which we have now under confideration. It was no trifling difcovery, that the contraction of the mufcles of animals puts their limbs in motion, though the original caufe of that contraction

[a] Book III. Chap. iv.

remains

·remains a fecret, and perhaps may always do fo; for the knowledge of thus much only has given rife to many fpeculations upon the force and artificial difpofition of the mufcles, and has opened no narrow profpect into the animal fabrick. The finding out, that the nerves are great agents in this action, leads us yet nearer to the original caufe, and yields us a wider view of the fubject. And each of thefe fteps affords us affiftance towards reftoring this animal motion, when impaired in our felves, by pointing out the feats of the injuries, to which it is obnoxious. To neglect all this, becaufe we can hitherto advance no farther, is plainly ridiculous. It is confeffed by all, that G A L I L E O greatly improved philofophy, by fhewing, as we fhall relate hereafter, that the power in bodies, which we call gravity, occafions them to move downwards with a velocity equably accelerated[a]; and that when any body is thrown forwards, the fame power obliges it to defcribe in its motion that line, which is called by geometers a parabola[b]: yet we are ignorant of the caufe, which makes bodies gravitate. But although we are unacquainted with the fpring, whence this power in nature is derived, neverthelefs we can eftimate its effects. When a body falls perpendicularly, it is known, how long time it takes in defcending from any height whatever: and if it be thrown forwards, we know the real path, which it defcribes; we can determine in what direction, and with what degree of fwiftnefs it muft be projected, in order to its ftriking againft any object defired; and we can alfo afcertain the very force, wherewith it will ftrike.

[a] Book I. Chap. 2. § 14.   [b] Ibid. § 85, &c.

Sir

Sir Isaac Newton has farther taught, that this power of gravitation extends up to the moon, and caufes that planet to gravitate as much towards the earth, as any of the bodies, which are familiar to us, would, if placed at the fame diftance [a] : he has proved likewife, that all the planets gravitate towards the fun, and towards one another ; and that their refpective motions follow from this gravitation. All this he has demon-ftrated upon indifputable geometrical principles, which cannot be rendered precarious for want of knowing what it is, which caufes thefe bodies thus mutually to gravitate: any more than we can doubt of the propenfity in all the bodies about us, to defcend towards the earth; or can call in queftion the fore-mentioned propofitions of Galileo, which are built upon that principle. And as Galileo has fhewn more fully, than was known before, what effects were produced in the motion of bodies by their gravitation towards the earth ; fo Sir Isaac Newton, by this his invention, has much advan-ced our knowledge in the celeftial motions. By difcovering that the moon gravitates towards the fun, as well as towards the earth; he has laid open thofe intricacies in the moon's motion, which no aftronomer, from obfervations only, could ever find out [b]: and one kind of heavenly bodies, the comets, have their motion now clearly afcertained ; whereof we had before no true knowledge at all [c].

16. Doubtless it might be expected, that fuch furprizing fuccefs fhould have filenced, at once, every cavil. But we

[a] See Book II. Ch. 3. § 3, 4. of this treatife.
[b] See Book II. Ch. 3. of this treatife.
[c] See Chap. 4.

have

have feen the contrary. For becaufe this philofophy profeffes
modeftly to keep within the extent of our faculties, and is
ready to confefs its imperfections, rather than to make any
fruitlefs attempts to conceal them, by feeking to cover the de-
fects in our knowledge with the vain oftentation of rafh and
groundlefs conjectures ; hence has been taken an occafion to
infinuate that we are led to miraculous caufes, and the occult
qualities of the fchools.

17. But the firft of thefe accufations is very extraordina-
ry. If by calling thefe caufes miraculous nothing more is
meant than only, that they often appear to us wonderful and
furprizing, it is not eafy to fee what difficulty can be raifed
from thence ; for the works of nature difcover every where
fuch proofs of the unbounded power, and the confummate
wifdom of their author, that the more they are known, the
more they will excite our admiration: and it is too manifeft
to be infifted on, that the common fenfe of the word mira-
culous can have no place here, when it implies what is above
the ordinary courfe of things.   The other imputation, that
thefe caufes are occult upon the account of our not perceiving
what produces them, contains in it great ambiguity.   That
fomething relating to them lies hid, the followers of this
philofophy are ready to acknowledge, nay defire it fhould
be carefully remarked, as pointing out proper fubjects for fu-
ture inquiry.   But this is very different from the proceeding
of the fchoolmen in the caufes called by them occult.   For
as their occult qualities were underftood to operate in a man-
ner occult, and not apprehended by us ; fo they were ob-
truded.

truded upon us for such original and essential properties in bodies, as made it vain to seek any farther cause; and a greater power was attributed to them, than any natural appearances authorized. For instance, the rise of water in pumps was ascribed to a certain abhorrence of a vacuum, which they thought fit to assign to nature. And this was so far a true observation, that the water does move, contrary to its usual course, into the space, which otherwise would be left void of any sensible matter; and, that the procuring such a vacuity was the apparent cause of the water's ascent. But while we were not in the least informed how this power, called an abhorrence of a vacuum, produced the visible effects; instead of making any advancement in the knowledge of nature, we only gave an artificial name to one of her operations: and when the speculation was pushed so beyond what any appearances required, as to have it concluded, that this abhorrence of a vacuum was a power inherent in all matter, and so unlimited as to render it impossible for a vacuum to exist at all; it then became a much greater absurdity, in being made the foundation of a most ridiculous manner of reasoning; as at length evidently appeared, when it came to be discovered, that this rise of the water followed only from the pressure of the air, and extended it self no farther, than the power of that cause. The scholastic stile in discoursing of these occult qualities, as if they were essential differences in the very substances, of which bodies consisted, was certainly very absurd; by reason it tended to discourage all farther inquiry. But no such ill consequences can follow from the considering of any natural causes, which confessedly are not traced up to

their

their firft original. How fhall we ever come to the knowledge of the feveral original caufes of things, otherwife than by ftoring up all intermediate caufes which we can difcover ? Are all the original and effential properties of matter fo very obvious, that none of them can efcape our firft view? This is not probable. It is much more likely, that, if fome of the effential properties are difcovered by our firft obfervations, a ftricter examination fhould bring more to light.

18. But in order to clear up this point concerning the effential properties of matter, let us confider the fubject a little diftinctly. We are to conceive, that the matter, out of which the univerfe of things is formed, is furnifhed with certain qualities and powers, whereby it is rendered fit to anfwer the purpofes, for which it was created. But every property, of which any particle of this matter is in it felf poffeffed, and which is not barely the confequence of the union of this particle with other portions of matter, we may call an effential property : whereas all other qualities or attributes belonging to bodies, which depend on their particular frame and compofition, are not effential to the matter, whereof fuch bodies are made ; becaufe the matter of thefe bodies will be deprived of thofe qualities, only by the diffolution of the body, without working any change in the original conftitution of one fingle particle of this mafs of matter. Extenfion we apprehend to be one of thefe effential properties, and impenetrability another. Thefe two belong univerfally to all matter; and are the principal ingredients in the idea, which this word matter ufually excites in the mind. Yet as the idea, marked

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by this name, is not purely the creature of our own understandings, but is taken for the reprefentation of a certain fubftance without us; if we fhould difcover, that every part of the fubftance, in which we find thefe two properties, fhould likewife be endowed univerfally with any other effential qualities; all thefe, from the time they come to our notice, muft be united under our general idea of matter. How many fuch properties there are actually in all matter we know not; thofe, of which we are at prefent apprized, have been found out only by our obfervations on things; how many more a farther fearch may bring to light, no one can fay; nor are we certain, that we are provided with fufficient methods of perception to difcern them all. Therefore, fince we have no other way of making difcoveries in nature, but by gradual inquiries into the properties of bodies; our firft ftep muft be to admit without diftinction all the properties, which we obferve; and afterwards we muft endeavour, as far as we are able, to diftinguifh between the qualities, wherewith the very fubftances themfelves are indued, and thofe appearances, which refult from the ftructure only of compound bodies. Some of the properties, which we obferve in things, are the attributes of particular bodies only; others univerfally belong to all, that fall under our notice. Whether fome of the qualities and powers of particular bodies, be derived from different kinds of matter entring their compofition, cannot, in the prefent imperfect ftate of our knowledge, abfolutely be decided; though we have not yet any reafon to conclude, but that all the bodies, with which we converfe, are framed out of the very fame kind of matter, and that their diftinct

<div align="right">quali-</div>

qualities are occasioned only by their structure; through the variety whereof the general powers of matter are caused to produce different effects. On the other hand, we should not hastily conclude, that whatever is found to appertain to all matter, which falls under our examination, must for that reason only be an essential property thereof, and not be derived from some unseen disposition in the frame of nature. Sir I s a a c N e w t o n has found reason to conclude, that gravity is a property universally belonging to all the perceptible bodies in the universe, and to every particle of matter, whereof they are composed. But yet he no where asserts this property to be essential to matter. And he was so far from having any design of establishing it as such, that, on the contrary, he has given some hints worthy of himself at a cause for it [a]; and expresly says, that he proposed those hints to shew, that he had no such intention [b].

19. I t appears from hence, that it is not easy to determine, what properties of bodies are essentially inherent in the matter, out of which they are made, and what depend upon their frame and composition. But certainly whatever properties are found to belong either to any particular systems of matter, or universally to all, must be considered in philosophy; because philosophy will be otherwise imperfect. Whether those properties can be deduced from some other appertaining to matter, either among those, which are already known, or among such as can be discovered by us, is afterwards to be sought for the farther improvement of our knowledge. But this

[a] At the end of his Optics. in Qu. 21.   [b] See the same treatise, in Advertisement 2.

inquiry

inquiry cannot properly have place in the deliberation about admitting any property of matter or bodies into philofophy; for that purpofe it is only to be confidered, whether the exiftence of fuch a property has been juftly proved or not. Therefore to decide what caufes of things are rightly received into natural philofophy, requires only a diftinct and clear conception of what kind of reafoning is to be allowed of as convincing, when we argue upon the works of nature.

20. THE proofs in natural philofophy cannot be fo abfolutely conclufive, as in the mathematics. For the fubjects of that fcience are purely the ideas of our own minds. They may be reprefented to our fenfes by material objects, but they are themfelves the arbitrary productions of our own thoughts; fo that as the mind can have a full and adequate knowledge of its own ideas, the reafoning in geometry can be rendered perfect. But in natural knowledge the fubject of our contemplation is without us, and not fo compleatly to be known: therefore our method of arguing muft fall a little fhort of abfolute perfection. It is only here required to fteer a juft courfe between the conjectural method of proceeding, againft which I have fo largely fpoke; and demanding fo rigorous a proof, as will reduce all philofophy to mere fcepticifm, and exclude all profpect of making any progrefs in the knowledge of nature.

21. THE conceffions, which are to be allowed in this fcience, are by Sir ISAAC NEWTON included under a very few fimple precepts.

<div align="right">22. THE.</div>

22. T H E firſt is, that more cauſes are not to be received into philoſophy, than are ſufficient to explain the appearances of nature. That this rule is approved of unanimouſly, is e-vident from thoſe expreſſions ſo frequent among all philoſo-phers, that nature does nothing in vain; and that a varie-ty of means, where fewer would ſuffice, is needleſs. And certainly there is the higheſt reaſon for complying with this rule. For ſhould we indulge the liberty of multiplying, without neceſſity, the cauſes of things, it would reduce all philoſophy to mere uncertainty; ſince the only proof, which we can have, of the exiſtence of a cauſe, is the ne-ceſſity of it for producing known effects. Therefore where one cauſe is ſufficient, if there really ſhould in nature be two, which is in the laſt degree improbable, we can have no poſſible means of knowing it, and conſequently ought not to take the liberty of imagining, that there are more than one.

23. T H E ſecond precept is the direct conſequence of the firſt, that to like effects are to be aſcribed the ſame cauſes. For inſtance, that reſpiration in men and in brutes is brought about by the ſame means; that bodies deſcend to the earth here in E U R O P E, and in A M E R I C A from the ſame principle; that the light of a culinary fire, and of the ſun have the ſame manner of production; that the reflection of light is effected in the earth, and in the planets by the ſame power; and the like.

24. T H E third of theſe precepts has equally evident rea-ſon for it. It is only, that thoſe qualities, which in the ſame body can neither be leſſened nor increaſed, and which belong

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to all bodies that are in our power to make trial upon, ought to be accounted the univerſal properties of all bodies what-ever.

25. In this precept is founded that method of arguing by induction, without which no progreſs could be made in na-tural philoſophy. For as the qualities of bodies become known to us by experiments only ; we have no other way of finding the properties of ſuch bodies, as are out of our reach to experiment upon, but by drawing concluſions from thoſe which fall under our examination. The only caution here required is, that the obſervations and experiments, we argue upon, be numerous enough, and that due regard be paid to all objections, that occur, as the Lord Bacon very judi-ciouſly directs[a]. And this admonition is ſufficiently com-plied with, when by virtue of this rule we aſcribe impene-trability and extenſion to all bodies, though we have no ſen-ſible experiment, that affords a direct proof of any of the ce-leſtial bodies being impenetrable ; nor that the fixed ſtars are ſo much as extended. For the more perfect our inſtru-ments are, whereby we attempt to find their viſible magni-tude, the leſs they appear ; inſomuch that all the ſenſible magnitude, which we obſerve in them, ſeems only to be an optical deception by the ſcattering of their light. However, I ſuppoſe no one will imagine they are without any magni-tude, though their immenſe diſtance makes it undiſcernable by us. After the ſame manner, if it can be proved, that all

[a] Nov. Org. Lib. i. Ax. 105.

E   bodies

bodies here gravitate towards the earth, in proportion to the quantity of folid matter in each; and that the moon gravitates to the earth likewife, in proportion to the quantity of matter in it; and that the fea gravitates towards the moon, and all the planets towards each other; and that the very comets have the fame gravitating faculty; we fhall have as great reafon to conclude by this rule, that all bodies gravitate towards each other. For indeed this rule will more ftrongly hold in this cafe, than in that of the impenetrability of bodies; becaufe there will more inftances be had of bodies gravitating, than of their being impenetrable.

25. THIS is that method of induction, whereon all phi-lofophy is founded; which our author farther inforces by this additional precept, that whatever is collected from this induction, ought to be received, notwithftanding any conje-ctural hypothefis to the contrary, till fuch times as it fhall be contradicted or limited by farther obfervations on nature.

J. Grifon del   J. Pine Sculp.
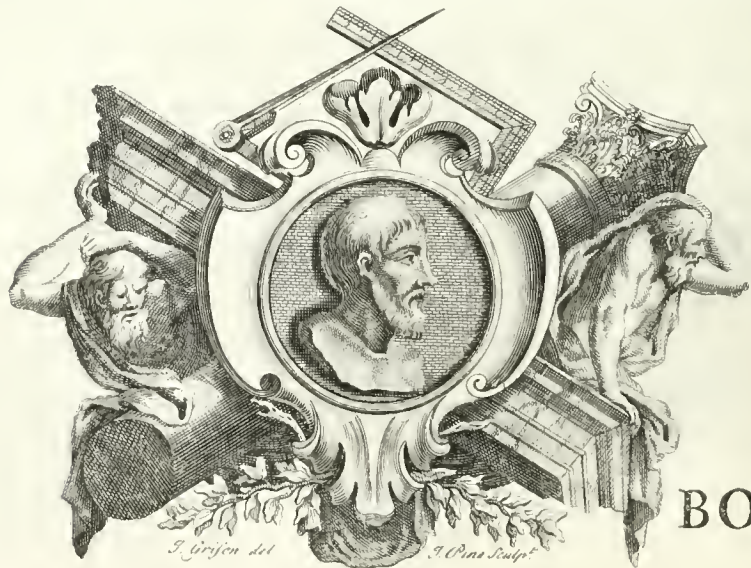
BOOK

BOOK I.

CONCERNING THE

MOTION of BODIES

IN GENERAL.

CHAP. I.
Of the LAWS of MOTION.

AVING thus explained Sir ISAAC NEWTON's method of reasoning in philosophy, I shall now proceed to my intended account of his discoveries. These are contained in two treatises. In one of them, the MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY, his chief design is to shew by what laws the heavenly

E 2   motions

motions are regulated; in the other, his Optics, he discourses
of the nature of light and colours, and of the action between
light and bodies.   This second treatise is wholly confined to
the subject of light: except some conjectures proposed at the
end concerning other parts of nature, which lie hitherto more
concealed.   In the other treatise our author was obliged to
smooth the way to his principal intention, by explaining ma-
ny things of a more general nature: for even some of the most
simple properties of matter were scarce well established at that
time.   We may therefore reduce Sir Isaac Newton's do-
ctrine under three general heads; and I shall accordingly di-
vide my account into three books.   In the first I shall speak
of what he has delivered concerning the motion of bodies,
without regard to any particular system of matter; in the se-
cond I shall treat of the heavenly motions; and the third
shall be employed upon light.

2. In the first part of my design, we must begin with an
account of the general laws of motion.

3. These laws are some universal affections and proper-
ties of matter drawn from experience, which are made use
of as axioms and evident principles in all our arguings upon the
motion of bodies.   For as it is the custom of geometers to
assume in their demonstrations some propositions, without
exhibiting the proof of them; so in philosophy, all our rea-
soning must be built upon some properties of matter, first ad-
mitted as principles whereon to argue.   In geometry these ax-
ioms are thus assumed, on account of their being so evident

as

as to make any proof in form needless.   But in philofophy no properties of bodies can be in this manner received for felf-evident; fince it has been obferved above, that we can conclude nothing concerning matter by any reafonings upon its nature and effence, but that we owe all the knowledge, we have thereof, to experience.   Yet when our obfervations on matter have inform'd us of fome of its properties, we may fecurely reafon upon them in our farther inquiries into nature. And thefe laws of motion, of which I am here to fpeak, are found fo univerfally to belong to bodies, that there is no motion known, which is not regulated by them.   Thefe are by Sir Isaac Newton reduced to three [a].

4. The firft law is, that all bodies have fuch an indifference to reft, or motion, that if once at reft they remain fo, till difturbed by fome power acting upon them :   but if once put in motion, they perfift in it ; continuing to move right forwards perpetually, after the power, which gave the motion, is removed ;   and alfo preferving the fame degree of velocity or quicknefs, as was firft communicated, not ftopping or remitting their courfe, till interrupted or otherwife difturbed by fome new power impreffed.

5. The fecond law of motion is, that the alteration of the ftate of any body, whether from reft to motion, or from motion to reft, or from one degree of motion to another, is always proportional to the force impreffed. A body at reft, when

[a] Princip. philof. pag. 13, 14.

acted

acted upon by any power, yields to that power, moving in
the same line, in which the power applied is directed ; and
moves with a lefs or greater degree of velocity, according to
the degree of the power ; fo that twice the power fhall com-
municate a double velocity, and three times the power a
threefold velocity. If the body be moving, and the power
impreffed act upon the body in the direction of its motion,
the body fhall receive an addition to its motion, as great as
the motion, into which that power would have put it from a
ftate of reft; but if the power impreffed upon a moving bo-
dy act directly oppofite to its former motion, that power fhall
then take away from the body's motion, as much as in the o-
ther cafe it would have added to it. Laftly, if the power be
impreffed obliquely, there will arife an oblique motion dif-
fering more or lefs from the former direction, according as
the new impreffion is greater or lefs. For example, if the bo-
dy A (in fig. 1.) be moving in the direction A B, and when it is
at the point A, a power be impreffed upon it in the direction
A C, the body fhall from henceforth neither move in its firft
direction A B, nor in the direction of the adventitious power,
but fhall take a courfe as A D between them : and if the
power laft impreffed be juft equal to that, which firft gave
to the body its motion ; the line A D fhall pafs in the middle
between A B and A C, dividing the angle under B A C into
two equal parts ; but if the power laft impreffed be greater
than the firft, the line AD fhall incline moft to AC ; whereas
if the laft impreffion be lefs than the firft, the line A D fhall
incline moft to A B. To be more particular, the fituation of
the

the line A D is always to be determined after this manner. Let A E be the fpace, which the body would have moved through in the line A B during any certain portion of time; provided that body, when at A, had received no fecond impulfe. Suppofe likewife, that A F is the part of the line A C, through which the body would have moved during an equal portion of time, if it had been at reft in A, when it received the impulfe in the direction A C: then if from E be drawn a line parallel to, or equidiftant from A C, and from F another line parallel to A B, thofe two lines will meet in the line A D.

6. The third and laft of thefe laws of motion is, that when any body acts upon another, the action of that body upon the other is equalled by the contrary reaction of that other body upon the firft.

7. These laws of motion are abundantly confirmed by this, that all the deductions made from them, in relation to the motion of bodies, how complicated foever, are found to agree perfectly with obfervation. This fhall be fhewn more at large in the next chapter. But before we proceed to fo diffufive a proof; I chufe here to point out thofe appearances of bodies, whereby the laws of motion are firft fuggefted to us.

8. Daily obfervation makes it appear to us, that any body, which we once fee at reft, never puts it felf into frefh

motion;

motion; but continues always in the fame place, till removed by fome power applied to it.

9. Again, whenever a body is once in motion, it continues in that motion fome time after the moving power has quitted it, and it is left to it felf. Now if the body continue to move but a fingle moment, after the moving power has left it, there can no reafon be affigned, why it fhould ever ftop without fome external force. For it is plain, that this continuance of the motion is caufed only by the body's having already moved, the fole operation of the power upon the body being the putting it in motion; therefore that motion continued will equally be the caufe of its farther motion, and fo on without end. The only doubt that can remain, is, whether this motion communicated continues intire, after the power, that caufed it, ceafes to act; or whether it does not gradually languifh and decreafe. And this fufpicion cannot be removed by a tranfient and flight obfervation on bodies, but will be fully cleared up by thofe more accurate proofs of the laws of motion, which are to be confidered in the next chapter.

10. Lastly, bodies in motion appear to affect a ftraight courfe without any deviation, unlefs when difturbed by fome adventitious power acting upon them. If a body be thrown perpendicularly upwards or downwards, it appears to continue in the fame ftraight line during the whole time of its motion. If a body be thrown in any other direction, it is found to deviate from the line, in which it began to move, more and
more

more continually towards the earth, whither it is directed
by its weight : but fince, when the weight of a body does
not alter the direction of its motion, it always moves in
a ftraight line, without doubt in this other cafe the body's
declining from its firft courfe is no more, than what is cau-
fed by its weight alone.   As this appears at firft fight to be
unqueftionable, fo we fhall have a very diftinct proof thereof
in the next chapter, where the oblique motion of bodies will
be particularly confidered.

11. THUS we fee how the firft of the laws of motion
agrees with what appears to us in moving bodies.   But
here occurs this farther confideration, that the real and ab-
folute motion of any body is not vifible to us : for we
are our felves alfo in conftant motion along with the
earth whereon we dwell; infomuch that we perceive bo-
dies to move fo far only, as their motion is different from
our own.   When a body appears to us to lie at reft, in
reality it only continues the motion, it has received, without
putting forth any power to change that motion.   If we
throw a body in the courfe or direction, wherein we are
carried our felves; fo much motion as we feem to have
given to the body, fo much we have truly added to the
motion, it had, while it appeared to us to be at reft. But
if we impel a body the contrary way, although the body
appears to us to have received by fuch an impulfe as much
motion, as when impelled the other way; yet in this cafe we
have taken from the body fo much real motion, as we feem
to have given it.   Thus the motion, which we fee in bodies,

F   is

is not their real motion, but only relative with refpect to us; and the forementioned obfervations only fhew us, that this firft law of motion has place in this relative or apparent motion. However, though we cannot make any obfervation immediately on the abfolute motion of bodies, yet by reafoning upon what we obferve in vifible motion, we can difcover the properties and effects of real motion.

12. With regard to this firft law of motion, which is now under confideration, we may from the foregoing obfervations moft truly collect, that bodies are difpofed to continue in the abfolute motion, which they have once received, without increafing or diminifhing their velocity. When a body appears to us to lie at reft, it really preferves without change the motion, which it has in common with our felves: and when we put it into vifible motion, and we fee it continue that motion; this proves, that the body retains that degree of its abfolute motion, into which it is put by our acting upon it: if we give it fuch an apparent motion, which adds to its real motion, it preferves that addition; and if our acting on the body takes off from its real motion, it continues afterwards to move with no more real motion, than we have left it.

13. Again, we do not obferve in bodies any difpofition or power within themfelves to change the direction of their motion; and if they had any fuch power, it would eafily be difcovered. For fuppofe a body by the ftructure or difpofition of its parts, or by any other circumftance in its make, was induced

dued with a power of moving it felf; this felf-moving prin-
ciple, which fhould be thus inherent in the body, and not
depend on any thing external, muft change the direction
wherein it would act, as often as the pofition of the body
was changed : fo that for inftance, if a body was lying be-
fore me in fuch a pofition, that the direction, wherein this
principle difpofes the body to move, was pointed directly from
me ; if I then gradually turned the body about, the direction
of this felf-moving principle would no longer be pointed di-
rectly from me, but would turn about along with the body.
Now if any body, which appears to us at reft, were furnifh-
ed with any fuch felf-moving principle ; from the body's ap-
pearing without motion we muft conclude, that this felf-mov-
ing principle lies directed the fame way as the earth is car-
rying the body ; and fuch a body might immediately be put
into vifible motion only by turning it about in any degree,
that this felf-moving principle might receive a different di-
rection.

14. From thefe confiderations it very plainly follows,
that if a body were once abfolutely at reft ; not being fur-
nifhed with any principle, whereby it could put it felf into
motion, it muft for ever continue in the fame place, till acted
upon by fomething external: and alfo that when a body is put
into motion, it has no power within it felf to make any
change in the direction of that motion ; and confequently
that the body muft move on ftraight forward without declin-
ing any way whatever.   But it has before been fhewn, that
bodies do not appear to have in themfelves any power to

F 2   change

change the velocity of their motion: therefore this firſt law of motion has been illuſtrated and confirmed, as much as can be from the tranſient obſervations, which have here been diſcourſed upon ; and in the next chapter all this will be farther eſtabliſhed by more correct obſervations.

15. But I ſhall now paſs to the ſecond law of motion ; wherein, when it is aſſerted, that the velocity, with which any body is moved by the action of a power upon it, is proportional to that power ; the degree of power is ſuppoſed to be meaſured by the greatneſs of the body, which it can move with a given celerity.   So that the ſenſe of this law is, that if any body were put into motion with that degree of ſwiftneſs, as to paſs in one hour the length of a thouſand yards ; the power, which would give the ſame degree of velocity to a body twice as great, would give this leſſer body twice the velocity, cauſing it to deſcribe in the ſame ſpace of an hour two thouſand yards.   But by a body twice as great as another, I do not here mean ſimply of twice the bulk, but one that contains a double quantity of ſolid matter.

16. Why the power, which can move a body twice as great as another with the ſame degree of velocity, ſhould be called twice as great as the power, which can give the leſſer body the ſame velocity, is evident.   For if we ſhould ſuppoſe the greater body to be divided into two equal parts, each equal to the leſſer body, each of theſe halves will require the ſame degree of power to move them with the velocity of the leſſer body, as the leſſer body it ſelf requires ; and therefore both

thoſe

thofe halves, or the whole greater body, will require the moving power to be doubled.

17. That the moving power being in this fenfe doubled, fhould juft double likewife the velocity of the fame body, feems near as evident, if we confider, that the effect of the power applied muft needs be the fame, whether that power be applied to the body at once, or in parts. Suppofe then the double power not applied to the body at once, but half of it firft, and afterwards the other half; it is not conceivable for what reafon the half laft applied fhould come to have a different effect upon the body, from that which is applied firft; as it muft have, if the velocity of the body was not juft doubled by the application of it. So far as experience can determine, we fee nothing to favour fuch a fuppofition. We cannot indeed (by reafon of the conftant motion of the earth) make trial upon any body perfectly at reft, whereby to fee whether a power applied in that cafe would have a different effect, from what it has, when the body is already moving; but we find no alteration in the effect of the fame power on account of any difference there may be in the motion of the body, when the power is applied. The earth does not always carry bodies with the fame degree of velocity; yet we find the vifible effects of any power applied to the fame body to be at all times the very fame: and a bale of goods, or other moveable body lying in a fhip is as eafily removed from place to place, while the fhip is under fail, if its motion be fteady, as when it is fixed at anchor.

18. Now

18. Now this experience is alone sufficient to shew to us the whole of this law of motion.

19. Since we find, that the same power will always produce the same change in the motion of any body, whether that body were before moving with a swifter or slower motion; the change wrought in the motion of a body depends only on the power applied to it, without any regard to the body's former motion: and therefore the degree of motion, which the body already possesses, having no influence on the power applied to disturb its operation, the effects of the same power will not only be the same in all degrees of motion of the body; but we have likewise no reason to doubt, but that a body perfectly at rest would receive from any power as much motion, as would be equivalent to the effect of the same power applied to that body already in motion.

20. Again, suppose a body being at rest, any number of equal powers should be successively applied to it; pushing it forward from time to time in the same course or direction. Upon the application of the first power the body would begin to move; when the second power was applied, it appears from what has been said, that the motion of the body would become double; the third power would treble the motion of the body; and so on, till after the operation of the last power the motion of the body would be as many times the motion, which the first power gave it, as there are powers in number. And the effect of this number of powers will be always the

fame,

fame, without any regard to the fpace of time taken up in applying them : fo that greater or leffer intervals between the application of each of thefe powers will produce no difference at all in their effects.   Since therefore the diftance of time between the action of each power is of no confequence; without doubt the effect will ftill be the fame, though the powers fhould all be applied at the very fame inftant ; or although a fingle power fhould be applied equal in ftrength to the collective force of all thefe powers.   Hence it plainly follows, that the degree of motion, into which any body will be put out of a ftate of reft by any power, will be proportional to that power.   A double power will give twice the velocity, a treble power three times the velocity, and fo on.   The foregoing reafoning will equally take place, though the body were not fuppofed to be at reft, when the powers began to be applied to it; provided the direction, in which the powers were applied, either confpired with the action of the body, or was directly oppofite to it.   Therefore if any power be applied to a moving body, and act upon the body either in the direction wherewith the body moves, fo as to accelerate the body ; or if it act directly oppofite to the motion of the body, fo as to retard it : in both thefe cafes the change of motion will be proportional to the power applied; nay, the augmentation of the motion in one cafe, and the diminution thereof in the other, will be equal to that degree of motion, into which the fame power would put the body, had it been at reft, when the power was applied.

21. FARTHER

21. Farther, a power may be fo applied to a moving body, as to act obliquely to the motion of the body. And the effects of fuch an oblique motion may be deduced from this obfervation; that as all bodies are continually moving along with the earth, we fee that the vifible effects of the fame power are always the fame, in whatever direction the power acts: and therefore the vifible effects of any power upon a body, which feems only to be at reft, is always to appearance the fame as the real effect would be upon a body truly at reft. Now fuppofe a body were moving along the line A B (in fig. 2.) and the eye accompanied it with an equal motion in the line C D equidiftant from A B; fo that when the body is at A, the eye fhall be at C, and when the body is advanced to E in the line A B, the eye fhall be advanced to F in the line C D, the diftances A E and C F being equal. It is evident, that here the body will appear to the eye to be at reft; and the line F E G drawn from the eye through the body fhall feem to the eye to be immoveable; though as the body and eye move forward together, this line fhall really alfo move; fo that when the body fhall be advanced to H and the eye to K, the line F E G fhall be transferred into the fituation K H L, this line K H L being equidiftant from F E G. Now if the body when at E were to receive an impulfe in the direction of the line F E G; while the eye is moving on from F to I, and carrying along with it the line F E G, the body will appear to the eye to move along this line F E G: for this is what has juft now been faid; that while bodies are moving along with the earth, and the fpectator's eye partakes of the fame motion, the effect of any power upon the body will appear to be what

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it would really have been, had the body been truly at reſt, when the power was applied. From hence it follows, that when the eye is advanced to K, the body will appear ſomewhere in the line KHL. Suppoſe it appear in M ; then it is manifeſt, from what has been premiſed at the beginning of this paragraph, that the diſtance HM is equal to what the body would have run upon the line EG, during the time, wherein the eye has paſſed from F to K, provided that the body had been at reſt, when acted upon in E. If it be farther asked, after what manner the body has moved from E to M ? I anſwer, through a ſtraight line ; for it has been ſhewn above in the explication of the firſt law of motion, that a moving body, from the time it is left to it ſelf, will proceed on in one continued ſtraight line.

22. IF EN be taken equal to HM and NM be drawn ; ſince HM is equidiſtant from EN, NM will be equidiſtant from EH. Therefore the effect of any power upon a moving body, when that power acts obliquely to the motion of the body, is to be determined in this manner. Suppoſe the body is moving along the ſtraight line AEB, if when the body is come to E, a power gives it an impulſe in the direction of the line EG, to find what courſe the body will afterwards take we muſt proceed thus. Take in EB any length EH, and in EG take ſuch a length EN, that if the body had been at reſt in E, the power applied to it would have cauſed it to move over EN in the ſame ſpace of time, as it would have employed in paſſing over EH, if the power had not acted at all upon it. Then draw HL equidiſtant from EG, and NM equidiſtant

G   from

from E B. After this, if a line be drawn from E to the point M, where thefe two lines meet, the line EM will be the courfe into which the body will be put by the action of the power upon it at E.

23. A MATHEMATICAL reader would here expect in fome particulars more regular demonftrations; but as I do not at prefent addrefs my felf to fuch, fo I hope, what I have now written will render my meaning evident enough to thofe, who are unacquainted with that kind of reafoning.

24. Now as we have been fhewing, that fome actual force is neceffary either to put bodies out of a ftate of reft into motion, or to change the motion, which they have once received; it is proper here to obferve, that this quality in bodies, whereby they preferve their prefent ftate, with regard to motion or reft, till fome active force difturb them, is called the VIS INERTIAE of matter : and by this property, matter, fluggifh and unactive of it felf, retains all the power impreffed upon it, and cannot be made to ceafe from action, but by the oppofition of as great a power, as that which firft moved it. By the degree of this VIS INERTIAE, or power of inactivity, as we fhall henceforth call it, we primarily judge of the quantity of folid matter in each body; for as this quality is inherent in all the bodies, upon which we can make any trial, we conclude it to be a property effential to all matter; and as we yet know no reafon to fuppofe, that bodies are compofed of different kinds of matter, we rather prefume, that the matter of all bodies is the fame; and that the degree of
this

this power of inactivity is in every body proportional to the quantity of the folid matter in it.   But although we have no abfolute proof, that all the matter in the univerfe is uniform, and poffeffes this power of inactivity in the fame degree; yet we can with certainty compare together the different degrees of this power of inactivity in different bodies.   Particularly this power is proportional to the weight of bodies, as Sir Isaac Newton has demonftrated [a].   However, notwithftanding that this power of inactivity in any body can be more certainly known, than the quantity of folid matter in it ; yet fince there is no reafon to fufpect that one is not proportional to the other, we fhall hereafter fpeak without hefitation of the quantity of matter in bodies, as the meafure of the degree of their power of inactivity.

25. This being eftablifhed, we may now compare the effects of the fame power upon different bodies, as hitherto we have fhewn the effects of different powers upon the fame body.   And here if we limit the word motion to the peculiar fenfe given to it in philofophy, we may comprehend all that is to be faid upon this head under one fhort precept ; that the fame power, to whatever body it is applied, will always produce the fame degree of motion.   But here motion does not fignify the degree of celerity or velocity with which a body moves, in which fenfe only we have hitherto ufed it ; but it is made ufe of particularly in philofophy to fignify the force with which a body moves: as if two bodies A and B be-

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[a] Princ. Philof. L. II. prop. 24. corol. 7.   See alfo B. II. Ch. 5. § 3. of this treatife.

ing

ing in motion, twice the force would be required to ftop A as to ftop B, the motion of A would be efteemed double the motion of B. In moving bodies, thefe two things are carefully to be diftinguifhed; their velocity or celerity, which is meafured by the fpace they pafs through during any determi-nate portion of time; and the quantity of their motion, or the force, with which they will prefs againft any refiftance. Which force, when different bodies move with the fame velocity, is proportional to the quantity of folid matter in the bodies; but if the bodies are equal, this force is proportional to their refpective velocities, and in other cafes it is proportional both to the quantity of folid matter in the body, and alfo to its velocity. To inftance in two bodies A and B: if A be twice as great as B, and they have both the fame velocity, the motion of A fhall be double the motion of B; and if the bodies be equal, and the velocity of A be twice that of B, the motion of A fhall likewife be double that of B; but if A be twice as large as B, and move twice as fwift, the motion of A will be four times the motion of B; and laftly, if A be twice as large as B, and move but half as faft, the degree of their motion fhall be the fame.

26. This is the particular fenfe given to the word motion by philofophers, and in this fenfe of the word the fame power always produces the fame quantity or degree of motion. If the fame power act upon two bodies A and B, the velocities, it fhall give to each of them, fhall be fo adjufted to the refpective bodies, that the fame degree of motion fhall be produced in each. If A be twice as great as B, its velocity fhall be half

that

that of B ; if A has three times as much folid matter as B, the velocity of A fhall be one third of the velocity of B ; and generally the velocity given to A fhall bear the fame proportion to the velocity given to B, as the quantity of folid matter contained in the body B bears to the quantity of folid matter contained in A.

27. T H E reafon of all this is evident from what has gone before.   If a power were applied to B,   which fhould bear the fame proportion to the power applied to A, as the body B bears to A,   the bodies B and A would both receive the fame velocity ;   and the velocity,   which B will receive from this power,   will bear the fame proportion to the velocity,   which it would receive from the action of the power applied to A, as the former of thefe powers bears to the latter: that is, the velocity,   which A receives from the power applied to it, will bear to the velocity,   which B would receive from the fame power,   the fame proportion as the body B bears to A.

28. F R O M hence we may now pafs to the third law of motion,   where this diftinction between the velocity of a body and its whole motion is farther neceffary to be regarded, as fhall immediately be fhewn ;   after having firft illuftrated the meaning of this law by a familiar inftance.   If a ftone or other load be drawn by a horfe; the load re-acts upon the horfe, as much as the horfe acts upon the load ;   for the harnefs, which is ftrained between them,   preffes againft the horfe as much as againft the load ;   and the progreffive motion of the
<div align="right">horfe</div>

horfe forward is hindred as much by the load, as the motion of the load is promoted by the endeavour of the horfe: that is, if the horfe put forth the fame ftrength, when loofened from the load, he would move himfelf forwards with greater fwiftnefs in proportion to the difference between the weight of his own body and the weight of himfelf and load together.

29. This inftance will afford fome general notion of the meaning of this law. But to proceed to a more philofophical explication: if a body in motion ftrike againft another at reft, let the body ftriking be ever fo fmall, yet fhall it communicate fome degree of motion to the body it ftrikes againft, though the lefs that body be in comparifon of that it impinges upon, and the lefs the velocity is, with which it moves, the fmaller will be the motion communicated. But whatever degree of motion it gives to the refting body, the fame it fhall lofe it felf. This is the neceffary confequence of the forementioned power of inactivity in matter. For fuppofe the two bodies equal, it is evident from the time they meet, both the bodies are to be moved by the fingle motion of the firft; therefore the body in motion by means of its power of inactivity retaining the motion firft given it, ftrikes upon the other with the fame force, wherewith it was acted upon it felf: but now both the bodies being to be moved by that force, which before moved one only, the enfuing velocity will be the fame, as if the power, which was applied to one of the bodies, and put it into motion, had been applied to both; whence it appears, that they will proceed forwards,

with

with half the velocity, which the body firſt in motion had, that is, the body firſt moved will have loſt half its motion, and the other will have gained exactly as much. This rule is juſt, provided the bodies keep contiguous after meeting, as they would always do, if it were not for a certain cauſe that often intervenes, and which muſt now be explained. Bodies upon ſtriking againſt each other, ſuffer an alteration in their figure, having their parts preſſed inwards by the ſtroke, which for the moſt part recoil again afterwards, the bodies endeavouring to recover their former ſhape. This power, whereby bodies are inabled to regain their firſt figure, is uſually called their elaſticity, and when it acts, it forces the bodies from each other, and cauſes them to ſeparate. Now the effect of this elaſticity in the preſent caſe is ſuch, that if the bodies are perfectly elaſtic, ſo as to recoil with as great a force as they are bent with, that they recover their figure in the ſame ſpace of time, as has been taken up in the alteration made in it by their compreſſion together; then this power will ſeparate the bodies as ſwiftly, as they before approached, and acting upon both equally, upon the body firſt in motion contrary to the direction in which it moves, and upon the other as much in the direction of its motion, it will take from the firſt, and add to the other equal degrees of velocity: ſo that the power being ſtrong enough to ſeparate them with as great a velocity, as they approached with, the firſt will be quite ſtopt, and that which was at reſt, will receive all the motion of the other. If the bodies are elaſtic in a leſs degree, the firſt will not loſe all its motion, nor will the other acquire the motion of the firſt, but fall as much ſhort of it, as the other retains.

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For this rule is never deviated from, that though the degree of elasticity determines how much more than half its velocity the body first in motion shall lose; yet in every case the loss in the motion of this body shall be transferred to the other, that other body always receiving by the stroke as much motion, as is taken from the first.

30. This is the case of a body striking directly against an equal body at rest, and the reasoning here used is fully confirmed by experience. There are many other cases of bodies impinging against one another: but the mention of these shall be reserved to the next chapter, where we intend to be more particular and diffusive in the proof of these laws of motion, than we have been here.

Chap. II.

Farther proofs of the Laws of Motion.

HAVING in the preceding chapter deduced the three laws of motion, delivered by our great philosopher, from the most obvious observations, that suggest them to us; I now intend to give more particular proofs of them, by recounting some of the discoveries which have been made in philosophy before Sir Isaac Newton. For as they were all collected by reasoning upon those laws; so the conformity of these discoveries to experience makes them so many proofs of the truth of the principles, from which they were derived.

2. Let us begin with the subject, which concluded the last chapter. Although the body in motion be not equal to the body at rest, on which it strikes; yet the motion after the stroke is to be estimated in the same manner as above. Let A (in fig. 3.) be a body in motion towards another body B lying at rest. When A is arrived at B, it cannot proceed farther without putting B into motion; and what motion it gives to B, it must lose it self, that the whole degree of motion of A and B together, if neither of the bodies be elastic, shall be equal, after the meeting of the bodies, to the single motion of A before the stroke. Therefore, from what has been said above, it is manifest, that as soon as the two bodies are met, they will move on together with a velocity, which will bear the same proportion to the original velocity of A, as the body A bears to the sum of both the bodies.

3. If the bodies are elastic, so that they shall separate after the stroke, A must lose a greater part of its motion, and the subsequent motion of B will be augmented by this elasticity, as much as the motion of A is diminished by it. The elasticity acting equally between both the bodies, it will communicate to each the same degree of motion; that is, it will separate the bodies by taking from the body A and adding to the body B different degrees of velocity, so proportioned to their respective quantities of matter, that the degree of motion, wherewith A separates from B, shall be equal to the degree of motion, wherewith B separates from A. It follows therefore, that the velocity taken from A by the elasticity bears to the velocity, which the same elasticity adds to B, the

H   same

fame proportion, as B bears to A : confequently the velocity, which the elafticity takes from A, will bear the fame proportion to the whole velocity, wherewith this elafticity caufes the two bodies to feparate from each other, as the body B bears to the fum of the two bodies A and B; and the velocity, which is added to B by the elafticity, bears to the velocity, wherewith the bodies feparate, the fame proportion, as the body A bears to the fum of the two bodies A and B. Thus is found, how much the elafticity takes from the velocity of A, and adds to the velocity of B; provided the degree of elafticity be known, whereby to determine the whole velocity wherewith the bodies feparate from each other after the ftroke [a].

4. After this manner is determined in every cafe the refult of a body in motion ftriking againft another at reft. The fame principles will alfo determine the effects, when both bodies are in motion.

5. Let two equal bodies move againft each other with equal fwiftnefs. Then the force, with which each of them preffes forwards, being equal when they ftrike; each preffing in its own direction with the fame energy, neither fhall furmount the other, but both be ftopt, if they be not elaftic: for if they be elaftic, they fhall from thence recover new motion, and recede from each other, as fwiftly as they met, if they be perfectly elaftic; but more flowly, if lefs fo. In the fame manner, if two bodies of unequal bignefs ftrike againft each other, and their velocities be fo related, that the velocity

[a] How this degree of elafticity is to be found by experiment, will be fhewn below in § 74.

of

of the leffer body fhall exceed the velocity of the greater in the fame proportion, as the greater body exceeds the leffer (for inftance, if one body contains twice the folid matter as the o- ther, and moves but half as faft) two fuch bodies will entire- ly fupprefs each other's motion, and remain from the time of their meeting fixed ; if, as before, they are not elaftic : but, if they are fo in the higheft degree, they fhall recede again, each with the fame velocity, wherewith they met.   For this elaftic power, as in the preceding cafe, fhall renew their mo- tion, and preffing equally upon both, fhall give the fame mo- tion to both ; that is, fhall caufe the velocity, which the leffer body receives, to bear the fame proportion to the velocity, which the greater receives, as the greater body bears to the leffer :   fo that the velocities fhall bear the fame proportion to each other after the ftroke, as before.   Therefore if the bodies, by being perfectly elaftic, have the fum of their velocities after the ftroke equal to the fum of their velocities before the ftroke, each body after the ftroke will receive its firft veloci- ty.   And the fame proportion will hold likewife between the velocities, wherewith they go off, though they are elaftic but in a lefs degree ; only then the velocity of each will be lefs in proportion to the defect of elafticity.

6. I f the velocities, wherewith the bodies meet, are not in the proportion here fuppofed ; but if one of the bodies, as A, has a fwifter velocity in comparifon to the velocity of the other ; then the effect of this excefs of velocity in the body A muft be joined to the effect now mentioned, after the manner of this following example.   Let A be twice as great as B, and

move with the fame fwiftnefs as B. Here A moves with twice that degree of fwiftnefs, which would anfwer to the forementioned proportion. For A being double to B, if it moved but with half the fwiftnefs, wherewith B advances, it has been juft now fhewn, that the two bodies upon meeting would ftop, if they were not elaftic ; and if they were elaftic, that they would each recoil, fo as to caufe A to return with half the velocity, wherewith B would return. But it is evident from hence, that B by encountring A will annul half its velocity, if the bodies be not elaftic; and the future motion of the bodies will be the fame, as if A had advanced againft B at reft with half the velocity here affigned to it. If the bodies be elaftic, the velocity of A and B after the ftroke may be thus difcovered. As the two bodies advance againft each other, the velocity, with which they meet, is made up of the velocities of both bodies added together. After the ftroke their elafticity will feparate them again. The degree of elafticity will determine what proportion the velocity, wherewith they feparate, muft bear to that, wherewith they meet. Divide this velocity, with which the bodies feparate into two parts, that one of the parts bear to the other the fame proportion, as the body A bears to B; and afcribe the leffer part to the greater body A, and the greater part of the velocity to the leffer body B. Then take the part afcribed to A from the common velocity, which A and B would have had after the ftroke, if they had not been elaftic ; and add the part afcribed to B to the fame common velocity. By this means the true velocities of A and B after the ftroke will be made known.

7. If

7. If the bodies are perfectly elaſtic, the great H u y g e n s has laid down this rule for finding their motion after concourſe[a].   Any ſtraight line C D (in fig. 4, 5.) being drawn, let it be divided in E, that C E bear the ſame proportion to E D, as the ſwiftneſs of A bore to the ſwiftneſs of B before the ſtroke.   Let the ſame line C D be alſo divided in F, that C F bear the ſame proportion to F D, as the body B bears to the body A.   Then F G being taken equal to F E, if the point G falls within the line C D, both the bodies ſhall recoil after the ſtroke, and the velocity, wherewith the body A ſhall return, will bear the ſame proportion to the velocity, wherewith B ſhall return, as G C bears to G D; but if the point G falls without the line C D, then the bodies after their concourſe ſhall both proceed to move the ſame way, and the velocity of A ſhall bear to the velocity of B the ſame proportion, that G C bears to G.D, as before.

8. If the body B had ſtood ſtill, and received the impulſe of the other body A upon it ; the effect has been already explained in the caſe, when the bodies are not elaſtic.   And when they are elaſtic, the reſult of their colliſion is found by combining the effect of the elaſticity with the other effect, in the ſame manner as in the laſt caſe.

9. When the bodies are perfectly elaſtic, the rule of H u y g e n s [b] here is to divide the line C D (fig. 6.) in E as before, and to take E G equal to E D.   And by theſe points

[a] In oper. poſthum. de Motu corpor. ex per-
cuſſion. prop. 9.

[b] In the above-cited place.

thus ;

thus found, the motion of each body after the ftroke is determined, as before.

10. In the next place, fuppofe the bodies A and B were both moving the fame way, but A with a fwifter motion, fo as to overtake B, and ftrike againft it. The effect of the percuffion or ftroke, when the bodies are not elaftic, is difcovered by finding the common motion, which the two bodies would have after the ftroke, if B were at reft, and A were to advance againft it with a velocity equal to the excefs of the prefent velocity of A above the velocity of B; and by adding to this common velocity thus found the velocity of B.

11. If the bodies are elaftic, the effect of the elafticity is to be united with this other, as in the former cafes.

12. When the bodies are perfectly elaftic, the rule of Huygens [a] in this cafe is to prolong CD (fig. 7.) and to take in it thus prolonged CE in the fame proportion to ED, as the greater velocity of A bears to the leffer velocity of B; after which FG being taken equal to FE, the velocities of the two bodies after the ftroke will be determined, as in the two preceding cafes.

13. Thus I have given the fum of what has been written concerning the effects of percuffion, when two bodies freely in motion ftrike directly againft each other; and the refults here fet down, as the confequence of our reafoning

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[a] In the place above-cited.

from

from the laws of motion, answer most exactly to experience. A particular set of experiments has been invented to make trial of these effects of percussion with the greatest exactness. But I must defer these experiments, till I have explained the nature of pendulums [a]. I shall therefore now proceed to describe some of the appearances, which are caused in bodies from the influence of the power of gravity united with the general laws of motion; among which the motion of the pendulum will be included.

14. THE most simple of these appearances is, when bodies fall down merely by their weight. In this case the body increases continually its velocity, during the whole time of its fall, and that in the very same proportion as the time increases. For the power of gravity acts constantly on the body with the same degree of strength: and it has been observed above in the first law of motion, that a body being once in motion will perpetually preserve that motion without the continuance of any external influence upon it: therefore, after a body has been once put in motion by the force of gravity, the body would continue that motion, though the power of gravity should cease to act any farther upon it; but, if the power of gravity continues still to draw the body down, fresh degrees of motion must continually be added to the body; and the power of gravity acting at all times with the same strength, equal degrees of motion will constantly be added in equal portions of time.

[a] These experiments are described in § 73.

15. THIS

15. This conclusion is not indeed absolutely true: for we shall find hereafter [a], that the power of gravity is not of the same strength at all distances from the center of the earth. But nothing of this is in the least sensible in any distance, to which we can convey bodies. The weight of bodies is the very same to sense upon the highest towers or mountains, as upon the level ground ; so that in all the observations we can make, the forementioned proportion between the velocity of a falling body and the time, in which it has been descending, obtains without any the least perceptible difference.

16. From hence it follows, that the space, through which a body falls, is not proportional to the time of the fall ; for since the body increases its velocity, a greater space will be passed over in the same portion of time at the latter part of the fall, than at the beginning. Suppose a body let fall from the point A (in fig. 8.) were to descend from A to B in any portion of time ; then if in an equal portion of time it were to proceed from B to C; I say, the space BC is greater than AB; so that the time of the fall from A to C being double the time of the fall from A to B, AC shall be more than double of AB.

17. The geometers have proved, that the spaces, through which bodies fall thus by their weight, are just in a duplicate or two-fold proportion of the times, in which the body has been falling. That is, if we were to take the line DE in the same proportion to AB, as the time, which the body has imployed in falling from A to C, bears to the time of the fall

[a] Book II. Chap. 5.

from

from A to B; then A C will be to D E in the fame proportion. In particular, if the time of the fall through A C be twice the time of the fall through A B; then D E will be twice A B, and A C twice D E; or A C four times A B. But if the time of the fall through A C had been thrice the time of the fall through A B; D E would have been treble of A B, and A C treble of D E; that is, A C would have been equal to nine times A B.

18. If a body fall obliquely, it will approach the ground by flower degrees, than when it falls perpendicularly. Suppofe two lines A B, A C (in fig. 9.) were drawn, one perpendicular, and the other oblique to the ground D E: then if a body were to defcend in the flanting line A C; becaufe the power of gravity draws the body directly downwards, if the line A C fupports the body from falling in that manner, it muft take off part of the effect of the power of gravity; fo that in the time, which would have been fufficient for the body to have fallen through the whole perpendicular line A B, the body fhall not have paffed in the line A C a length equal to A B; confequently the line A C being longer than A B, the body fhall moft certainly take up more time in paffing through A C, than it would have done in falling perpendicularly down through A B.

19. The geometers demonftrate, that the time, in which the body will defcend through the oblique ftraight line A C, bears the fame proportion to the time of its defcent through the perpendicular A B, as the line it felf A C bears to A B. And in refpect to the velocity, which the body will have ac-

quired

quired in the point C, they likewife prove, that the length of
the time imployed in the defcent through A C fo compenfates
the diminution of the influence of gravity from the obliquity
of this line, that though the force of the power of gravity on
the body is oppofed by the obliquity of the line A C, yet the
time of the body's defcent fhall be fo much prolonged, that
the body fhall acquire the very fame velocity in the point C,
as it would have got at the point B by falling perpendicularly
down.

20. If a body were to defcend in a crooked line, the time
of its defcent cannot be determined in fo fimple a manner;
but the fame property, in relation to the velocity, is demon-
ftrated to take place in all cafes: that is, in whatever line the
body defcends, the velocity will always be anfwerable to the
perpendicular height, from which the body has fell. For in-
ftance, fuppofe the body A (in fig. 10.) were hung by a
ftring to the pin B. If this body were let fall, till it came to
the point C perpendicularly under B, it will have moved from
A to C in the arch of a circle. Then the horizontal line A D
being drawn, the velocity of the body in C will be the fame,
as if it had fallen from the point D directly down to C.

21. If a body be thrown perpendicularly upward with a-
ny force, the velocity, wherewith the body afcends, fhall
continually diminifh, till at length it be wholly taken away;
and from that time the body will begin to fall down again,
and pafs over a fecond time in its defcent the line, wherein it
afcended; falling through this line with an increafing veloci-
ty in fuch a manner, that in every point thereof, through
which

which it falls, it fhall have the very fame velocity, as it had in the fame place, when it afcended; and confequently fhall come down into the place, whence it firft afcended, with the velocity which was at firft given to it. Thus if a body were thrown perpendicularly up in the line A B (in fig. 11.) with fuch a force, as that it fhould ftop at the point B, and there begin to fall again; when it fhall have arrived in its defcent to any point as C in this line, it fhall there have the fame velocity, as that wherewith it paffed by this point C in its afcent; and at the point A it fhall have gained as great a velocity, as that wherewith it was firft thrown upwards. As this is demonftrated by the geometrical writers; fo, I think, it will appear evident, by confidering only, that while the body defcends, the power of gravity muft act over again, in an inverted order, all the influence it had on the body in its afcent; fo as to give again to the body the fame degrees of velocity, which it had taken away before.

22. AFTER the fame manner, if the body were thrown upwards in the oblique ftraight line C A (in fig. 9.) from the point C, with fuch a degree of velocity as juft to reach the point A; it fhall by its own weight return again through the line A C by the fame degrees, as it afcended.

23. AND laftly, if a body were thrown with any velocity in a line continually incurvated upwards, the like effect will be produced upon its return to the point, whence it was thrown. Suppofe for inftance, the body A (in fig. 12.) were hung by a ftring A B. Then if this body be impelled any

way,

way, it muft move in the arch of a circle. Let it receive fuch an impulfe, as fhall caufe it to move in the arch A C ; and let this impulfe be of fuch ftrength, that the body may be carried from A as far as D, before its motion is overcome by its weight : I fay here, that the body forthwith returning from D, fhall come again into the point A with the fame velocity, as that wherewith it began to move.

24. It will be proper in this place to obferve concerning the power of gravity, that its force upon any body does not at all depend upon the fhape of the body ; but that it continues conftantly the fame without any variation in the fame body, whatever change be made in the figure of the body : and if the body be divided into any number of pieces, all thofe pieces fhall weigh juft the fame, as they did, when united together in one body : and if the body be of a uniform contexture, the weight of each piece will be proportional to its bulk. This has given reafon to conclude, that the power of gravity acts upon bodies in proportion to the quantity of matter in them. Whence it fhould follow, that all bodies muft fall from equal heights in the fame fpace of time. And as we evidently fee the contrary in feathers and fuch like fubftances, which fall very flowly in comparifon of more folid bodies; it is reafonable to fuppofe, that fome other caufe concurs to make fo manifeft a difference. This caufe has been found by particular experiments to be the air. The experiments for this purpofe are made thus. They fet up a very tall hollow glafs; within which near the top they lodge a feather and fome very ponderous body, ufually a piece of gold,

this

this metal being the moſt weighty of any body known to us. This glaſs they empty of the air contained within it, and by moving a wire, which paſſes through the top of the glaſs, they let the feather and the heavy body fall together ; and it is always found, that as the two bodies begin to deſcend at the ſame time, ſo they accompany each other in the fall, and come to the bottom at the very ſame inſtant, as near as the eye can judge. Thus, as far as this experiment can be depended on, it is certain, that the effect of the power of gravity upon each body is proportional to the quantity of ſolid matter, or to the power of inactivity in each body. For in the limited ſenſe, which we have given above to the word motion, it has been ſhewn, that the ſame force gives to all bodies the ſame degree of motion, and different forces communicate different degrees of motion proportional to the reſpective powers [a]. In this caſe, if the power of gravity were to act equally upon the feather, and upon the more ſolid body, the ſolid body would deſcend ſo much ſlower than the feather, as to have no greater degree of motion than the feather: but as both bodies deſcend with equal ſwiftneſs, the degree of motion in the ſolid body is greater than in the feather, bearing the ſame proportion to it, as the quantity of matter in the ſolid body to the quantity of matter in the feather. Therefore the effect of gravity on the ſolid body is greater than on the feather, in proportion to the greater degree of motion communicated; that is, the effect of the power of gravity on the ſolid body bears the ſame proportion to its effect on the feather, as the quanti-

[a] Chap. I. § 25, 26, 27, compared with § 15, &c.

ty

ty of matter in the folid body bears to the quantity of matter in the feather. Thus it is the proper deduction from this experiment, that the power of gravity acts not on the furface of bodies only, but penetrates the bodies themfelves moft intimately, and operates alike on every particle of matter in them. But as the great quicknefs, with which the bodies fall, leaves it fomething uncertain, whether they do defcend abfolutely in the fame time, or only fo nearly together, that the difference in their fwift motion is not difcernable to the eye; this property of the power of gravity, which has here been deduced from this experiment, is farther confirmed by pendulums, whofe motion is fuch, that a very minute difference would become fufficiently fenfible. This will be farther difcourfed on in another place [a]; but here I fhall make ufe of the principle now laid down to explain the nature of what is called the center of gravity in bodies.

25. The center of gravity is that point, by which if a body be fufpended, it fhall hang at reft in any fituation. In a globe of a uniform texture the center of gravity is the fame with the center of the globe; for as the parts of the globe on every fide of its center are fimilarly difpofed, and the power of gravity acts alike on every part; it is evident, that the parts of the globe on each fide of the center are drawn with equal force, and therefore neither fide can yield to the other; but the globe, if fupported at its center, muft of neceffity hang at reft. In like manner, if two equal bodies A and B (in

[a] Book II. Chap. 5. § 3.

fig. 13.)

fig. 13.) be hung at the extremities of an inflexible rod C D, which fhould have no weight; thefe bodies, if the rod be fupported at its middle E, fhall equiponderate; and the rod remain without motion. For the bodies being equal and at the fame diftance from the point of fupport E, the power of gravity will act upon each with equal ftrength, and in all re-fpects under the fame circumftances; therefore the weight of one cannot overcome the weight of the other. The weight of A can no more furmount the weight of B, than the weight of B can furmount the weight of A. Again, fuppofe a bo-dy as A B (in fig. 14.) of a uniform texture in the form of a roller, or as it is more ufually called a cylinder, lying hori-zontally. If a ftraight line be drawn between C and D, the centers of the extreme circles of this cylinder; and if this ftraight line, commonly called the axis of the cylinder, be divided into two equal parts in E : this point E will be the center of gravity of the cylinder. The cylinder being a uni-form figure, the parts on each fide the point E are equal, and fituated in a perfectly fimilar manner; therefore this cylin-der, if fupported at the point E, muft hang at reft, for the fame reafon as the inflexible rod above-mentioned will remain without motion, when fufpended at its middle point. And it is evident, that the force applied to the point E, which would uphold the cylinder, muft be equal to the cylinder's weight. Now fuppofe two cylinders of equal thicknefs A B and C D to be joined together at C B, fo that the two axis's E F, and F G lie in one ftraight line. Let the axis E F be di-vided into two equal parts at H, and the axis F G into two

<div align="right">equal.</div>

equal parts at I.   Then becaufe the cylinder A B would be
upheld at reft by a power applied in H equal to the weight of
this cylinder, and the cylinder C D would likewife be upheld
by a power applied in I equal to the weight of this cylinder ;
the whole cylinder A D will be fupported by thefe two powers:
but the whole cylinder may likewife be fupported by a power
applied to K, the middle point of the whole axis E G, provided
that power be equal to the weight of the whole cylinder.   It
is evident therefore, that this power applied in K will produce
the fame effect, as the two other powers applied in H and I. It
is farther to be obferved, that H K is equal to half F G, and
K I equal to half E F; for E K being equal to half E G, and E H
equal to half E F, the remainder H K muft be equal to half
the remainder F G ; fo likewife G K being equal to half G E,
and G I equal to half G F, the remainder I K muft be equal to
half the remainder E F.   It follows therefore, that H K bears
the fame proportion to K I, as F G bears to E F.   Befides, I
believe, my readers will perceive, and it is demonftrated in
form by the geometers, that the whole body of the cylinder
C D bears the fame proportion to the whole body of the cy-
linder A B, as the axis F G bears to the axis E F [a].   But hence
it follows, that in the two powers applied at H and I, the
power applied at H bears the fame proportion to the power
applied at I, as K I bears to K H.   Now fuppofe two ftrings
H L and I M extended upwards, one from the point H and the
other from I, and to be laid hold on by two powers, one
ftrong enough to hold up the cylinder A B, and the other of

[a] See Euclid's Elements, Book XII. prop. 13.

ftrength

strength sufficient to support the cylinder C D.   Here as these
two powers uphold the whole cylinder, and therefore pro-
duce an effect, equal to what would have been produced by
a power applied to the point K of sufficient force to sustain the
whole cylinder : it is manifest, that if the cylinder be taken
away, the axis only being left, and from the point K a string,
as K N, be extended, which shall be drawn down by a power
equivalent to the weight of the cylinder, this power shall act
against the other two powers, as much as the cylinder acted
against them; and consequently these three powers shall be
upon a balance, and hold the axis H I fixed between them.
But if these three powers preserve a mutual balance, the
two powers applied to the strings H L and I M are a balance
to each other; the power applied to the string H L bearing
the same proportion to the power applied to the string I M,
as the distance I K bears to the distance K H.   Hence it far-
ther appears, that if an inflexible rod A B (in fig. 15.) be
suspended by any point C not in the middle thereof; and if
at A the end of the shorter arm be hung a weight, and at B
the end of the longer arm be also hung a weight less than
the other, and that the greater of these weights bears to the
lesser the same proportion, as the longer arm of the rod bears
to the shorter; then these two weights will equiponderate :
for a power applied at C equal to both these weights will sup-
port without motion the rod thus charged; since here no-
thing is changed from the preceding case but the situa-
tion of the powers, which are now placed on the contra-
ry sides of the line, to which they are fixed.   Also for the

K   same

fame reafon, if two weights A and B (in fig. 16.) were con-
nected together by an inflexible rod C D, drawn from C the
center of gravity of A to D the center of gravity of B ; and
if the rod C D were to be fo divided in E, that the part D E
bear the fame proportion to the other part C E, as the weight
A bears to the weight B : then this rod being fupported at E
will uphold the weights, and keep them at reft without mo-
tion. This point E, by which the two bodies A and B will be
fupported, is called their common center of gravity. And if
a greater number of bodies were joined together, the point, by
which they could all be fupported, is called the common center
of gravity of them all. Suppofe (in fig. 17.) there were three
bodies A, B, C, whofe refpective centers of gravity were joined
by the three lines D E, D F, E F: the line D E being fo divided
in G, that D G bear the fame proportion to G E, as B bears to
A ; G is the center of gravity common to the two bodies A
and B ; that is, a power equal to the weight of both the bo-
dies applied to G would fupport them, and the point G is
preffed as much by the two weights A and B, as it would be,
if they were both hung together at that point. Therefore,
if a line be drawn from G to F, and divided in H, fo that G H
bear the fame proportion to H F, as the weight C bears to
both the weights A and B, the point H will be the common
center of gravity of all the three weights ; for H would be
their common center of gravity, if both the weights A and B
were hung together at G, and the point G is preffed as much
by them in their prefent fituation, as it would be in that cafe.
In the fame manner from the common center of thefe three
weights,

weights, you might proceed to find the common center, if a fourth weight were added, and by a gradual progrefs might find the common center of gravity belonging to any number of weights whatever.

26. A s all this is the obvious confequence of the propofition laid down for affigning the common center of gravity of any two weights, by the fame propofition the center of gravity of all figures is found.   In a triangle, as A B C (in fig. 18.) the center of gravity lies in the line drawn from the middle point of any one of the fides to the oppofite angle, as the line B D is drawn from D the middle of the line A C to the oppofite angle B [a] ; fo that if from the middle of either of the other fides, as from the point E in the fide A B, a line be drawn, as E C, to the oppofite angle ; the point F, where this line croffes the other line B D, will be the center of gravity of the triangle [b].   Likewife D F is equal to half F B, and E F equal to half F C [c].   In a hemifphere, as A B C (fig. 19.) if from D the center of the bafe the line D B be erected perpendicular to that bafe, and this line be fo divided in E, that D E be equal to three fifths of B E, the point E is the center of gravity of the hemifphere [d].

27. I t will be of ufe to obferve concerning the center of gravity of bodies ; that fince a power applied to this center alone can fupport a body againft the power of gravity, and

[a] Archimed. de æquipond. prop. 11.
[b] Ibid. prop. 12.
[c] Lucas Valerius De centr. gravit. folid. L. I.
prop. 2.
[d] Idem L. II. prop. 2.

hold

hold it fixed at rest; the effect of the power of gravity on a body is the same, as if that whole power were to exert itself on the center of gravity only. Whence it follows, that, when the power of gravity acts on a body suspended by any point, if the body is so suspended, that the center of gravity of the body can descend; the power of gravity will give motion to that body, otherwise not: or if a number of bodies are so connected together, that, when any one is put into motion, the rest shall, by the manner of their being joined, receive such motion, as shall keep their common center of gravity at rest; then the power of gravity shall not be able to produce any motion in these bodies, but in all other cases it will. Thus, if the body A B (in fig. 20,21.) whose center of gravity is C, be hung on the point A, and the center C be perpendicularly under A (as in fig. 20.) the weight of the body will hold it still without motion, because the center C cannot descend any lower. But if the body be removed into any other situation, where the center C is not perpendicularly under A (as in fig. 21.) the body by its weight will be put into motion towards the perpendicular situation of its center of gravity. Also if two bodies A, B (in fig. 22.) be joined together by the rod C D lying in an horizontal situation, and be supported at the point E; if this point be the center of gravity common to the two bodies, their weight will not put them into motion; but if this point E is not their common center of gravity, the bodies will move; that part of the rod C D descending, in which the common center of gravity is found. So in like manner, if these two bodies were connected together by any more complex contrivance; yet

if

if one of the bodies cannot move without fo moving the other, that their common center of gravity fhall reft, the weight of the bodies will not put them in motion, otherwife it will.

28. I shall proceed in the next place to fpeak of the mechanical powers. Thefe are certain inftruments or machines, contrived for the moving great weights with fmall force; and their effects are all deducible from the obfervation we have juft been making. They are ufually reckoned in number five; the lever, the wheel and axis, the pulley, the wedge, and the fcrew; to which fome add the inclined plane. As thefe inftruments have been of very ancient ufe, fo the celebrated Archimedes feems to have been the firft, who difcovered the true reafon of their effects. This, I think, may be collected from what is related of him, that fome expreffions, which he ufed to denote the unlimited force of thefe inftruments, were received as very extraordinary paradoxes: whereas to thofe, who had underftood the caufe of their great force, no expreffions of that kind could have appeared furprizing.

29. All the effects of thefe powers may be judged of by this one rule, that, when two weights are applied to any of thefe inftruments, the weights will equiponderate, if, when put into motion, their velocities will be reciprocally proportional to their refpective weights. And what is faid of weights, muft of neceffity be equally underftood of any other forces

equi-

equivalent to weights, fuch as the force of a man's arm, a ftream of water, or the like.

30. But to comprehend the meaning of this rule, the reader muft know, what is to be underftood by reciprocal proportion; which I fhall now endeavour to explain, as diftinctly as I can; for I fhall be obliged very frequently to make ufe of this term. When any two things are fo related, that one increafes in the fame proportion as the other, they are directly proportional. So if any number of men can perform in a determined fpace of time a certain quantity of any work, fuppofe drain a fifh-pond, or the like; and twice the number of men can perform twice the quantity of the fame work, in the fame time; and three times the number of men can perform as foon thrice the work; here the number of men and the quantity of the work are directly proportional. On the other hand, when two things are fo related, that one decreafes in the fame proportion, as the other increafes, they are faid to be reciprocally proportional. Thus if twice the number of men can perform the fame work in half the time, and three times the number of men can finifh the fame in a third part of the time; then the number of men and the time are reciprocally proportional. We fhewed above [a] how to find the common center of gravity of two bodies, there the diftances of that common center from the centers of gravity of the two bodies are reciprocally proportional to the refpective bodies. For CE in fig. 16. being in the fame pro-

[a] § 25.

portion

portion to E D, as B bears to A ; C E is fo much greater in proportion than E D, as A is lefs in proportion than B.

31. N o w this being underſtood, the reaſon of the rule here ſtated will eaſily appear.   For if theſe two bodies were put in motion, while the point E reſted, the velocity, where- with A would move, would bear the ſame proportion to the velocity, wherewith B would move, as E C bears to E D. The velocity therefore of each body, when the common center of gravity reſts, is reciprocally proportional to the body. But we have ſhewn above[a], that if two bodies are ſo connected to- gether, that the putting them in motion will not move their common center of gravity ; the weight of thoſe bodies will not produce in them any motion.   Therefore in any of theſe mechanical engines, if, when the bodies are put into motion, their velocities are reciprocally proportional to their reſpective weights,   whereby the common center of gravity would re- main at reſt ; the bodies will not receive any motion from their weight, that is, they will equiponderate.   But this perhaps will be yet more clearly conceived by the particular deſcrip- tion of each mechanical power.

32. T h e lever was firſt named above. This is a bar made uſe of to ſuſtain and move great weights.   The bar is ap- plied in one part to ſome ſtrong ſupport ; as the bar A B (in fig. 23, 24.) is applied at the point C to the ſupport D.   In ſome other part of the bar, as E, is applied the weight to be ſuſtained or moved ; and in a third place, as F, is applied ano- ther weight or equivalent force, which is to ſuſtain or move

the

[a] § 27.

the weight at E.   Now here, if, when the lever should be
put in motion, and turned upon the point C, the velocity,
wherewith the point F would move, bears the same propor-
tion to the velocity, wherewith the point E would move, as
the weight at E bears to the weight or force at F ; then the
lever thus charged will have no propensity to move either
way.   If the weight or other force at F be not so great as to
bear this proportion, the weight at E will not be sustained ;
but if the force at F be greater than this, the weight at E will
be surmounted.   This is evident from what has been said
above [a], when the forces at E and F are placed (as in fig. 23.)
on different sides of the support D.   It will appear also equal-
ly manifest in the other case, by continuing the bar BC in
fig. 24. on the other side of the support D, till CG be equal
to CF, and by hanging at G a weight equivalent to the power
at F; for then, if the power at F were removed, the two
weights at G and E would counterpoize each other, as in
the former case : and it is evident, that the point F will
be lifted up by the weight at G with the same degree of
force, as by the other power applied to F ; since, if the
weight at E were removed, a weight hung at F equal to
that at G would balance the lever, the distances CG and
CF being equal.

33. If the two weights, or other powers, applied to the
lever do not counterbalance each other ; a third power may
be applied in any place proposed of the lever, which shall

hold

hold the whole in a juſt counterpoize. Suppoſe (in fig. 25.) the two powers at E and F did not equiponderate, and it were required to apply a third power to the point G, that might be ſufficient to balance the lever. Find what power in F would juſt counterbalance the power in E ; then if the difference between this power and that, which is actually applied at F, bear the ſame proportion to the third power to be applied at G, as the diſtance C G bears to C F ; the lever will be coun- terpoized by the help of this third power, if it be ſo applied as to act the ſame way with the power in F, when that power is too ſmall to counterbalance the power in E ; but other- wiſe the power in G muſt be ſo applied, as to act againſt the power in F. In like manner, if a lever were charged with three, or any greater number of weights or other powers, which did not counterpoize each other, another power might be applied in any place propoſed, which ſhould bring the whole to a juſt balance. And what is here ſaid concerning a plurality of powers, may be equally applied to all the following caſes.

34. If the lever ſhould conſiſt of two arms making an angle at the point C (as in fig. 26.) yet if the forces are ap- plied perpendicularly to each arm, the ſame proportion will hold between the forces applied, and the diſtances of the cen- ter, whereon the lever reſts, from the points to which they are applied. That is, the weight at E will be to the force in F in the ſame proportion, as C F bears to C E.

35. But whenever the forces applied to the lever act ob- liquely to the arm, to which they are applied (as in fig. 27.)

<div align="center">L</div>

<div align="right">then</div>

then the ftrength of the forces is to be eftimated by lines let
fall from the center of the lever to the directions, wherein the
forces act.   To balance the levers in fig. 27, the weight or
other force at F will bear the fame proportion to the weight
at E, as the diftance C E bears to C G the perpendicular let fall
from the point C upon the line, which denotes the direction,
wherein the force applied to F acts : for here, if the lever be
put into motion, the power applied to F will begin to move in
the direction of the line F G; and therefore its firft motion will
be the fame, as the motion of the point G.

36. W H E N two weights hang upon a lever, and the point,
by which the lever is fupported, is placed in the middle be-
tween the two weights, that the arms of the lever are both
of equal length ; then this lever is particularly called a ba-
lance ; and equal weights equiponderate as in common fcales.
When the point of fupport is not equally diftant from both
weights, it conftitutes that inftrument for weighing, which
is called a fteelyard. Though both in common fcales, and the
fteelyard, the point, on which the beam is hung, is not ufu-
ally placed juft in the fame ftraight line with the points, that
hold the weights, but rather a little above (as in fig. 28.)
where the lines drawn from the point C, whereon the beam
is fufpended, to the points E and F, on which the weights are
hung, do not make abfolutely one continued line. If the
three points E, C, and F were in one ftraight line, thofe weights,
which equiponderated, when the beam hung horizontally,
would alfo equiponderate in any other fituation. But we
fee in thefe inftruments, when they are charged with weights,
which

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 10.

Fig. 13.

Fig. 14.

Fig. 16.

Fig. 19.

Fig. 17.

Fig. 18.

Fig. 22.

Fig. 23.

Fig. 24.

Fig. 27.

Fig. 25.
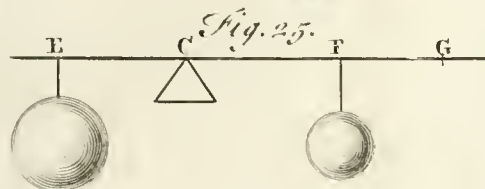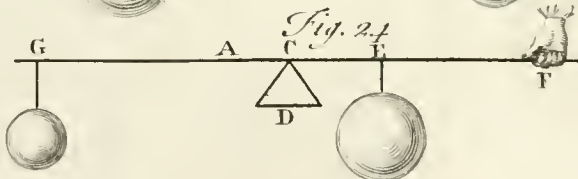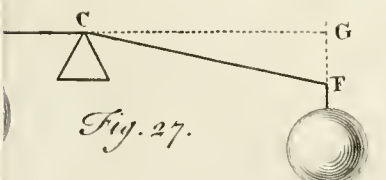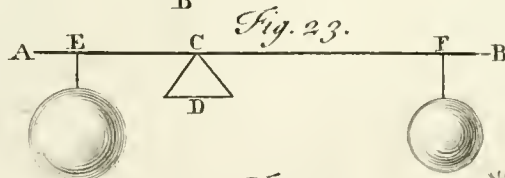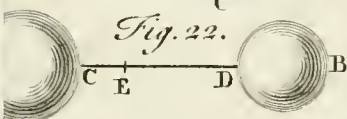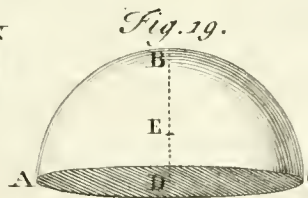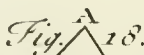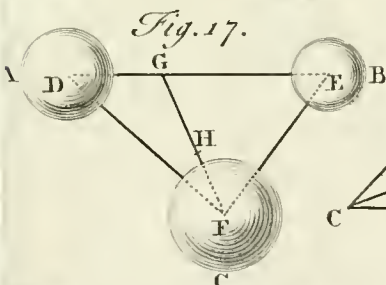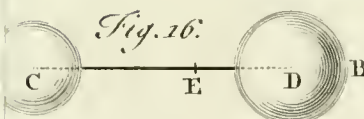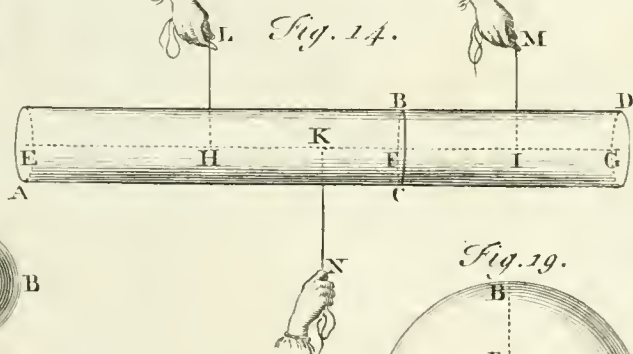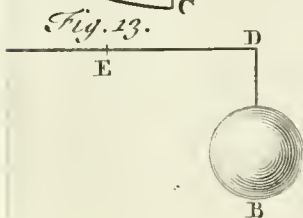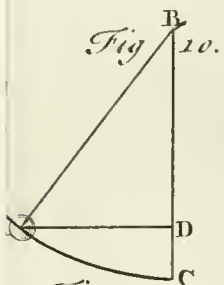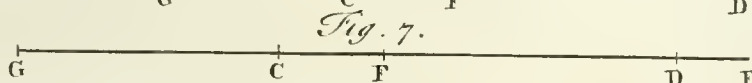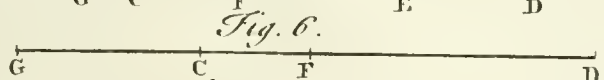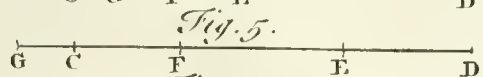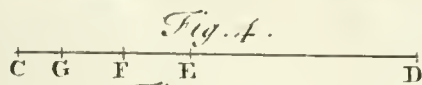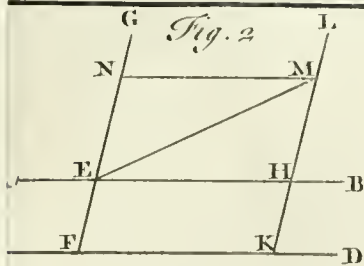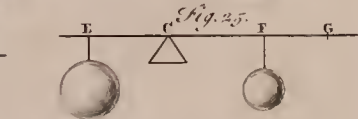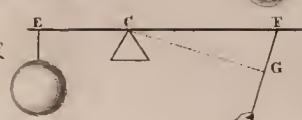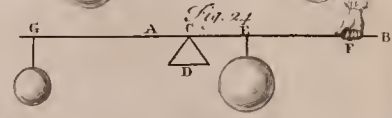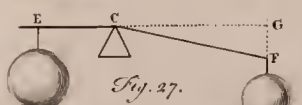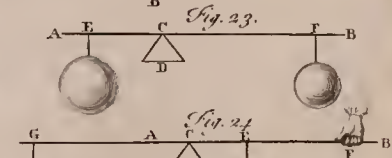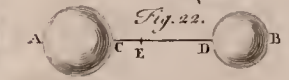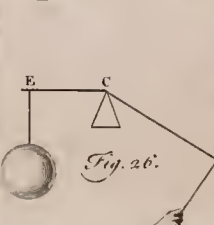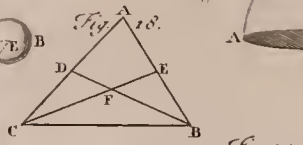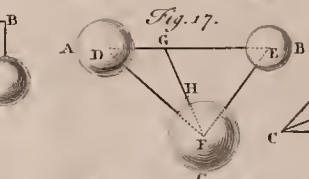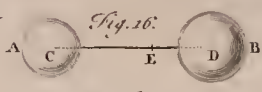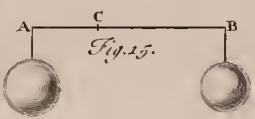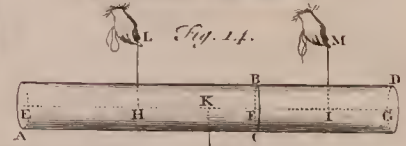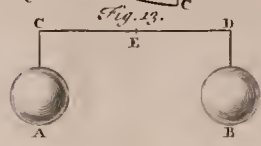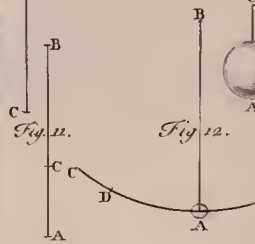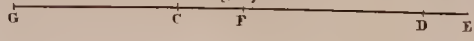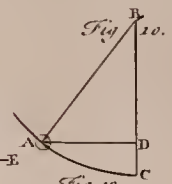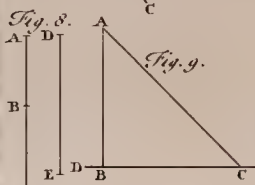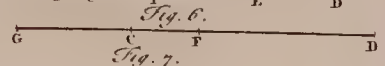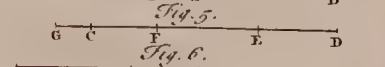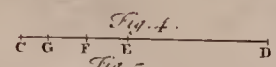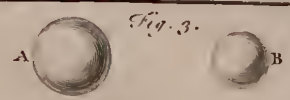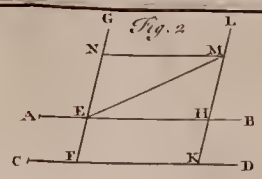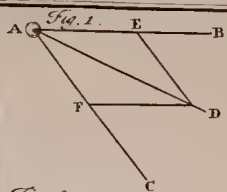
which equiponderate with the beam hanging horizontally; that, if the beam be inclined either way, the weight moſt elevated ſurmounts the other, and deſcends, cauſing the beam to ſwing, till by degrees it recovers its horizontal poſition. This effect ariſes from the forementioned ſtructure: for by this ſtructure theſe inſtruments are levers compoſed of two arms, which make an angle at the point of ſupport (as in fig. 29, 30.) the firſt of which repreſents the caſe of the common balance, the ſecond the caſe of the ſteelyard. In the firſt, where CE and CF are equal, equal weights hung at E and F will equiponderate, when the points E and F are in an horizontal ſituation. Suppoſe the lines EG and FH to be perpendicular to the horizon, then they will denote the directions, wherein the forces applied to E and F act. Therefore the proportion between the weights at E and F, which ſhall equiponderate, are to be judged of by perpendiculars, as CI, CK, let fall from C upon EG and FH: ſo that the weights being equal, the lines CI, CK, muſt be equal alſo, when the weights equiponderate. But I believe my readers will eaſily ſee, that ſince CE and CF are equal, the lines CI and CK will be equal, when the points E and F are horizontally ſituated.

37. If this lever be ſet into any other poſition (as in fig. 31.) then the weight, which is raiſed higheſt, will outweigh the other. Here, if the point F be raiſed higher than E, the perpendicular CK will be longer than CI: and therefore the weights would equiponderate, if the weight at F

were

were lefs than the weight at E.   But the weight at F is equal to that at E; therefore is greater, than is neceffary to counterbalance the weight at E, and confequently will outweigh it, and draw the beam of the lever down.

38. In like manner in the cafe of the fteelyard (fig.32.) if the weights at E and F are fo proportioned, as to equiponderate, when the points E and F are horizontally fituated; then in any other fituation of this lever the weight, which is raifed higheft, will preponderate. That is, if in the horizontal fituation of the points E and F the weight at F bears the fame proportion to the weight at E, as CI bears to CK; then, if the point F be raifed higher than E (as in fig. 32.) the weight at F fhall bear a greater proportion to the weight at E, than CI bears to CK.

39. Farther a lever may be hung upon an axis, and then the two arms of the lever need not be continuous, but fixed to different parts of this axis; as in fig. 33, where the axis AB is fupported by its two extremities A and B. To this axis one arm of the lever is fixed at the point C, the other at the point D. Now here, if a weight be hung at E, the extremity of that arm, which is fixed to the axis at the point C; and another weight be hung at F, the extremity of the arm, which is fixed on the axis at D; then thefe weights will equiponderate, when the weight at E bears the fame proportion to the weight at F, as the arm DF bears to CE.

40. This

40. This is the cafe, if both the arms are perpendicular to the axis, and lie (as the geometers exprefs themfelves) in the fame plane ; or, in other words, if the arms are fo fixed perpendicularly upon the axis, that, when one of them lies horizontally, the other fhall alfo be horizontal. If either arm ftand not perpendicular to the axis; then, in determining the proportion between the weights, inftead of the length of that arm, you muft ufe the perpendicular let fall upon the axis from the extremity of that arm. If the arms are not fo fixed as to become horizontal, at the fame time ; the method of affigning the proportion between the weights is analogous to that made ufe of above in levers, which make an angle at the point, whereon they are fupported.

41. From this cafe of the lever hung on an axis, it is eafy to make a tranfition to another mechanical power, the wheel and axis.

42. This inftrument is a wheel fixed on a roller, the roller being fupported at each extremity fo as to turn round freely with the wheel, in the manner reprefented in fig. 34, where A B is the wheel, C D the roller, and E F its two fupports. Now fuppofe a weight G hung by a cord wound round the roller, and another weight H hung by a cord wound about the wheel the contrary way : that thefe weights may fupport each other, the weight H muft bear the fame proportion to the weight G, as the thicknefs of the roller bears to the diameter of the wheel.

43. Suppose

43. Suppose the line kl to be drawn through the middle of the roller; and from the place of the roller, where the cord, on which the weight G hangs, begins to leave the roller, as at m, let the line mn be drawn perpendicularly to kl; and from the point, where the cord holding the weight H begins to leave the wheel, as at o, let the line op be drawn perpendicular to kl. This being done, the two lines op and mn reprefent two arms of a lever fixed on the axis kl; confequently the weight H will bear to the weight G the fame proportion, as mn bears to op. But mn bears the fame proportion to op, as the thicknefs of the roller bears to the diameter of the wheel; for mn is half the thicknefs of the roller, and op half the diameter of the wheel.

44. If the wheel be put into motion, and turned once round, that the cord, on which the weight G hangs, be wound once more round the axis; then at the fame time the cord, whereon the weight H hangs, will be wound off from the wheel one circuit. Therefore the velocity of the weight G will bear the fame proportion to the velocity of the weight H, as the circumference of the roller to the circumference of the wheel. But the circumference of the roller bears the fame proportion to the circumference of the wheel, as the thicknefs of the roller bears to the diameter of the wheel, confequently the velocity of the weight G bears to the velocity of the weight H the fame proportion, as the thicknefs of the roller bears to the diameter of the wheel, which is the proportion that the weight H bears to the weight G. Therefore as before in the lever, fo here alfo the general rule laid

down

down above is verified, that the weights equiponderate, when their velocities would be reciprocally proportional to their respective weights.

45. In like manner, if on the same axis two wheels of different sizes are fixed (as in fig. 35.) and a weight hung on each; the weights will equiponderate, if the weight hung on the greater wheel bear the same proportion to the weight hung on the lesser, as the diameter of the lesser wheel bears to the diameter of the greater.

46. It is usual to join many wheels together in the same frame, which by the means of certain teeth, formed in the circumference of each wheel, shall communicate motion to each other. A machine of this nature is represented in fig. 36. Here A B C is a winch, upon which is fixed a small wheel D indented with teeth, which move in the like teeth of a larger wheel E F fixed on the axis G H. Let this axis carry another wheel I, which shall move in like manner a greater wheel K L fixed on the axis M N. Let this axis carry another small wheel O, which after the same manner shall turn about a larger wheel P Q fixed on the roller R S, on which a cord shall be wound, that holds a weight, as T. Now the proportion required between the weight T and a power applied to the winch at A sufficient to support the weight, will most easily be estimated, by computing the proportion, which the velocity of the point A would bear to the velocity of the weight. If the winch be turned round, the point A will describe a circle as A V. Suppose the wheel E F to have ten times the number of teeth, as
the

the wheel D; then the winch muft turn round ten times to carry the wheel E F once round. If the wheel K L has alfo ten times the number of teeth, as I, the wheel I muft turn round ten times to carry the wheel K L once round ; and confequently the winch A B C muft turn round an hundred times to turn the wheel K L once round. Laftly, if the wheel P Q has ten times the number of teeth, as the wheel O, the winch muft turn about one thoufand times in order to turn the wheel P Q, or the roller R S once round. Therefore here the point A muft have gone over the circle A V a thoufand times, in order to lift the weight T through a fpace equal to the circumference of the roller R S : whence it follows, that the power applied at A will balance the weight T, if it bear the fame proportion to it, as the circumference of the roller to one thoufand times the circle A V ; or the fame proportion as half the thicknefs of the roller bears to one thoufand times A B.

47. I shall now explain the effect of the pulley. Let a weight hang by a pulley, as in fig. 37. Here it is evident, that the power A, by which the weight B is fupported, muft be equal to the weight ; for the cord C D is equally ftrained between them ; and if the weight B move, the power A muft move with equal velocity. The pulley E has no other effect, than to permit the power A to act in another direction, than it muft have done, if it had been directly applied to fupport the weight without the intervention of any fuch inftrument.

48. Again, let a weight be fupported, as in fig. 38 ; where the weight A is fixed to the pulley B, and the cord, by which

which the weight is upheld, is annexed by one extremity to a hook C, and at the other end is held by the power D. Here the weight is fupported by a cord doubled; infomuch that although the cord were not ftrong enough to hold the weight fingle, yet being thus doubled it might fupport it. If the end of the cord held by the power D were hung on the hook C, as well as the other end; then, when both ends of the cord were tied to the hook, it is evident, that the hook would bear the whole weight; and each end of the ftring would bear againft the hook with the force of half the weight only, feeing both ends together bear with the force of the whole. Hence it is evident, that, when the power D holds one end of the weight, the force, which it muft exert to fupport the weight, muft be equal to juft half the weight. And the fame proportion between the weight and power might be collected from comparing the refpective velocities, with which they would move ; for it is evident, that the power muft move through a fpace equal to twice the diftance of the pulley from the hook, in order to lift the pulley up to the hook.

49. It is equally eafy to eftimate the effect, when many pulleys are combined together, as in fig. 39, 40; in the firft of which the under fet of pulleys, and confequently the weight is held by fix ftrings; and in the latter figure by five: therefore in the firft of thefe figures the power to fupport the weight, muft be one fixth part only of the weight, and in the latter figure the power muft be one fifth part.

M

50. There

50. There are two other ways of supporting a weight by pulleys, which I shall particularly consider.

51. One of these ways is represented in fig. 41. Here the weight being connected to the pulley B, a power equal to half the weight A would support the pulley C, if applied immediately to it. Therefore the pulley C is drawn down with a force equal to half the weight A. But if the pulley D were to be immediately supported by half the force, with which the pulley C is drawn down, this pulley D will uphold the pulley C ; so that if the pulley D be upheld with a force equal to one fourth part of the weight A, that force will support the weight. But, for the same reason as before, if the power in E be equal to half the force necessary to uphold the pulley D; this pulley, and consequently the weight A, will be upheld : therefore, if the power in E be one eighth part of the weight A, it will support the weight.

52. Another way of applying pulleys to a weight is represented in fig. 42. To explain the effect of pulleys thus applied, it will be proper to consider different weights hanging, as in fig. 43. Here, if the power and weights balance each other, the power A is equal to the weight B ; the weight C is equal to twice the power A, or the weight B ; and for the same reason the weight D is equal to twice the weight C, or equal to four times the power A. It is evident therefore, that all the three weights B, C, D together are equal to seven times the power A. But if these three weights were joined in one, they would produce the case of fig. 40 : so that in that figure the
weight

weight A, where there are three pulleys, is seven times the power B. If there had been but two pulleys, the weight would have been three times the power ; and if there had ben four pulleys, the weight would have been fifteen times the power.

53. The wedge is next to be considered. The form of this instrument is sufficiently known. When it is put under any weight (as in fig. 44.) the force, with which the wedge will lift the weight, when drove under it by a blow upon the end A B, will bear the same proportion to the force, wherewith the blow would act on the weight, if directly applied to it; as the velocity, which the wedge receives from the blow, bears to the velocity, wherewith the weight is lifted by the wedge.

54. The screw is the fifth mechanical power. There are two ways of applying this instrument. Sometimes it is screwed into a hole, as in fig. 45, where the screw A B is screwed through the plank C D. Sometimes the screw is applied to the teeth of a wheel, as in fig. 46, where the thred of the screw A B turns in the teeth of a wheel C D. In both these cases, if a bar, as A E, be fixed to the end A of the screw ; the force, wherewith the end B of the screw in fig. 45 is forced down, and the force, wherewith the teeth of the wheel C D in fig. 44 are held, bears the same proportion to the power applied to the end E of the bar ; as the velocity, wherewith the end E will move, when the screw is turned, bears to the velocity, wherewith the end B of the screw in fig. 43, or the teeth of the wheel C D in fig. 46, will be moved.

M 2   55. The

55. The inclined plane affords also a means of raising a weight with less force, than what is equal to the weight it self. Suppose it were required to raise the globe A (in fig. 47.) from the ground BC up to the point, whose perpendicular height from the ground is ED. If this globe be drawn along the slant DF, less force will be required to raise it, than if it were lifted directly up. Here if the force applied to the globe bear the same proportion only to its weight, as ED bears to FD, it will be sufficient to hold up the globe; and therefore any addition to that force will put it in motion, and draw it up; unless the globe, by pressing against the plane, whereon it lies, adhere in some degree to the plane. This indeed it must always do more or less, since no plane can be made so absolutely smooth as to have no inequalities at all; nor yet so infinitely hard, as not to yield in the least to the pressure of the weight. Therefore the globe cannot be laid on such a plane, whereon it will slide with perfect freedom, but they must in some measure rub against each other; and this friction will make it necessary to imploy a certain degree of force more, than what is necessary to support the globe, in order to give it any motion. But as all the mechanical powers are subject in some degree or other to the like impediment from friction; I shall here only shew what force would be necessary to sustain the globe, if it could lie upon the plane without causing any friction at all. And I say, that if the globe were drawn by the cord GH, lying parallel to the plane DF; and the force, wherewith the cord is pulled, bear the same proportion to the weight of the globe, as ED bears to DF;

this

this force will fuftain the globe.   In order to the making
proof of this, let the cord G H be continued on, and turned
over the pulley I,   and let the weight K be hung to it.
Now I fay,   if this weight bears the fame proportion to
the globe A,   as D E bears to D F,   the weight will fupport
the globe.   I think it is very manifeft, that the center of the
globe A will lie in one continued line with the cord H G. Let
L be the center of the globe, and M the center of gravity of
the weight K.   In the firft place let the weight hang fo, that
a line drawn from L to M fhall lie horizontally ;   and I fay,
if the globe be moved either up or down the plane D F,   the
weight will fo move along with it,   that the center of gravity
common to both the weights fhall continue in this line L M,
and therefore fhall in no cafe defcend.   To prove this more
fully, I fhall depart a little from the method of this treatife,
and make ufe of a mathematical propofition or two : but they
are fuch, as any perfon, who has read EUCLID's ELEMENTS,
will fully comprehend; and are in themfelves fo evident, that,
I believe, my readers, who are wholly ftrangers to geometri-
cal writings, will make no difficulty of admitting them. This
being premifed, let the globe be moved up, till its center be
at G, then will M the center of gravity of the weight K be
funk to N ; fo that M N fhall be equal to G L.   Draw N G
croffing the line M L in O ; then I fay, that O is the common
center of gravity of the two weights in this their new fitua-
tion.   Let G P be drawn perpendicular to M L ; then G L will
bear the fame proportion to G P, as D F bears to D E ;   and
M N being equal to G L,   M N will bear the fame proportion

to

to G P, as D F bears to D E.   But N O bears the fame propor-
tion to O G, as M N bears to G P; confequently N O will bear
the fame proportion to O G, as D F bears to D E.   In the laft
place, the weight of the globe A bears the fame proportion to
the other weight K, as D F bears to D E;   therefore N O bears
the fame proportion to O G, as the weight of the globe A bears
to the weight K.   Whence it follows, that, when the center
of the globe A is in G, and the center of gravity of the weight
K is in N, O will be the center of gravity common to both
the weights.   After the fame manner, if the globe had been
caufed to defcend, the common center of gravity would have
been found in this line M L.   Since therefore no motion of
the globe either way will make the common center of gravity
defcend, it is manifeft, from what has been faid above, that
the weights A and K counterpoize each other.

56. I shall now confider the cafe of pendulums.   A
pendulum is made by hanging a weight to a line, fo that it
may fwing backwards and forwards.   This motion the geo-
meters have very carefully confidered, becaufe it is the moft
commodious inftrument of any for the exact meafurement of
time.

57. I have obferved already [a], that if a body hanging
perpendicularly by a ftring, as the body A (in fig. 48.) hangs
by the ftring A B, be put fo into motion, as to be made to a-
fcend up the circular arch A C;   then as foon as it has arrived

[a] § 23.

at

at the higheft point, to which the motion, that the body has received, will carry it; it will immediately begin to defcend, and at A will receive again as great a degree of motion, as it had at firft. This motion therefore will carry the body up the arch A D, as high as it afcended before in the arch A C. Confequently in its return through the arch D A it will acquire again at A its original velocity, and advance a fecond time up the arch A C as high as at firft; by this means continuing without end its reciprocal motion. It is true indeed, that in fact every pendulum, which we can put in motion, will gradually leffen its fwing, and at length ftop, unlefs there be fome power conftantly applied to it, whereby its motion fhall be renewed; but this arifes from the refiftance, which the body meets with both from the air, and the ftring by which it is hung: for as the air will give fome obftruction to the progrefs of the body moving through it; fo alfo the ftring, whereon the body hangs, will be a farther impediment; for this ftring muft either flide on the pin, whereon it hangs, or it muft bend to the motion of the weight; in the firft there muft be fome degree of friction, and in the latter the ftring will make fome refiftance to its inflection. However, if all refiftance could be removed, the motion of a pendulum would be perpetual.

58. But to proceed, the firft property, I fhall take notice of in this motion, is, that the greater arch the pendulous body moves through, the greater time it takes up: though the length of time does not increafe in fo great a proportion as the arch. Thus if C D be a greater arch, and E F a leffer, where C A is equal to A D, and E A equal to A F; the body,

when

when it swings through the greater arch C D, shall take up in its swing from C to D a longer time than in swinging from E to F, when it moves only in that lesser arch ; or the time in which the body let fall from C will descend through the arch C A is greater than the time, in which it will descend through the arch E A, when let fall from E.   But the first of these times will not hold the same proportion to the latter, as the first arch C A bears to the other arch E A; which will appear thus.   Let C G and E H be two horizontal lines.   It has been remarked above [a], that the body in falling through the arch C A will acquire as great a velocity at the point A, as it would have gained by falling directly down through G A ; and in falling through the arch E A it will acquire in the point A only that velocity, which it would have got in falling through H A.   Therefore, when the body descends through the greater arch C A, it shall gain a greater velocity, than when it passes only through the lesser; so that this greater velocity will in some degree compensate the greater length of the arch.

59. The increase of velocity, which the body acquires in falling from a greater height, has such an effect, that, if straight lines be drawn from A to C and E, the body would fall through the longer straight line C A just in the same time, as through the shorter straight line E A.   This is demonstrated by the geometers, who prove, that if any circle, as ABCD (fig. 49.) be placed in a perpendicular situation; a body shall fall obliquely through every line, as A B drawn from the lowest point A in the circle to any other point in the circum-

[a] § 20.

ference

ference juft in the fame time, as would be imployed by the body in falling perpendicularly down through the diameter C A.   But the time in which the body will defcend through the arch, is different from the time, which it would take up in falling through the line A B.

60. I T has been thought by fome, that becaufe in very fmall arches this correfpondent ftraight line differs but little from the arch itfelf; therefore the defcent through this ftraight line would be performed in fuch fmall arches nearly in the fame time as through the arches themfelves : fo that if a pendulum were to fwing in fmall arches, half the time of a fingle fwing would be nearly equal to the time, in which a body would fall perpendicularly through twice the length of the pendulum.   That is, the whole time of the fwing, according to this opinion, will be four fold the time required for the body to fall through half the length of the pendulum ; becaufe the time of the body's falling down twice the length of the pendulum is half the time required for the fall through one quarter of this fpace, that is through half the pendulum's length.   However there is here a miftake ; for the whole time of the fwing, when the pendulum moves through fmall arches, bears to the time required for a body to fall down through half the length of the pendulum very nearly the fame proportion, as the circumference of a circle bears to its diameter ; that is very nearly the proportion of 355 to 113, or little more than the proportion of 3 to 1. If the pendulum takes fo great a fwing, as to pafs over an arch equal to one fixth part of the whole circumference of the

circle,

circle, it will fwing 115 times, while it ought according to this proportion to have fwung 117 times ; fo that, when it fwings in fo large an arch, it lofes fomething lefs than two fwings in an hundred. If it fwing through $\frac{1}{10}$ only of the circle, it fhall not lofe above one vibration in 160. If it fwing in $\frac{1}{20}$ of the circle, it fhall lofe about one vibration in 690. If its fwing be confined to $\frac{1}{40}$ of the whole circle, it fhall lofe very little more than one fwing in 2600. And if it take no greater a fwing than through $\frac{1}{60}$ of the whole circle, it fhall not lofe one fwing in 5800.

61. Now it follows from hence, that, when pendulums fwing in fmall arches, there is very nearly a conftant proportion obferved between the time of their fwing, and the time, in which a body would fall perpendicularly down through half their length. And we have declared above, that the fpaces, through which bodies fall, are in a two fold proportion of the times, which they take up in falling [a]. Therefore in pendulums of different lengths, fwinging throug hfmall arches, the lengths of the pendulums are in a two fold or duplicate proportion of the times, they take in fwinging ; fo that a pendulum of four times the length of another fhall take up twice the time in each fwing, one of nine times the length will make one fwing only for three fwings of the fhorter, and fo on.

62. This proportion in the fwings of different pendulums not only holds in fmall arches ; but in large ones alfo,

§ 17.

provided

provided they be fuch, as the geometers call fimilar ; that
is, if the arches bear the fame proportion to the whole cir-
cumferences of their refpective circles.   Suppofe (in fig. 48.)
A B, C D to be two pendulums.   Let the arch E F be defcrib-
ed by the motion of the pendulum A B, and the arch G H
be defcribed by the pendulum C D ; and let the arch E F bear
the fame proportion to the whole circumference, which
would be formed by turning the pendulum A B quite round
about the point A, as the arch G H bears to the whole cir-
cumference, that would be formed by turning the pendu-
lum C D quite round the point C.   Then I fay, the propor-
tion, which the length of the pendulum A B bears to the
length of the pendulum C D, will be two fold of the propor-
tion, which the time taken up in the defcription of the arch
E F bears to the time employed in the defcription of the arch
G H.

63. T<small>HUS</small> pendulums, which fwing in very fmall arches,
are nearly an equal meafure of time. But as they are not fuch
an equal meafure to geometrical exactnefs; the mathematicians
have found out a method of caufing a pendulum fo to fwing,
that, if its motion were not obftructed by any refiftance, it
would always perform each fwing in the fame time, whether
it moved through a greater, or a leffer fpace.   This was firft
difcovered by the great H<small>UYGENS</small>, and is as follows.   Up-
on the ftraight line A B (in fig. 49.) let the circle C D E be fo
placed, as to touch the ftraight line in the point C.   Then let
this circle roll along upon the ftraight line A B, as a coach-
wheel rolls along upon the ground.   It is evident, that, as

foon

foon as ever the circle begins to move, the point C in the circle will be lifted off from the ftraight line A B; and in the motion of the circle will defcribe a crooked courfe, which is reprefented by the line CFGH. Here the part CH of the ftraight line included between the two extremities C and H of the line CFGH will be equal to the whole circumference of the circle CDE; and if CH be divided into two equal parts at the point I, and the ftraight line IK be drawn perpendicular to CH, this line IK will be equal to the diameter of the circle CDE. Now in this line if a body were to be let fall from the point H, and were to be carried by its weight down the line HGK, as far as the point K, which is the loweft point of the line CFGH; and if from any other point G a body were to be let fall in the fame manner; this body, which falls from G, will take juft the fame time in coming to K, as the body takes up, which falls from H. Therefore if a pendulum can be fo hung, that the ball fhall move in the line AGFE, all its fwings, whether long or fhort, will be performed in the fame time; for the time, in which the ball will defcend to the point K, is always half the time of the whole fwing. But the ball of a pendulum will be made to fwing in this line by the following means. Let KI (in fig. 52.) be prolonged upwards to L, till IL is equal to IK. Then let the line LMH equal and like to KH be applied, as in the figure between the points L and H, fo that the point which in this line LMH anfwers to the point H in the line KH fhall be applied to the point L, and the point anfwering to the point K fhall be applied to the point H. Alfo let fuch another line LNC be applied between L and C in the fame

<div align="right">manner.</div>

manner.   This preparation being made; if a pendulum be hung at the point L of fuch a length, that the ball thereof fhall reach to K; and if the ftring fhall continually bend a-gainft the lines H M L and L N C, as the pendulum fwings to and fro; by this means the ball fhall conftantly keep in the line C K H.

64. Now in this pendulum, as all the fwings, whether long or fhort, will be performed in the fame time; fo the time of each will exactly bear the fame proportion to the time re-quired for a body to fall perpendicularly down, through half the length of the pendulum, that is from I to K, as the cir-cumference of a circle bears to its diameter.

65. It may from hence be underftood in fome meafure, why, when pendulums fwing in circular arches, the times of their fwings are nearly equal, if the arches are fmall, though thofe arches be of very unequal lengths ; for if with the fe-midiameter L K the circular arch O K P be defcribed, this arch in the lower part of it will differ very little from the line C K H.

66. It may not be amifs here to remark, that a body will fall in this line C K H (fig. 53.) from C to any other point, as Q or R in a fhorter fpace of time, than if it moved through the ftraight line drawn from C to the other point; or through any other line whatever, that can be drawn be-tween thefe two points.

67. But

67. But as I have obſerved, that the time, which a pendulum takes in ſwinging, depends upon its length ; I ſhall now ſay ſomething concerning the way, in which this length of the pendulum is to be eſtimated. If the whole ball of the pendulum could be crouded into one point, this length, by which the motion of the pendulum is to be computed, would be the length of the ſtring or rod. But the ball of the pendulum muſt have a ſenſible magnitude, and the ſeveral parts of this ball will not move with the ſame degree of ſwiftneſs ; for thoſe parts, which are fartheſt from the point, whereon the pendulum is ſuſpended, muſt move with the greateſt velocity. Therefore to know the time in which the pendulum ſwings, it is neceſſary to find that point of the ball, which moves with the ſame degree of velocity, as if the whole ball were to be contracted into that point.

68. This point is not the center of gravity, as I ſhall now endeavour to ſhew. Suppoſe the pendulum A B (in fig. 54.) compoſed of an inflexible rod A C and ball C B, to be fixed on the point A, and lifted up into an horizontal ſituation. Here if the rod were not fixed to the point A, the body C B would deſcend directly with the whole force of its weight ; and each part of the body would move down with the ſame degree of ſwiftneſs. But when the rod is fixed at the point A, the body muſt fall after another manner ; for the parts of the body muſt move with different degrees of velocity, the parts more remote from A deſcending with a ſwifter motion, than the parts nearer to A ; ſo that the body will receive a kind of rolling motion while it deſcends. But it has

been
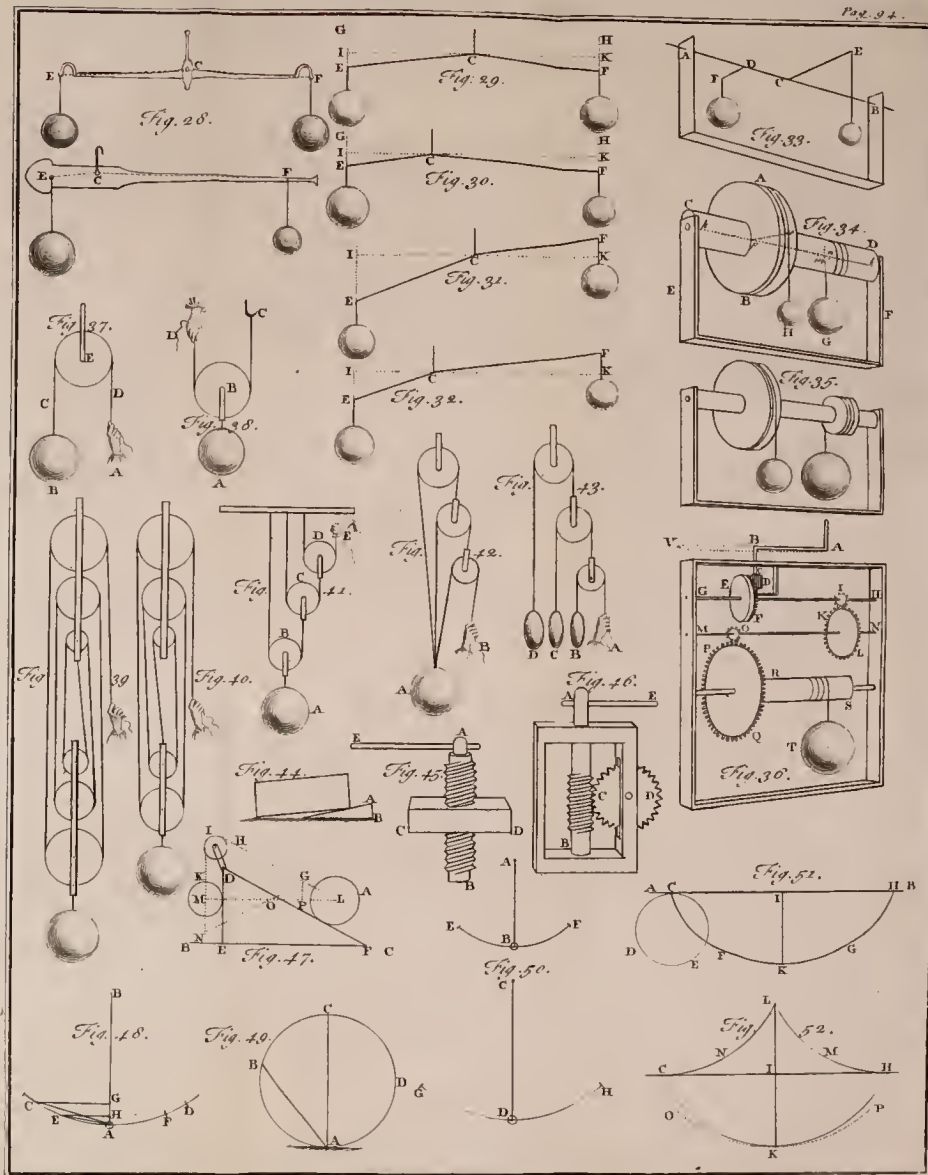
been obferved above, that the effect of gravity upon any bo-
dy is the fame, as if the whole force were exerted on the bo-
dy's center of gravity ᵃ.   Since therefore the power of gravity
in drawing down the body muft alfo communicate to it the
rolling motion juft defcribed ; it feems evident, that the cen-
ter of gravity of the body cannot be drawn down as fwiftly,
as when the power of gravity has no other effect to produce
on the body, than merely to draw it downward.   If there-
fore the whole matter of the body C B could be crouded into
its center of gravity, fo that being united into one point, this
rolling motion here mentioned might give no hindrance to
its defcent ; this center would defcend fafter, than it can now
do.   And the point, which now defcends as faft, as if the
whole matter of the body C B were crouded into it, will be
farther removed from the point A, than the center of gravity
of the body C B.

69. AGAIN, fuppofe the pendulum A B (in fig. 55.) to
hang obliquely.   Here the power of gravity will operate lefs
upon the ball of the pendulum, than before : but the line D E
being drawn fo, as to ftand perpendicular to the rod A C of
the pendulum ; the force of gravity upon the body CB,
now it is in this fituation, will produce the fame effect, as
if the body were to glide down an inclined plane in the po-
fition of D E.   But here the motion of the body, when the
rod is fixed to the point A, will not be equal to the uninter-
rupted defcent of the body down this plane ; for the body

§ 27.

will

will here alfo receive the fame kind of rotation in its motion, as before ; fo that the motion of the center of gravity will in like manner be retarded ; and the point, which here defcends with that degree of fwiftnefs, which the body would have, if not hindered by being fixed to the point A ; that is, the point, which defcends as faft, as if the whole body were crouded into it, will be as far removed from the point A, as before.

70. This point, by which the length of the pendulum is to be eftimated, is called the center of ofcillation. And the mathematicians have laid down general directions, whereby to find this center in all bodies. If the globe A B (in fig. 56.) be hung by the ftring C D, whofe weight need not be regarded, the center of ofcillation is found thus. Let the ftraight line drawn from C to D be continued through the globe to F. That it will pafs through the center of the globe is evident. Suppofe E to be this center of the globe ; and take the line G of fuch a length, that it fhall bear the fame proportion to E D, as E D bears to E C. Then E H being made equal to $\frac{2}{5}$ of G, the point H fhall be the center of ofcillation [a]. If the weight of the rod C D is too confiderable to be neglected, divide C D (fig. 57) in I, that D I be equal to $\frac{1}{3}$ part of C D; and take K in the fame proportion to C I, as the weight of the globe A B to the weight of the rod C D. Then having found H, the center of ofcillation of the globe, as before, divide I K in L, fo that I L fhall bear the fame pro-

[a] Hugen. Horolog. ofcillat. pag. 141, 142.

portion

portion to L H, as the line C H bears to K ; and L ſhall be the center of oſcillation of the whole pendulum.

71. Th is computation is made upon ſuppoſition, that the center of oſcillation of the rod C D, if that were to ſwing alone without any other weight annexed, would be the point I. And this point would be the true center of oſcillation, ſo far as the thickneſs of the rod is not to be regarded.   If any one chuſes to take into conſideration the thickneſs of the rod, he muſt place the center of oſcillation thereof ſo much below the point I, that eight times the diſtance of the center from the point I ſhall bear the ſame proportion to the thickneſs of the rod, as the thickneſs of the rod bears to its length C D [a].

72. It has been obſerved above, that when a pendulum ſwings in an arch of a circle, as here in fig. 58, the pendulum A B ſwings in the circular arch C D; if you draw an horizontal line, as E F, from the place whence the pendulum is let fall, to the line A G, which is perpendicular to the horizon: then the velocity, which the pendulum will acquire in coming to the point G, will be the ſame, as any body would acquire in falling directly down from F to G. Now this is to be underſtood of the circular arch, which is deſcribed by the center of oſcillation of the pendulum.   I ſhall here farther obſerve, that if the ſtraight line E G be drawn from the point, whence the pendulum falls, to the loweſt point of the arch ; in the ſame or in equal pendulums the velocity, which the

[a] See Hugen. Horolog. Oſcillat. p. 142.

O   pendulum

pendulum acquires in G, is proportional to this line : that is, if the pendulum, after it has defcended from E to G, be taken back to H, and let fall from thence, and the line H G be drawn ; the velocity, which the pendulum fhall acquire in G by its defcent from H, fhall bear the fame proportion to the velocity, which it acquires in falling from E to G, as the ftraight line H G bears to the ftraight line E G.

73. We may now proceed to thofe experiments upon the percuffion of bodies, which I obferved above might be made with pendulums.   This expedient for examining the effects of percuffion was firft propofed by our late great architect Sir Christopher Wren.   And it is as follows. Two balls, as A and B (in fig. 59.) either equal or unequal, are hung by two ftrings from two points C and D, fo that, when the balls hang down without motion, they fhall juft touch each other, and the ftrings be parallel.   Here if one of thefe balls be removed to any diftance from its perpendicular fituation, and then let fall to defcend and ftrike against the other; by the laft preceding paragraph it will be known, with what velocity this ball fhall return into its firft perpendicular fituation, and confequently with what force it fhall ftrike againft the other ball; and by the height to which this other ball afcends after the ftroke, the velocity communicated to this ball will be difcovered.   For inftance, let the ball A be taken up to E, and from thence be let fall to ftrike againft B, paffing over in its defcent the circular arch E F. By this impulfe let B fly up to G, moving through the circular arch H G.   Then E I and G K being drawn horizontally, the

the ball A will ftrike againft B with the velocity, which it would acquire in falling directly down from I ; and the ball B has received a velocity, wherewith, if it had been thrown directly upward, it would have afcended up to K.   Likewife if ftraight lines be drawn from E to F and from H to G, the velocity of A, wherewith it ftrikes, will bear the fame proportion to the velocity, which B has received by the blow, as the ftraight line E F bears to the ftraight line H G.   In the fame manner by noting the place to which A afcends after the ftroke, its remaining velocity may be compared with that, wherewith it ftruck againft B.   Thus may be experimented the effects of the body A ftriking againft B at reft.   If both the bodies are lifted up, and fo let fall as to meet and impinge againft each other juft upon the coming of both into their perpendicular fituation ;   by obferving the places into which they move after the ftroke, the effects of their percuffion in all thefe cafes may be found in the fame manner as before.

74. Sir Isaac Newton has defcribed thefe experiments; and has fhewn how to improve them to a greater exactnefs by making allowance for the refiftance, which the air gives to the motion of the balls [a].   But as this refiftance is exceeding fmall, and the manner of allowing for it is delivered by himfelf in very plain terms, I need not enlarge upon it here.   I fhall rather fpeak to a difcovery, which he made by thefe experiments upon the elafticity of bodies.   It has been explained above [b], that when two bodies ftrike, if they be not elaftic,

[a] Princip. Philof. pag. 22.   [b] Chap. 1. § 29.

O 2   they

they remain contiguous after the ſtroke; but that if they are elaſtic, they ſeparate, and that the degree of their elaſticity determines the proportion between the celerity wherewith they ſeparate, and the celerity wherewith they meet. Now our author found, that the degree of elaſticity appeared in the ſame bodies always the ſame, with whatever degree of force they ſtruck; that is, the celerity wherewith they ſeparated, always bore the ſame proportion to the celerity wherewith they met: ſo that the elaſtic power in all the bodies, he made trial upon, exerted it ſelf in one conſtant proportion to the compreſſing force. Our author made trial with balls of wool bound up very compact, and found the celerity with which they receded, to bear about the proportion of 5 to 9 to the celerity wherewith they met; and in ſteel he found nearly the ſame proportion; in cork the elaſticity was ſomething leſs; but in glaſs much greater; for the celerity, wherewith balls of that material ſeparated after percuſſion, he found to bear the proportion of 15 to 16 to the celerity wherewith they met [a].

75. I SHALL finiſh my diſcourſe on pendulums, with this farther obſervation only, that the center of oſcillation is alſo the center of another force. If a body be fixed to any point, and being put in motion turns round it; the body, if uninterrupted by the power of gravity or any other means, will continue perpetually to move about with the ſame equable motion. Now the force, with which ſuch a body

[a] Princip. Philoſ. pag. 25.

moves,

moves, is all united in the point, which in relation to the power of gravity is called the center of ofcillation. Let the cylinder A B C D (in fig. 60.) whofe axis is E F, be fixed to the point E. And fuppofing the point E to be that on which the cylinder is fufpended, let the center of ofcillation be found in the axis E F, as has been explained above [a]. Let G be that center: then I fay, that the force, wherewith this cylinder turns round the point E, is fo united in the point G, that a fufficient force applied in that point fhall ftop the motion of the cylinder, in fuch a manner, that the cylinder fhould immediately remain without motion, though it were to be loofened from the point E at the fame inftant, that the impediment was applied to G: whereas, if this impediment had been applied to any other point of the axis, the cylinder would turn upon the point, where the impediment was applied. If the impediment had been applied between E and G, the cylinder would fo turn on the point, where the impediment was applied, that the end B C would continue to move on the fame way it moved before along with the whole cylinder; but if the impediment were applied to the axis farther off from E than G, the end A D of the cylinder would ftart out of its prefent place that way in which the cylinder moved. From this property of the center of ofcillation, it is alfo called the center of percuffion. That excellent mathematician, Dr. Brook Taylor, has farther improved this doctrine concerning the center of percuffion, by fhewing, that if through this point G a line, as G H I, be drawn perpendicular to E F, and lying

[a] § 71.

in

in the courfe of the body's motion; a fufficient power appli-
ed to any point of this line will have the fame effect, as the
like power applied to G[a]: fo that as we before fhewed the
center of percuffion within the body on its axis; by this means
we may find this center on the furface of the body alfo, for
it will be where this line H I croffes that furface.

76. I shall now proceed to the laft kind of motion, to
be treated on in this place, and fhew what line the power of
gravity will caufe a body to defcribe, when it is thrown for-
wards by any force.   This was firft difcovered by the great
GALILEO, and is the principle, upon which engineers
fhould direct the fhot of great guns.   But as in this cafe bo-
dies defcribe in their motion one of thofe lines, which in geo-
metry are called conic fections; it is neceffary here to pre-
mife a defcription of thofe lines.   In which I fhall be the
more particular, becaufe the knowledge of them is not only
neceffary for the prefent purpofe, but will be alfo required
hereafter in fome of the principal parts of this treatife.

77. THE firft lines confidered by the ancient geometers
were the ftraight line and the circle.   Of thefe they compof-
ed various figures, of which they demonftrated many proper-
ties, and refolved divers problems concerning them.   Thefe
problems they attempted always to refolve by the defcribing
ftraight lines and circles.   For inftance, let a fquare A B C D
(fig. 6 I.) be propofed,   and let it be required to make ano-

[a] See Method. Increment. prop. 25.

ther

ther fquare in any affigned proportion to this. Prolong one fide, as D A, of this fquare to E, till A E bear the fame proportion to A D, as the new fquare is to bear to the fquare A C. If the oppofite fide B C of the fquare A C be alfo prolonged to F, till B F be equal to A E, and E F be afterwards drawn, I fuppofe my readers will eafily conceive, that the figure ABFE will bear to the fquare A B C D the fame proportion, as the line A E bears to the line A D. Therefore the figure A B F E will be equal to the new fquare, which is to be found, but is not it felf a fquare, becaufe the fide A E is not of the fame length with the fide E F. But to find a fquare equal to the figure A B F E you muft proceed thus. Divide the line D E into two equal parts in the point G, and to the center G with the interval G D defcribe the circle D H E I ; then prolong the line A B, till it meets the circle in K ; and make the fquare AKLM, which fquare will be equal to the figure ABFE, and bear to the fquare A B C D the fame proportion, as the line A E bears to A D.

78. I SHALL not proceed to the proof of this, having only here fet it down as a fpecimen of the method of refolving geometrical problems by the defcription of ftraight lines and circles. But there are fome problems, which cannot be refolved by drawing ftraight lines or circles upon a plane. For the management therefore of thefe they took into confideration folid figures, and of the folid figures they found that, which is called a cone, to be the moft ufeful.

79. A.

79. A cone is thus defined by Euclide in his elements of geometry [a].   If to the ftraight line A B (in fig. 62.) another ftraight line, as A C, be drawn perpendicular, and the two extremities B and C be joined by a third ftraight line compofing the triangle A C B (for fo every figure is called, which is included under three ftraight lines :) then the two points A and B being held fixed, as two centers, and the triangle A C B being turned round upon the line A B, as on an axis; the line A C will defcribe a circle, and the figure A C B will defcribe a cone, of the form reprefented by the figure B C D E F (fig. 63.) in which the circle C D E F is ufually called the bafe of the cone, and B the vertex.

80. Now by this figure may feveral problems be refolved, which cannot by the fimple defcription of ftraight lines and circles upon a plane.   Suppofe for inftance, it were required to make a cube, which fhould bear any affigned proportion to fome other cube named.   I need not here inform my readers, that a cube is the figure of a dye.   This problem was much celebrated among the ancients, and was once inforced by the command of an oracle.   This problem may be performed by a cone thus.   Firft make a cone from a triangle, whofe fide A C fhall be half the length of the fide B C. Then on the plane A B C D (fig. 64.) let the line E F be exhibited equal in length to the fide of the cube propofed; and let the line F G be drawn perpendicular to E F, and of fuch a length, that it bear the fame proportion to E F, as the

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[a] Lib. XI. Def.

cube

cube to be fought is required to bear to the cube propofed. Through the points E, F, and G let the circle F H I be defcribed. Then let the line E F be prolonged beyond F to K, that F K be equal to F E, and let the triangle F K L, having all its fides F K, K L, L F equal to each other, be hung down perpendicularly from the plane A B C D. After this, let another plane M N O P be extended through the point L, fo as to be equidiftant from the former plane A B C D, and in this plane let the line Q L R be drawn fo, as to be equidiftant from the line E F K. All this being thus prepared, let fuch a cone, as was above directed to be made, be fo applied to the plane M N O P, that it touch this plane upon the line Q R, and that the vertex of the cone be applied to the point L. This cone, by cutting through the firft plane A B C D, will crofs the circle F H I before defcribed. And if from the point S, where the furface of this cone interfects the circle, the line S T be drawn fo, as to be equidiftant from the line E F; the line F T will be equal to the fide of the cube fought : that is, if there be two cubes or dyes formed, the fide of one being equal to E F, and the fide of the other equal to F T; the former of thefe cubes fhall bear the fame proportion to the latter, as the line E F bears to F G.

81. INDEED this placing a cone to cut through a plane is not a practicable method of refolving problems. But when the geometers had difcovered this ufe of the cone, they applied themfelves to confider the nature of the lines, which will be produced by the interfection of the furface of a cone

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and a plane ; whereby they might be enabled both to reduce thefe kinds of folutions to practice, and alfo to render their demonftrations concife and elegant.

82. Whenever the plane, which cuts the cone, is equi-diftant from another plane, that touches the cone on the fide; (which is the cafe of the prefent figure;) the line, wherein the plane cuts the furface of the cone, is called a parabola. But if the plane, which cuts the cone, be fo inclined to this other, that it will pafs quite through the cone (as in fig. 65.) fuch a plane by cutting the cone produces the figure called an ellipfis, in which we fhall hereafter fhew the earth and other planets to move round the fun. If the plane, which cuts the cone, recline the other way (as in fig. 66.) fo as not to be parallel to any plane, whereon the cone can lie, nor yet to cut quite through the cone; fuch a plane fhall produce in the cone a third kind of line, which is called an hyperbola. But it is the firft of thefe lines named the parabola, wherein bodies, that are thrown obliquely, will be carried by the force of gravity; as I fhall here proceed to fhew, after having firft directed my readers how to defcribe this fort of line upon a plane, by which the form of it may be feen.

83. To any ftraight line A B (fig. 67.) let a ftraight ruler C D be fo applied, as to ftand againft it perpendicularly. Upon the edge of this ruler let another ruler E F be fo placed, as to move along upon the edge of the firft ruler C D, and keep al-ways perpendicular to it. This being fo difpofed, let any point, as G, be taken in the line A B, and let a ftring equal

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in length to the ruler E F be faftened by one end to the point G, and by the other to the extremity F of the ruler E F. Then if the ftring be held down to the ruler E F by a pin H, as is reprefented in the figure ; the point of this pin, while the ruler E F moves on the ruler C D, fhall defcribe the line I K L, which will be one part of the curve line, whofe defcription we were here to teach : and by applying the rulers in the like manner on the other fide of the line A B, we may defcribe the other part I M of this line.   If the diftance C G be equal to half the line E F in fig. 64, the line M I L will be that very line, wherein the plane A B C D in that figure cuts the cone.

84. T H E line A I is called the axis of the parabola M I L, and the point G is called the focus.

85. N o w by comparing the effects of gravity upon falling bodies, with what is demonftrated of this figure by the geometers, it is proved, that every body thrown obliquely is carried forward in one of thefe lines, the axis whereof is perpendicular to the horizon.

86. T H E geometers demonftrate, that if a line be drawn to touch a parabola in any point, as the line A B (in fig. 68.) touches the parabola C D, whofe axis is Y Z, in the point E ; and feveral lines F G, H I, K L be drawn parallel to the axis of the parabola : then the line F G will be to H I in the duplicate proportion of E F to E H, and F G to K L in the duplicate proportion of E F to E K ; likewife H I to K L in the duplicate proportion of E H to E K.   What is to be underftood by duplicate or two-fold

pro-

proportion, has been already explained [a].   Accordingly I mean here, that if the line M be taken to bear the same proportion to E H, as E H bears to E F,   H I will bear the same proportion to F G, as M bears to E F ; and if the line N bears the same proportion to E K, as E K bears to E F, K L will bear the same proportion to F G, as N bears to E F ; or if the line O bear the same proportion to E K, as E K bears to E H, K L will bear the same proportion to H I, as O bears to E H.

87. This property is essential to the parabola, being so connected with the nature of the figure, that every line possessing this property is to be called by this name.

88. Now suppose a body to be thrown from the point A (in fig. 69.) towards B in the direction of the line A B. This body, if left to it self, would move on with a uniform motion through this line A B.   Suppose the eye of a spectator to be placed at the point C just under the point A ;   and let us imagine the earth to be so put into motion along with the body, as to carry the spectator's eye along the line C D parallel to A B; and that the eye should move on with the same velocity, wherewith the body would proceed in the line A B, if it were to be left to move without any disturbance from its gravitation towards the earth.   In this case if the body moved on without being drawn towards the earth, it would appear to the spectator to be at rest.   But if the power of gravity exerted it self on the body, it would appear to the spe-

[a] Chap. 2. § 17.

ctator

ctator to fall directly down.   Suppose at the distance of time, wherein the body by its own progressive motion would have moved from A to E, it should appear to the spectator to have fallen through a length equal to E F : then the body at the end of this time will actually have arrived at the point F. If in the space of time, wherein the body would have moved by its progressive motion from A to G, it would have appeared to the spectator to have fallen down the space G H : then the body at the end of this greater interval of time will be arrived at the point H.   Now if the line A F H I be that, through which the body actually passes ;   from what has here been said, it will follow, that this line is one of those, which I have been describing under the name of the parabola.   For the distances E F, G H, through which the body is seen to fall, will increase in the duplicate proportion of the times [a] ;   but the lines A E, A G will be proportional to the times wherein they would have been described by the single progressive motion of the body :   therefore the lines E F, G H will be in the duplicate proportion of the lines A F, A G; and the line A F H I possesses the property of the parabola.

89. If the earth be not supposed to move along with the body, the case will be a little different.   For the body being constantly drawn directly towards the center of the earth, the body in its motion will be drawn in a direction a little oblique to that, wherein it would be drawn by the earth in motion, as before supposed.   But the distance to the center of the

[a] See above Ch. 2. § 17.

earth

earth bears so vast a proportion to the greatest length, to which we can throw bodies, that this obliquity does not merit any regard. From the sequel of this discourse it may indeed be collected, what line the body being thrown thus would be found to describe, allowance being made for this obliquity of the earth's action[a]. This is the discovery of Sir Is. Newton; but has no use in this place. Here it is abundantly sufficient to consider the body as moving in a parabola.

90. The line, which a projected body describes, being thus known, practical methods have been deduced from hence for directing the shot of great guns to strike any object desired. This work was first attempted by Galileo, and soon after farther improved by his scholar Torricelli; but has lately been rendred more complete by the great Mr. Cotes, whose immature death is an unspeakable loss to mathematical learning. If it be required to throw a body from the point A (in fig. 70.) so as to strike the point B; through the points A, B draw the straight line CD, and erect the line AE perpendicular to the horizon, and of four times the height, from which a body must fall to acquire the velocity, wherewith the body is intended to be thrown. Through the points A and E describe a circle, that shall touch the line CD in the point A. Then from the point B draw the line BF perpendicular to the horizon, interfecting the circle in the points G and H. This being done, if the body be projected directly towards either of these points G or H, it shall fall upon the point B; but with this difference, that, if it be thrown

[a] From B. II. Ch. 3.

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in the direction A G, it fhall fooner arrive at B, than if it were projected in the direction A H. When the body is projected in the direction A G; the time, it will take up in arriving at B, will bear the fame proportion to the time, wherein it would fall down through one fourth part of A E, as A G bears to half A E. But when the body is thrown in the direction of A H, the time of its paffing to B will bear the fame proportion to the time, wherein it would fall through one fourth part of A E, as A H bears to half A E.

91. If the line A I be drawn fo as to divide the angle under E A D in the middle, and the line I K be drawn perpendicular to the horizon; this line will touch the circle in the point I, and if the body be thrown in the direction A I, it will fall upon the point K: and this point K is the fartheft point in the line A D, which the body can be made to ftrike, without increafing its velocity.

92. THE velocity, wherewith the body every where moves, may be found thus. Suppofe the body to move in the parabola A B (fig. 71.) Erect A C perpendicular to the horizon, and equal to the height, from which a body muft fall to acquire the velocity, wherewith the body fets out from A. If you take any points as D and E in the parabola, and draw D F and E G parallel to the horizon; the velocity of the body in D will be equal to what a body will acquire in falling down by its own weight through C F, and in E the velocity will be the fame, as would be acquired in falling through C G. Thus the body moves floweft at the higheft point H of the parabola; and at equal diftances from this point will

move

move with equal fwiftnefs, and defcend from that higheft point through the line H B altogether like to the line A H in which it afcended; abating only the refiftance of the air, which is not here confidered.   If the line H I be drawn from the higheft point H parallel to the horizon, A I will be equal to ¼ of B G in fig. 70, when the body is projected in the direction A G, and equal to ¼ of B H, when the body is thrown in the direction A H provided A D be drawn horizontally.

93. Thus I have recounted the principal difcoveries, which had been made concerning the motion of bodies by Sir Isaac Newton's predeceffors; all thefe difcoveries, by being found to agree with experience, contributing to eftablifh the laws of motion, from whence they were deduced. I fhall therefore here finifh what I had to fay upon thofe laws; and conclude this chapter with a few words concerning the diftinction which ought to be made between abfolute and relative motion.   For fome have thought fit to confound them together; becaufe they obferve the laws of motion to take place here on the earth, which is in motion, after the fame manner as if it were at reft.   But Sir Isaac Newton has been careful to diftinguifh between the relative and abfolute confideration both of motion and time[a].   The aftronomers anciently found it neceffary to make this diftinction in time. Time confidered in it felf paffes on equably without relation to any thing external, being the proper meafure of the continuance and duration of all things.   But it is moft frequently conceived of by us under a relative view to fome fucceffion in

[a] Prin. Philof. pag. 7, &c.

fenfible

sensible things, of which we take cognizance. The succession of the thoughts in our own minds is that, from whence we receive our first idea of time, but is a very uncertain measure thereof; for the thoughts of some men flow on much more swiftly, than the thoughts of others; nor does the same person think equally quick at all times. The motions of the heavenly bodies are more regular; and the eminent division of time into night and day, made by the sun, leads us to measure our time by the motion of that luminary: nor do we in the affairs of life concern our selves with any inequality, which there may be in that motion; but the space of time which comprehends a day and night is rather supposed to be always the same. However astronomers anciently found these spaces of time not to be always of the same length, and have taught how to compute their differences. Now the time, when so equated as to be rendered perfectly equal, is the true measure of duration, the other not. And therefore this latter, which is absolutely true time, differs from the other, which is only apparent. And as we ordinarily make no distinction between apparent time, as measured by the sun, and the true; so we often do not distinguish in our usual discourse between the real, and the apparent or relative motion of bodies; but use the same words for one, as we should for the other. Though all things about us are really in motion with the earth; as this motion is not visible, we speak of the motion of every thing we see, as if our selves and the earth stood still. And even in other cases, where we discern the motion of bodies, we often speak of them not in relation to the whole motion we see, but with regard to other

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bodies

bodies, to which they are contiguous. If any body were lying on a table ; when that table fhall be carried along, we fay the body refts upon the table, or perhaps abfolutely, that the body is at reft. However philofophers muft not reject all diftinction between true and apparent motions, any more than aftronomers do the diftinction between true and vulgar time ; for there is as real a difference between them, as will appear by the following confideration. Suppofe all the bodies of the univerfe to have their courfes ftopped, and reduced to perfect reft. Then fuppofe their prefent motions to be again reftored ; this cannot be done without an actual impreffion made upon fome of them at leaft. If any of them be left untouched, they will retain their former ftate, that is, ftill remain at reft ; but the other bodies, which are wrought upon, will have changed their former ftate of reft, for the contrary ftate of motion. Let us now fuppofe the bodies left at reft to be annihilated, this will make no alteration in the ftate of the moving bodies ; but the effect of the impreffion, which was made upon them, will ftill fubfift. This fhews the motion they received to be an abfolute thing, and to have no neceffary dependence upon the relation which the body faid to be in motion has to any other body [a].

94. Besides abfolute and relative motion are diftinguifhable by their Effects. One effect of motion is, that bodies, when moved round any center or axis, acquire a certain.

[a] See Newton. princip. philof. pag. 9. lin. 30.

power,

power, by which they forcibly press themselves from that center or axis of motion. As when a body is whirled about in a sling, the body presses against the sling, and is ready to fly out as soon as liberty is given it. And this power is proportional to the true, not relative motion of the body round such a center or axis. Of this Sir Isaac Newton gives the following instance [a]. If a pail or such like vessel near full of water be suspended by a string of sufficient length, and be turned about till the string be hard twisted. If then as soon as the vessel and water in it are become still and at rest, the vessel be nimbly turned about the contrary way the string was twisted, the vessel by the strings untwisting it self shall continue its motion a long time. And when the vessel first begins to turn, the water in it shall receive little or nothing of the motion of the vessel, but by degrees shall receive a communication of motion, till at last it shall move round as swiftly as the vessel it self. Now the definition of motion, which Des Cartes has given us upon this principle of making all motion meerly relative, is this: that motion, is a removal of any body from its vicinity to other bodies, which were in immediate contact with it, and are considered as at rest [b]. And if this be compared with what he soon after says, that there is nothing real or positive in the body moved, for the sake of which we ascribe motion to it, which is not to be found as well in the contiguous bodies, which are considered as at rest [c]; it will follow from thence, that we may consider the vessel as at rest

[a] Princip. Philof. pag. 10.
[b] Renat. Des Cart. Princ. Philof. Part. II. § 25.
[c] Ibid. § 30.

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and

and the water as moving in it : and the water in refpect of the veffel has the greateft motion, when the veffel firft begins to turn, and lofes this relative motion more and more, till at length it quite ceafes. But now, when the veffel firft begins to turn, the furface of the water remains fmooth and flat, as before the veffel began to move; but as the motion of the veffel communicates by degrees motion to the water, the furface of the water will be obferved to change, the water fubfiding in the middle and rifing at the edges: which elevation of the water is caufed by the parts of it preffing from the axis, they move about; and therefore this force of receding from the axis of motion depends not upon the relative motion of the water within the veffel, but on its abfolute motion ; for it is leaft, when that relative motion is greateft, and greateft, when that relative motion is leaft, or none at all.

95. Thus the true caufe of what appears in the furface of this water cannot be affigned, without confidering the water's motion within the veffel. So alfo in the fyftem of the world, in order to find out the caufe of the planetary motions, we muft know more of the real motions, which belong to each planet, than is abfolutely neceffary for the ufes of aftronomy. If the aftronomer fhould fuppofe the earth to ftand ftill, he could afcribe fuch motions to the celeftial bodies, as fhould anfwer all the appearances; though he would not account for them in fo fimple a manner, as by attributing motion to the earth. But the motion of the earth muft of neceffity be confidered, before the real caufes, which actuate the planetary fyftem, can be difcovered.

Chap.

Chap. III.

Of CENTRIPETAL FORCES.

WE have juft been defcribing in the preceding chapter the effects produced on a body in motion, from its being continually acted upon by a power always equal in ftrength, and operating in parallel directions [a]. But bodies may be acted upon by powers, which in different places fhall have different degrees of force, and whofe feveral directions fhall be varioufly inclined to each other. The moft fimple of thefe in refpect to direction is, when the power is pointed conftantly to one center. This is truly the cafe of that power, whofe effects we defcribed in the foregoing chapter; though the center of that power is fo far removed, that the fubject then before us is moft conveniently to be confidered in the light, wherein we have placed it: But Sir ISAAC NEWTON has confidered very particularly this other cafe of powers, which are conftantly directed to the fame center. It is upon this foundation, that all his difcoveries in the fyftem of the world are raifed. And therefore, as this fubject bears fo very great a fhare in the philofophy, of which I am difcourfing, I think it proper in this place to take a fhort view of fome of the general effects of thefe powers, before we come to apply them particularly to the fyftem of the world.

[a] § 85, &c.

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2. These powers or forces are by Sir Isaac Newton called centripetal; and their firſt effect is to cauſe the body, on which they act, to quit the ſtraight courſe, wherein it would proceed if undiſturbed, and to deſcribe an incurvated line, which ſhall always be bent towards the center of the force. It is not neceſſary, that ſuch a power ſhould cauſe the body to approach that center. The body may continue to recede from the center of the power, notwithſtanding its being drawn by the power; but this property muſt always belong to its motion, that the line, in which it moves, will continually be concave towards the center, to which the power is directed. Suppoſe A (in fig. 72.) to be the center of a force. Let a body in B be moving in the direction of the ſtraight line B C, in which line it would continue to move, if undiſturbed; but being attracted by the centripetal force towards A, the body muſt neceſſarily depart from this line B C, and being drawn into the curve line B D, muſt paſs between the lines A B and B C. It is evident therefore, that the body in B being gradually turned off from the ſtraight line B C, it will at firſt be convex toward the line B C, and conſequently concave towards the point A: for theſe centripetal powers are ſuppoſed to be in ſtrength proportional to the power of gravity, and, like that, not to be able after the manner of an impulſe to turn the body ſenſibly out of its courſe into a different one in an inſtant, but to take up ſome ſpace of time in producing a viſible effect. That the curve will always continue to have its concavity towards A may thus appear. In the line B C near to B take any point as E, from which the line E F G may be ſo

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drawn, as to touch the curve line B D in some point as F. Now when the body is come to F, if the centripetal power were immediately to be suspended, the body would no longer continue to move in a curve line, but being left to it self would forthwith reassume a straight course; and that straight course would be in the line F G : for that line is in the direction of the body's motion at the point F.   But the centripetal force continuing its energy, the body will be gradually drawn from this line F G so as to keep in the line F D, and make that line near the point F to be convex toward F G, and concave toward A.   After the same manner the body may be followed on in its course through the line B D, and every part of that line be shewn to be concave toward the point A.

3. This then is the constant character belonging to those motions, which are carried on by centripetal forces; that the line, wherein the body moves, is throughout concave towards the center of the force.   In respect to the successive distances of the body from the center there is no general rule to be laid down ;   for the distance of the body from the center may either increase, or decrease, or even keep always the same. The point A (in fig. 73.) being the center of a centripetal force, let a body at B set out in the direction of the straight line B C perpendicular to the line A B drawn from A to B.   It will be easily conceived, that there is no other point in the line B C so near to A, as the point B; that A B is the shortest of all the lines, which can be drawn from A to any part of the line B C; all other lines, as A D,. or A E, drawn from A to the line B C being longer than A B.   Hence it follows, that the body set-

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ting out from B, if it moved in the line B C, it would recede more and more from the point A.   Now as the operation of a centripetal force is to draw a body towards the center of the force : if such a force act upon a resting body, it must necessarily put that body so into motion, as to cause it to move towards the center of the force : if the body were of it self moving towards that center, the centripetal force would accelerate that motion, and cause it to move faster down: but if the body were in such a motion, as being left to itself it would recede from this center, it is not necessary, that the action of a centripetal power upon it should immediately compel the body to approach the center, from which it would otherwise have receded ; the centripetal power is not without effect, if it cause the body to recede more slowly from that center, than otherwise it would have done.   Thus in the case before us, the smallest centripetal power, if it act on the body, will force it out of the line B C, and cause it to pass in a bent line between B C and the point A, as has been before explained.   When the body, for instance, has advanced to the line A D, the effect of the centripetal force discovers it self by having removed the body out of the line B C, and brought it to cross the line A D somewhere between A and D: suppose at F.   Now A D being longer than A B, A F may also be longer than A B.   The centripetal power may indeed be so strong, that A F shall be shorter than A B ; or it may be so evenly balanced with the progressive motion of the body, that A F and A B shall be just equal : and in this last case, when the centripetal force is of that strength, as constantly to draw the body as much toward

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the center, as the progreſſive motion would carry it off, the body will deſcribe a circle about the center A, this center of the force being alſo the center of the circle.

4. I F the body, inſtead of ſetting out in the line B C perpendicular to A B, had ſet out in another line B G more inclined towards the line A B, moving in the curve line B H; then as the body, if it were to continue its motion in the line B G, would for ſome time approach the center A; the centripetal force would cauſe it to make greater advances toward that center. But if the body were to ſet out in the line B I reclined the other way from the perpendicular B C, and were to be drawn by the centripetal force into the curve line B K; the body, notwithſtanding any centripetal force, would for ſome time recede from the center; ſince ſome part at leaſt of the curve line B K lies between the line B I and the perpendicular BC.

5. T H U S far we have explained ſuch effects, as attend every centripetal force. But as theſe forces may be very different in regard to the different degrees of ſtrength, wherewith they act upon bodies in different places; I ſhall now proceed to make mention in general of ſome of the differences attending theſe centripetal motions.

6. T o reaſſume the conſideration of the laſt mentioned caſe. Suppoſe a centripetal power directed toward the point A (in fig. 74.) to act on a body in B, which is moving in the direction of the ſtraight line B C, the line B C reclining off from A B. If from A the ſtraight lines A D, A E, A F are

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drawn at pleafure to the line C B; the line C B being prolong-
ed beyond B to G, it appears that A D is inclined to the line
G C more obliquely, than A B is inclined to it, A E is inclin-
ed more obliquely than A D, and A F more than A E. To
fpeak more correctly, the angle under A D G is lefs than that
under A B G, the angle under A E G lefs than that under
A D G, and the angle under A F G lefs than that under A E G.
Now fuppofe the body to move in the curve line B H I K.
Then it is here likewife evident, that the line B H I K be
ing concave towards A, and convex towards the line B C,
it is more and more turned off from the line B C; fo
that in the point H the line A H will be lefs obliquely inclin-
ed to the curve line B H I K, than the fame line A H D is inclin-
ed to B C at the point D ; at the point I the inclination of the
line A I to the curve line will be more different from the in-
clination of the fame line A I E to the line B C, at the point E;
and in the points K and F the difference of inclination will be
ftill greater ; and in both the inclination at the curve will be
lefs oblique, than at the ftraight line B C. But the ftraight
line A B is lefs obliquely inclined to B G, than A D is inclined
towards D G: therefore although the line A H be lefs oblique-
ly inclined towards the curve H B, than the fame line A H D is
inclined towards D G ; yet it is poffible, that the inclination
at H may be more oblique, than the inclination at B. The in-
clination at H may indeed be lefs oblique than the other, or
they may be both the fame. This depends upon the degree
of ftrength, wherewith the centripetal force exerts it felf,
during the paffage of the body from B to H. After the fame
manner the inclinations at I and K depend entirely on the de-
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gree of ftrength, wherewith the centripetal force acts on the body in its paffage from H to K: if the centripetal force be weak enough, the lines A H and A I drawn from the center A to the body at H and at I fhall be more obliquely inclined to the curve, than the line A B is inclined towards B G. The centripetal force may be of that ftrength as to render all thefe inclinations equal, or if ftronger, the inclinations at I and K will be lefs oblique than at B. Sir Isaac Newton has particularly fhewn, that if the centripetal power decreafes after a certain manner with the increafe of diftance, a body may defcribe fuch a curve line, that all the lines drawn from the center to the body fhall be equally inclined to that curve line [a]. But I do not here enter into any particulars, my prefent intention being only to fhew, that it is poffible for a body to be acted upon by a force continually drawing it down towards a center, and yet that the body fhall continue to recede from that center; for here as long as the lines A H, A I, &c drawn from the center A to the body do not become lefs oblique to the curve, in which the body moves; fo long fhall thofe lines perpetually increafe, and confequently the body fhall more and more recede from the center.

7. But we may obferve farther, that if the centripetal power, while the body increafes its diftance from the center, retain fufficient ftrength to make the lines drawn from the center to the body to become at length lefs oblique to the curve; then if this diminution of the obliquity continue, till

[a] Princip. Philof. Lib. I. prop. 9.

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at laſt the line drawn from the center to the body ſhall ceaſe to be obliquely inclined to the curve, and ſhall become perpendicular thereto; from this inſtant the body ſhall no longer recede from the center, but in its following motion it ſhall again deſcend, and ſhall deſcribe a curve line in all reſpects like to that, which it has deſcribed already; provided the centripetal power, every where at the ſame diſtance from the center, acts with the ſame ſtrength. So we obſerved in the preceding chapter, that, when the motion of a projectile became parallel to the horizon, the projectile no longer aſcended, but forthwith directed its courſe downwards, deſcending in a line altogether like that, wherein it had before aſcended [a].

8. This return of the body may be proved by the following propoſition: that if the body in any place, ſuppoſe at I, were to be ſtopt, and be thrown directly backward with the velocity, wherewith it was moving forward in that point I; then the body, by the action of the centripetal force upon it, would move back again over the path I H B, in which it had before advanced forward, and would arrive again at the point B in the ſame ſpace of time, as was taken up in its paſſage from B to I; the velocity of the body at its return to the point B being the ſame, as that wherewith it firſt ſet out from that point. To give a full demonſtration of this propoſition, would require that uſe of mathematics, which I here purpoſe to avoid; but, I believe, it will appear in great meaſure evident from the following conſiderations..

[a] § 92.

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9. Suppose (in fig. 75.) that a body were carried after the following manner through the bent figure A B C D E F, compofed of the ftraight lines AB, BC, CD, DE, EF.   Firft let it be moving in the line A B, from A towards B, with any uniform velocity.   At B let the body receive an impulfe directed toward fome point, as G, taken within the concavity of the figure.   Now whereas this body, when once moving in the ftraight line A B, will continue to move on in this line, fo long as it fhall be left to it felf; but being difturbed at the point B in its motion by the impulfe, which there acts upon it, it will be turned out of this line A B into fome other ftraight line, wherein it will afterwards continue to move, as long as it fhall be left to itfelf.   Therefore let this impulfe have ftrength fufficient to turn the body into the line B C.   Then let the body move on undifturbed from B to C, but at C let it receive another impulfe pointed toward the fame point G, and of fufficient ftrength to turn the body into the line C D.   At D let a third impulfe, directed like the reft to the point G, turn the body into the line D E. And at E let another impulfe, directed likewife to the point G, turn the body into the line E F. Now, I fay, if the body while moving in the line E F be ftopt, and turned back again in this line with the fame velocity, as that wherewith it was moving forward in this line; then by the repetition of the former impulfe at E the body will be turned into the line E D, and move in it from E to D with the fame velocity as before it moved with from D to E ; by the repetition of the impulfe at D, when the body fhall have returned to that point, it will be turned into the line D C ; and by the repetition of the other impulfes at C and B

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the body will be brought back again into the line B A, with the velocity, wherewith it firſt moved in that line.

10. This I prove as follows. Let D E and F E be continued beyond E. In D E thus continued take at pleaſure the length E H, and let H I be ſo drawn, as to be equidiſtant from the line G E. Then, by what has been written upon the ſecond law of motion [a], it follows, that after the impulſe on the body in E it will move through E I in the ſame time, as it would have imployed in moving from E to H, with the velocity which it had in the line D E. In F E prolonged take E K equal to E I, and draw K L equidiſtant from G E. Then, becauſe the body is thrown back in the line F E with the ſame velocity as that wherewith it went forward in that line; if, when the body was returned to E, it were permitted to go ſtraight on, it would paſs through E K in the ſame time, as it took up in paſſing through E I, when it went forward in the line E F. But, if at the body's return to the point E, ſuch an impulſe directed toward the point D were to be given it, whereby it ſhould be turned into the line D E; I ſay, that the impulſe neceſſary to produce this effect muſt be equal to that, which turned the body out of the line D E into E F; and that the velocity, with which the body will return into the line E D, is the ſame, as that wherewith it before moved through this line from D to E. Becauſe E K is equal to E I, and K L and H I, being each equidiſtant from G E, are by conſequence equidiſtant from each other; it follows, that the two

[a] Ch. II. § 22.

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triangular figures I E H and K E L are altogether like and equal to each other.   If I were writing to mathematicians, I might refer them to some propofitions in the elements of EUCLID for the proof of this [a] : but as I do not here addrefs my felf to fuch, fo I think this affertion will be evident enough without a proof in form ;   at leaft I muft defire my readers to receive it as a propofition true in geometry.   But thefe two triangular figures being altogether like each other and equal; as E K is equal to E I,   fo E L is equal to E H,   and K L equal to H I. Now the body after its return to E being turned out of the line F E into E D by an impulfe acting upon it in E, after the manner above expreffed;   the body will receive fuch a velocity by this impulfe, as will carry it through E L in the fame time, as it would have imployed in paffing through E K,   if it had gone on in that line undifturbed.   And it has already been obferved,   that the time, in which the body would pafs over E K with the velocity wherewith it returns, is equal to the time it took up in going forward from E to I;   that is, equal to the time, in which it would have gone through E H with the velocity, wherewith it moved from D to E.   Therefore the time, in which the body will pafs through E L after its return into the line E D, is the fame, as would have been taken up by the body in paffing through E H with the velocity,   wherewith the body firft moved in the line D E.   Since therefore E L and E H are equal, the body returns into the line D E with the velocity, which it had before in that line.   Again I fay, the fecond impulfe in E is equal to the firft.   By what has

[a] Viz. L. I. prop. 30, 29, & 26.

been

been said on the second law of motion concerning the effect of oblique impulses[a], it will be understood, that the impulse in E, whereby the body was turned out of the line D E into the line E F, is of such strength, that if the body had been at rest, when this impulse had acted upon it, this impulse would have communicated so much motion to the body, as would have carried it through a length equal to H I, in the time wherein the body would have passed from E to H, or in the time wherein it passed from E to I. In the same manner, on the return of the body, the impulse in E, whereby the body is turned out of the line F E into E D, is of such strength, that if it had acted on the body at rest, it would have caused the body to move through a length equal to K L, in the same time, as the body would imploy in passing through E K with the velocity, wherewith it returns in the line F E. Therefore the second impulse, had it acted on the body at rest, would have caused it to move through a length equal to K L in the same space of time, as would be taken up by the body in passing through a length equal to H I, were the first impulse to act on the body when at rest. That is, the effects of the first and second impulse on the body when at rest would be the same; for K L and H I are equal : consequently the second impulse is equal to the first.

11. Thus if the body be returned through F E with the velocity, wherewith it moved forward ; we have shewn how by the repetition of the impulse, which acted on it at E, the

* Ch. II. § 21, 22.

body

body will return again into the line D E with the velocity, which it had before in that line.   By the fame procefs of rea-foning it may be proved, that, when the body is returned back to D, the impulfe, which before acted on the body at that point, will throw the body into the line D C with the ve-locity, which it firft had in that line; and the other impulfes being fuccefively repeated, the body will at length be brought back again into the line B A with the velocity, wherewith it fet out in that line.

12. T H U S thefe impulfes, by acting over again in an invert-ed order all their operation on the body, bring it back again through the path, in which it had proceeded forward.   And this obtains equally, whatever be the number of the ftraight lines, whereof this curve figure is compofed.   Now by a me-thod of reafoning, which Sir I s a a c N e w t o n makes great ufe of, and which he introduced into geometry, thereby greatly inriching that fcience.ᵃ; we might make a tranfition from this figure compofed of a number of ftraight lines to a figure of one continued curvature, and from a number of fe-parate impulfes repeated at diftinct intervals to a continual centripetal force, and fhew, that, becaufe what has been here advanced holds univerfally true, whatever be the num-ber of ftraight lines, whereof the curve figure A C F is com-pofed, and howfoever frequently the impulfes at the angles of this figure are repeated; therefore the fame will ftill remain true, although this figure fhould be converted into one of a continued curvature, and thefe diftinct impulfes fhould be

ᵃ viz. His doctrine of prime and ultimate ratios.

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changed into a continual centripetal force. But as the explaining this method of reasoning is foreign to my present design; so I hope my readers, after what has been said, will find no difficulty in receiving the proposition laid down above: that, if the body, which has moved through the curve line B H I (in fig. 74.) from B to I, when it is come to I, be thrown directly back with the same velocity as that, wherewith it proceeded forward, the centripetal force, by acting over again all its operation on the body, shall bring the body back again in the line I H B: and as the motion of the body in its course from B to I was every where in such a manner oblique to the line drawn from the center to the body, that the centripetal power acted in some degree against the body's motion, and gradually diminished it; so in the return of the body, the centripetal power will every where draw the body forward, and accelerate its motion by the same degrees, as before it retarded it.

13. This being agreed, suppose the body in K to have the line A K no longer obliquely inclined to its motion. In this case, if the body be turned back, in the manner we have been considering, it must be directed back perpendicularly to A K. But if it had proceeded forward, it would likewise have moved in a direction perpendicular to A K ; consequently, whether it move from this point K backward or forward, it must describe the same kind of course. Therefore since by being turned back it will go over again the line K I H B; if it be permitted to go forward, the line K L, which it shall describe, will be altogether similar to the line K H B.

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14. In like manner we may determine the nature of the motion, if the line, wherein the body sets out, be inclined (as in fig. 76.) down toward the line B A drawn between the body and the center. If the centripetal power so much increases in strength, as the body approaches, that it can bend the path, in which the body moves, to that degree, as to cause all the lines as A H, A I, A K to remain no less oblique to the motion of the body, than A B is oblique to B C; the body shall continually more and more approach the center. But if the centripetal power increases in so much less a degree, as to permit the line drawn from the center to the body, as it accompanies the body in its motion, at length to become more and more erect to the curve wherein the body moves, and in the end, suppose at K, to become perpendicular thereto; from that time the body shall rise again. This is evident from what has been said above; because for the very same reason here also the body shall proceed from the point K to describe a line altogether similar to the line, in which it has moved from B to K. Thus, as it was observed of the pendulum in the preceding chapter[a], that all the time it approaches towards being perpendicular to the horizon, it more and more descends; but, as soon as it is come into that perpendicular situation, it immediately rises again by the same degrees, as it descended by before: so here the body more and more approaches the center all the time it is moving from B to K; but thence forward it rises from the center again by the same degrees, as it approached by before.

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15. I f (in fig. 77.) the line B C be perpendicular to A B ; then it has been obferved above [a], that the centripetal power may be fo balanced with the progreffive motion of the body, that the body may keep moving round the center A conftantly at the fame diftance ; as a body does, when whirled about any point, to which it is tyed by a ftring. If the centripetal power be too weak to produce this effect, the motion of the body will prefently become oblique to the line drawn from itfelf to the center, after the manner of the firft of the two cafes, which we have been confidering. If the centripetal power be ftronger, than what is required to carry the body in a circle, the motion of the body will prefently fall in with the fecond of the cafes, we have been confidering.

16. I f the centripetal power fo change with the change of diftance, that the body, after its motion has become oblique to the line drawn from itfelf to the center, fhall again become perpendicular thereto ; which we have fhewn to be poffible in both the cafes treated of above ; then the body fhall in its fubfequent motion return again to the diftance of A B, and from that diftance take a courfe fimilar to the former : and thus, if the body move in a fpace free from all refiftance, which has been here all along fuppofed ; it fhall continue in a perpetual motion about the center, defcending and afcending alternately therefrom. If the body fetting out from B (in fig. 78.) in the line B C perpendicular to A B, defcribe the line B D E, which in D fhall be oblique to the line A D, but in E

fhall

ſhall again become erect to AE drawn from the body in E to the center A; then from this point E the body ſhall deſcribe the line EFG altogether like to the line BDE, and at G ſhall be at the ſame diſtance from A, as it was at B. But likewiſe the line AG ſhall be erect to the body's motion. Therefore the body ſhall proceed to deſcribe from G the line GHI altogether ſimilar to the line GFE, and at I have the ſame diſtance from the center, as it had at E; and alſo have the line AI erect to its motion: ſo that its following motion muſt be in the line IKL ſimilar to IHG, and the diſtance AL equal to AG. Thus the body will go on in a perpetual round without ceaſing, alternately inlarging and contracting its diſtance from the center.

17. IF it ſo happen, that the point E fall upon the line BA continued beyond A; then the point G will fall on B, I on E, and L alſo on B; ſo that the body will deſcribe in this caſe a ſimple curve line round the center A, like the line BDEF in fig. 79, in which it will continually revolve from B to E and from E to B without end.

18. IF AE in fig. 78 ſhould happen to be perpendicular to AB, in this caſe alſo a ſimple line will be deſcribed; for the point G will fall on the line BA prolonged beyond A, the point I on the line AE prolonged beyond A, and the point L on B: ſo that the body will deſcribe a line like the curve line BEGI in fig. 80, in which the oppoſite points B and G are equally diſtant from A, and the oppoſite points E and I are alſo equally diſtant from the ſame point A.

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19. In other cases the line described will have a more complex figure.

20. Thus we have endeavoured to shew how a body, while it is constantly attracted towards a center, may notwithstanding by its progressive motion keep it self from falling down to that center; but describe about it an endless circuit, sometimes approaching toward that center, and at other times as much receding from the same.

21. But here we have supposed, that the centripetal power is of equal strength every where at the same distance from the center. And this is the case of that centripetal power, which will hereafter be shewn to be the cause, that keeps the planets in their courses. But a body may be kept on in a perpetual circuit round a center, although the centripetal power have not this property. Indeed a body may by a centripetal force be kept moving in any curve line whatever, that shall have its concavity turned every where towards the center of the force.

22. To make this evident I shall first propose the case of a body moving through the incurvated figure ABCDE (in fig. 81.) which is composed of the straight lines A B, B C, C D, D E, and E A; the motion being carried on in the following manner. Let the body first move in the line A B with any uniform velocity. When it is arrived at the point B, let it receive an impulse directed toward any point F taken within the figure; and let the impulse be of that strength as to turn the body out

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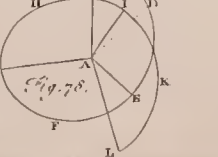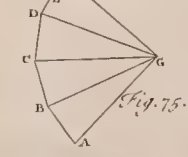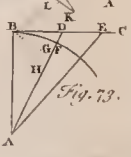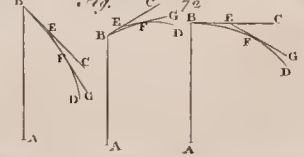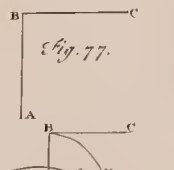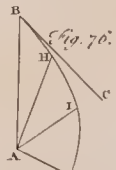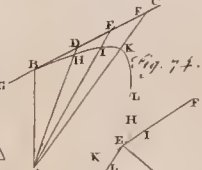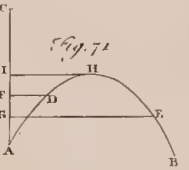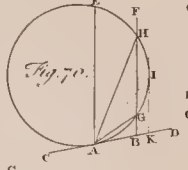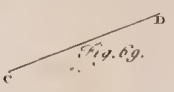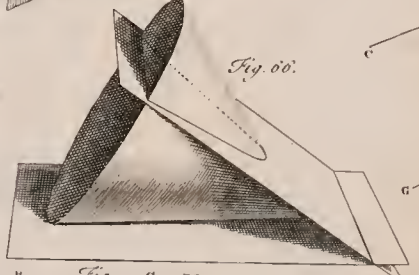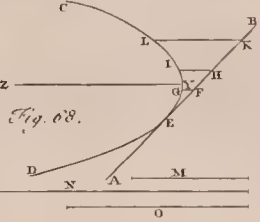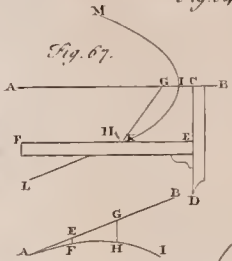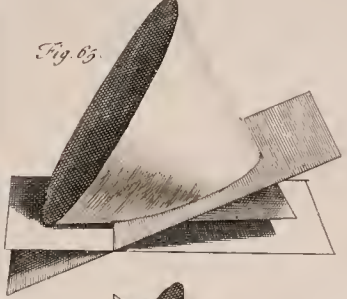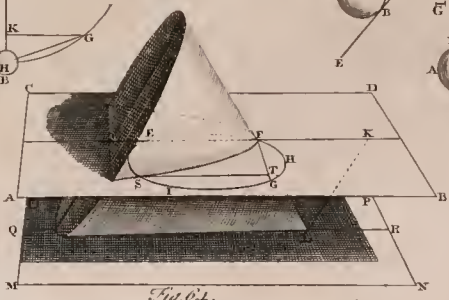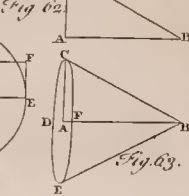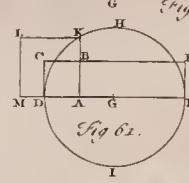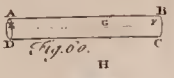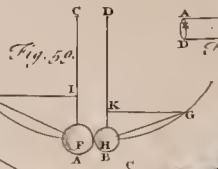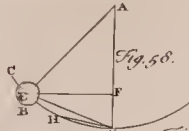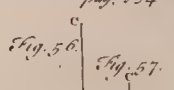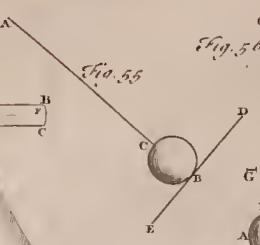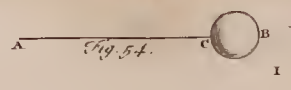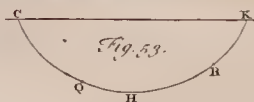
of the line A B into the line B C.   The body after this im-
pulfe, while left to itfelf, will continue moving in the line B C.
At C let the body receive another impulfe directed towards
the fame point F, of fuch ftrength, as to turn the body from
the line B C into the line C D.   At D let the body by another
impulfe, directed likewife to the point F, be turned out of the
line C D into D E.   And at E let another impulfe, directed to-
ward the point F, turn the body from the line D E into E A.
Thus we fee how a body may be carried through the figure
ABCDE by certain impulfes directed always toward the fame
center, only by their acting on the body at proper intervals,
and with due degrees of ftrength.

23. But farther, when the body is come to the point A, if
it there receive another impulfe directed like the reft toward the
point F, and of fuch a degree of ftrength as to turn the body
into the line A B, wherein it firft moved ; I fay that the body
fhall return into this line with the fame velocity, as it had at firft.

24. Let A B be prolonged beyond B at pleafure, fuppofe to
G ; and from G let G H be drawn, which if produced fhould
always continue equidiftant from B F, or, according to the
more ufual phrafe, let G H be drawn parallel to B F.   Then
it appears, from what has been faid upon the fecond law of
motion[a], that in the time, wherein the body would have moved
from B to G, had it not received a new impulfe in B, by the
means of that impulfe it will have acquired a velocity, which
will carry it from B to H.   After the fame manner, if C I be

[a] Ch. 2. § 22.

taken

taken equal to B H, and I K be drawn equidiftant from or pa-
rallel to C F; the body will have moved from C to K with the
velocity, which it has in the line C D, in the fame time, as it
would have employed in moving from C to I with the velocity,
it had in the line B C.   Therefore fince C I and B H are equal,
the body will move through C K in the fame time, as it would
have taken up in moving from B to G with the original velo-
city, wherewith it moved through the line A B.   Again, D L
being taken equal to C K and L M drawn parallel to D F; for
the fame reafon as before the body will move through DM with
the velocity, which it has in the line D E, in the fame time,
as it would imploy in moving through B G with its original ve-
locity.   In the laft place, if E N be taken equal to D M, and
N O be drawn parallel to E F; likewife if A P be taken equal
to E O, and P Q be drawn parallel to A F: then the body with
the velocity, wherewith it returns into the line A B, will pafs
through A Q in the fame time, as it would have imployed in
paffing through B G with its original velocity.   Now as all
this follows directly from what has above been delivered, con-
cerning the effect of oblique impulfes impreffed upon bodies
in motion; fo we muft here obferve farther, that it can be
proved by geometry, that A Q will always be equal to B G.
The proof of this I am obliged, from the nature of my pre-
fent defign, to omit; but this geometrical propofition being
granted, it follows, that the body has returned into the line
A B with the velocity, which it had, when it firft moved in
that line; for the velocity, with which it returns into the line
A B, will carry it over the line A Q in the fame time, as would

3   have

have been taken up in its paffing over an equal line B G with the original velocity.

25. T h u s we have found, how a body may be carried round the figure A B C D E by the action of certain impulfes upon it, which fhould all be pointed toward one center. And we likewife fee, that when the body is brought back again to the point, whence it firft fet out ; if it there meet with an impulfe fufficient to turn it again into the line, wherein it moved at firft, its original velocity will be again reftored ; and by the repetition of the fame impulfes, the body will be carried again in the fame round.   Therefore if thefe impulfes, which act on the body at the points B, C, D, E, and A, continue always the fame, the body will make round this figure innumerable revolutions.

26. The proof, which we have here made ufe of, holds the fame in any number of ftraight lines, whereof the figure A B D fhould be compofed ; and therefore by the method of reafoning referred to above [a] we are to conclude, that what has here been faid upon this rectilinear figure, will remain true, if this figure were changed into one of a continued curvature, and inftead of diftinct impulfes acting by intervals at the angles of this figure, we had a continual centripetal force.   We have therefore fhewn, that a body may be carried round in any curve figure A B C (fig. 82.) which fhall every where be concave towards any one point as D, by the continual action

[a] § 12.

3   T   cf

of a centripetal power directed to that point, and when it is returned to the point, from whence it set out, it shall recover again the velocity, with which it departed from that point. It is not indeed always neceſſary, that it ſhould return again into its firſt courſe ; for the curve line may have ſome ſuch figure as the line A B C D B E in fig. 83. In this curve line, if the body ſet out from B in the direction B F, and moved through the line B C D, till it returned to B; here the body would not enter again into the line B C D, becauſe the two parts B D and B C of the curve line make an angle at the point B: ſo that the centripetal power, which at the point B could turn the body from the line B F into the curve, will not be able to turn the body into the line B C from the direction, in which it returns to the point B ; a forceable impulſe muſt be given the body in the point B to produce that effect.

27. I F at the point B, whence the body ſets out, the curve line return into it ſelf (as in fig. 82 ;) then the body, upon its arrival again at B, may return into its former courſe, and thus make an endleſs circuit about the center of the centripetal power.

28. W H A T has here been ſaid, I hope, will in ſome meaſure enable my readers to form a juſt idea of the nature of theſe centripetal motions.

29. I H A V E not attempted to ſhew, how to find particularly, what kind of centripetal force is neceſſary to carry a body in any curve line propoſed. This is to be deduced from the degree-

gree of curvature, which the figure has in each point of it, and requires a long and complex mathematical reasoning. However I shall speak a little to the first proposition, which Sir Isaac Newton lays down for this purpose. By this proposition, when a body is found moving in a curve line, it may be known, whether the body be kept in its course by a power always pointed toward the same center; and if it be so, where that center is placed.   The proposition is this: that if a line be drawn from some fixed point to the body, and remaining by one extream united to that point, it be carried round along with the body; then, if the power, whereby the body is kept in its course, be always pointed to this fixed point as a center, this line will move over equal spaces in equal portions of time.   Suppose a body were moving through the curve line A B C D (in fig. 84.) and passed over the arches A B, B C, C D in equal portions of time; then if a point, as E, can be found, from whence the line E A being drawn to the body in A, and accompanying the body in its motion, it shall make the spaces E A B, E B C, and E C D equal, over which it passes, while the body describes the arches A B, B C, and C D: and if this hold the same in all other arches, both great and small, of the curve line A B C D, that these spaces are always equal, where the times are equal; then is the body kept in this line by a power always pointed to E as a center.

30. The principle, upon which Sir Isaac Newton has demonstrated this, requires but small skill in geometry to comprehend.   I shall therefore take the liberty to close the pre-

sent

fent chapter with an explication of it; becaufe fuch an example will give the cleareft notion of our author's method of applying mathematical reafoning to thefe philofophical fubjects.

31. He reafons thus. Suppofe a body fet out from the point A (in fig. 85.) to move in the ftraight line A B; and after it had moved for fome time in that line, it were to receive an impulfe directed to fome point as C.   Let it receive that impulfe at D; and thereby be turned into the line D E; and let the body after this impulfe take the fame length of time in paffing from D to E, as it imployed in the paffing from A to D.   Then the ftraight lines C A, C D, and C E being drawn, Sir Isaac Newton proves, that the and triangular fpaces C A D and C D E are equal.   This he does in the following manner.

32. Let E F be drawn parallel to C D. Then, from what has been faid upon the fecond law of motion [a], it is evident, that fince the body was moving in the line A B, when it received the impulfe in the direction D C; it will have moved after that impulfe through the line D E in the fame time, as it would have taken up in moving through D F, provided it had received no difturbance in D. But the time of the body's moving from D to E is fuppofed to be equal to the time of its moving through A D; therefore the time, which the body would have imployed in moving through D F, had it not been difturbed in D, is equal to the time, wherein it moved through A D: confequently D F is equal in length to A D; for if the

[a] Ch. 1. fect. 21, 22.

body

body had gone on to move through the line A B without in-
terruption, it would have moved through all parts thereof
with the fame velocity, and have paffed over equal parts of
that line in equal portions of time.   Now C F being drawn,
fince A D and D F are equal, the triangular fpace C D F is equal
to the triangular fpace C A D.   Farther, the line E F being
parallel to C D, it is proved by E u c l i d, that the triangle
C E D is equal to the triangle C F D [a]: therefore the triangle
C E D is equal to the triangle C A D.

33. A f t e r the fame manner, if the body receive at E ano-
ther impulfe directed toward the point C, and be turned by
that impulfe into the line E G ; if it move afterwards from E to
G in the fame fpace of time, as was taken up by its motion from
D to E, or from A to D; then C G being drawn, the triangle
C E G is equal to C D E.   A third impulfe at G directed as the
two former to C, whereby the body fhall be turned into the
line G H, will have alfo the like effect with the reft.   If the
body move over G H in the fame time, as it took up in mov-
ing over E G, the triangle C G H will be equal to the triangle
C E G.   Laftly, if the body at H be turned by a frefh impulfe
directed toward C into the line H I, and at I by another im-
pulfe directed alfo to C be turned into the line I K; and if the
body move over each of the lines H I, and I K in the fame
time, as it imployed in moving over each of the preceding
lines A D, D E, E G, and G H : then each of the triangles
C H I, and C I K will be equal to each of the preceding. Like-

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[a] Elem. Book I. p. 37.

wife

wife as the time, in which the body moves over A D E, is equal to the time of its moving over E G H, and to the time of its moving over H I K ; the fpace C A D E will be equal to the fpace C E G H, and to the fpace C H I K. In the fame manner as the time, in which the body moved over A D E G is equal to the time of its moving over G H I K, fo the fpace C A D E G will be equal to the fpace C G H I K.

34. F R O M this principle Sir I s a a c N e w t o n demonftrates the propofition mentioned above, by that method of arguing introduced by him into geometry, whereof we have before taken notice [a], by making according to the principles of that method a tranfition from this incurvated figure compofed of ftraight lines, to a figure of continued curvature ; and by fhewing, that fince equal fpaces are defcribed in equal times in this prefent figure compofed of ftraight lines, the fame relation between the fpaces defcribed and the times of their defcription will alfo have place in a figure of one continued curvature. He alfo deduces from this propofition the reverfe of it ; and proves, that whenever equal fpaces are continually defcribed ; the body is acted upon by a centripetal force directed to the center, at which the fpaces terminate.

[a] § 12.

C H A P.

Chap. IV.

Of the RESISTANCE of FLUIDS.

BEFORE the caufe can be difcovered, which keeps the planets in motion, it is neceffary firft to know, whether the fpace, wherein they move, is empty and void, or filled with any quantity of matter. It has been a prevailing opinion, that all fpace contains in it matter of fome kind or other ; fo that where no fenfible matter is found, there was yet a fubtle fluid fubftance by which the fpace was filled up; even fo as to make an abfolute plenitude. In order to examine this opinion, Sir Isaac Newton has largely confidered the effects of fluids upon bodies moving in them.

2. THESE effects he has reduced under thefe three heads. In the firft place he fhews how to determine in what manner the refiftance, which bodies fuffer, when moving in a fluid, gradually increafes in proportion to the fpace, they defcribe in any fluid ; to the velocity, with which they defcribe it ; and to the time they have been in motion. Under the fecond head he confiders what degree of refiftance different bodies moving in the fame fluid undergo, according to the different proportion between the denfity of the fluid and the denfity of the body. The denfities of bodies, whether fluid or folid, are meafured by the quantity of matter, which is comprehended under the fame magnitude ; that body being

the

the moſt denſe or compact, which under the ſame bulk contains the greateſt quantity of ſolid matter, or which weighs moſt, the weight of every body being obſerved above to be proportional to the quantity of matter in it [a].   Thus water is more denſe than cork or wood, iron more denſe than water, and gold than iron.   The third particular Sir Is. Newton conſiders concerning the reſiſtance of fluids is the influence, which the diverſity of figure in the reſiſted body has upon its reſiſtance.

3. For the more perfect illuſtration of the firſt of theſe heads, he diſtinctly ſhews the relation between all the particulars ſpecified upon three different ſuppoſitions.   The firſt is, that the ſame body be reſiſted more or leſs in the ſimple proportion to its velocity ; ſo that if its velocity be doubled, its reſiſtance ſhall become threefold.   The ſecond is of the reſiſtance increaſing in the duplicate proportion of the velocity ; ſo that, if the velocity of a body be doubled, its reſiſtance ſhall be rendered four times ; and if the velocity be trebled, nine times as great as at firſt.   But what is to be underſtood by duplicate proportion has been already explained [b].   The third ſuppoſition is, that the reſiſtance increaſes partly in the ſingle proportion of the velocity, and partly in the duplicate proportion thereof.

4. In all theſe ſuppoſitions, bodies are conſidered under two reſpects, either as moving, and oppoſing themſelves

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[a] Ch 1 § 24.   [b] Ch. 2 ſelect 17.

againſt

againſt the fluid by that power alone, which is eſſential to them, of reſiſting to the change of their ſtate from reſt to motion, or from motion to reſt, which we have above called their power of inactivity ; or elſe, as deſcending or aſcending, and ſo having the power of gravity combined with that other power.   Thus our author has ſhewn in all thoſe three ſuppoſitions, in what manner bodies are reſiſted in an uniform fluid, when they move with the aforeſaid progreſſive motion [a] ;   and what the reſiſtance is, when they aſcend or deſcend perpendicularly [b].   And if a body aſcend or deſcend obliquely, and the reſiſtance be ſingly proportional to the velocity, it is ſhewn how the body is reſiſted in a fluid of an uniform denſity, and what line it will deſcribe [c], which is determined by the meaſurement of the hyperbola, and appears to be no other than that line, firſt conſidered in particular by Dr. B A R R O W [d], which is now commonly known by the name of the logarithmical curve.   In the ſuppoſition that the reſiſtance increaſes in the duplicate proportion of the velocity, our author has not given us the line which would be deſcribed in an uniform fluid ; but has inſtead thereof diſcuſſed a problem, which is in ſome ſort the reverſe ; to find the denſity of the fluid at all altitudes, by which any given curve line may be deſcribed ; which problem is ſo treated by him, as to be applicable to any kind of reſiſtance whatever [e].   But here not unmindful of practice, he ſhews that a body in a fluid of uniform denſity, like the

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[a] Newt. Princ. L. II. prop. 2 ; 5, 6, 7 ; 11, 12.
[b] Prop. 3 ; 8, 9 ; 13, 14.
[c] Prop. 4.
[d] Prælect. Geometr. pag. 123.
[e] Newton. Princ. Lib. II. prop. 10.

air,

air, will defcribe a line, which approaches towards an hyperbola; that is, its motion will be nearer to that curve line than to the parabola. And confequent upon this remark, he fhews how to determine this hyperbola by experiment, and briefly refolves the chief of thofe problems relating to projectiles, which are in ufe in the art of gunnery, in this curve[a]; as Torricelli and others have done in the parabola[b], whofe inventions have been explained at large above[c].

5. Our author has alfo handled diftinctly that particular fort of motion, which is defcribed by pendulums[d]; and has likewife confidered fome few cafes of bodies moving in refifting fluids round a center, to which they are impelled by a centripetal force, in order to give an idea of thofe kinds of motions[e].

6. The treating of the refiftance of pendulums has given him an opportunity of inferting into another part of his work fome fpeculations upon the motions of them without refiftance, which have a very peculiar elegance; where in he treats of them as moved by a gravitation acting in the law, which he fhews to belong to the earth below its furface[f]; performing in this kind of gravitation, where the force is proportional to the diftance from the center, all that Huygens had before done in the common fuppofition of its being uniform, and acting in parallel lines[g].

[a] Newton. Princ. Lib II prop. 10. in fchol.
[b] Torricelli de motu gravium.
[c] Ch 2 § 85, &c.
[d] Newt. Princ. L. II. fect. 6.
[e] L. II. fect 4.
[f] See B. II. Ch 6. § 7. of this treatife.
[g] Lib. I. fect. 10.

7. Huy-

7. Huygens at the end of his treatise of the cause of gravity[a] informs us, that he likewise had carried his speculations on the first of these suppositions, of the resistance in fluids being proportional to the velocity of the body, as far as our author.   But finding by experiment that the second was more conformable to nature,   he afterwards made some progrefs in that, till he was stopt, by not being able to execute to his wish what related to the perpendicular descent of bodies;   not observing that the measurement of the curve line,   he made use of to explain it by,   depended on the hyperbola.   Which oversight may well be pardoned in that great man,   considering that our author had not been pleased at that time to communicate to the publick his admirable discourse of the QUADRATURE or MEASUREMENT OF CURVE LINES,   with which he has since obliged the world: for without the use of that treatise,   it is I think no injury even to our author's unparalleled abilities to believe,   it would not have been easy for himself to have succeeded so happily in this and many other parts of his writings.

8. What Huygens found by experiment, that bodies were in reality resisted in the duplicate proportion of their velocity,   agrees with the reasoning of our author [b], who distinguishes the resistance, which fluids give to bodies by the tenacity of their parts, and the friction between them and the body, from that, which arises from the power of inactivity, with which the constituent particles of fluids are endued like all

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[a] De la Pesanteur, pag. 169, and the following. | [b] Newton. Princ. L. II. prop. 4. schol.

other

other portions of matter, by which power the particles of fluids like other bodies make refiftance againft being put into motion.

9. The refiftance, which arifes from the friction of the body againft the parts of the fluid, muft be very inconfiderable; and the refiftance, which follows from the tenacity of the parts of fluids, is not ufually very great, and does not depend much upon the velocity of the body in the fluid; for as the parts of the fluid adhere together with a certain degree of force, the refiftance, which the body receives from thence, cannot much depend upon the velocity, with which the body moves; but like the power of gravity, its effect muft be proportional to the time of its acting.   This the reader may find farther explained by Sir Isaac Newton himfelf in the poftfcript to a difcourfe publifhed by me in the philo-sophical transactions, N° 371.   The principal refiftance, which moft fluids give to bodies, arifes from the power of inactivity in the parts of the fluids, and this depends upon the velocity, with which the body moves, on a double account. In the firft place, the quantity of the fluid moved out of place by the moving body in any determinate fpace of time is proportional to the velocity, wherewith the body moves; and in the next place, the velocity with which each particle of the fluid is moved, will alfo be proportional to the velocity of the body: therefore fince the refiftance, which any body makes againft being put into motion, is proportional both to the quantity of matter moved and the velocity it is moved with; the refiftance, which a fluid gives on this account, will be doubly increafed with the increafe of the velocity in the moving body;
that

that is, the refiftance will be in a two-fold or duplicate propor-
tion of the velocity, wherewith the body moves through the
fluid.

10. FARTHER it is moft manifeft, that this latter kind
of refiftance increafing with the increafe of velocity, even
in a greater degree than the velocity it felf increafes, the
fwifter the body moves, the lefs proportion the other fpecies
of refiftance will bear to this : nay that this part of the refift-
ance may be fo much augmented by a due increafe of velo-
city, till the former refiftances fhall bear a lefs proportion to
this, than any that might be affigned. And indeed expe-
rience fhews, that no other refiftance, than what arifes from
the power of inactivity in the parts of the fluid, is of mo-
ment, when the body moves with any confiderable fwiftnefs.

11. THERE is befides thefe yet another fpecies of refift-
ance, found only in fuch fluids, as, like our air, are elaftic.
Elaft[i]city belongs to no fluid known to us befide the air. By
this property any quantity of air may be contracted into a
lefs fpace by a forcible preffure, and as foon as the com-
preffing power is removed, it will fpring out again to its
former dimenfions. The air we breath is held to its prefent
denfity by the weight of the air above us. And as this in-
cumbent weight, by the motion of the winds, or other cauf-
es, is frequently varied (which appears by the barometer;)
fo when this weight is greateft, we breath a more denfe air
than at other times. To what degree the air would expand
it felf by its fpring, if all preffure were removed, is not
known,

known, nor yet into how narrow a compaſs it is capable of being compreſſed. Mr. Boyle found it by experiment capable both of expanſion and compreſſion to ſuch a degree, that he could cauſe a quantity of air to expand it ſelf over a ſpace ſome hundred thouſand times greater, than the ſpace to which he could confine the ſame quantity [a]. But I ſhall treat more fully of this ſpring in the air hereafter [b]. I am now only to conſider what reſiſtance to the motion of bodies ariſes from it.

12. But before our author ſhews in what manner this cauſe of reſiſtance operates, he propoſes a method, by which fluids may be rendered elaſtic, demonſtrating that if their particles be provided with a power of repelling each other, which ſhall exert it ſelf with degrees of ſtrength reciprocally proportional to the diſtances between the centers of the particles; that then ſuch fluids will obſerve the ſame rule in being compreſſed, as our air does, which is this, that the ſpace, into which it yields upon compreſſion, is reciprocally proportional to the compreſſing weight [c]. The term reciprocally proportional has been explained above [d]. And if the centrifugal force of the particles acted by other laws, ſuch fluids would yield in a different manner to compreſſion [e].

13. Whether the particles of the air be endued with ſuch a power, by which they can act upon each other out of contact, our author does not determine; but leaves that

[a] See his Tract on the admirable rarifaction of air
    Book II. Ch. 6.

[c] Princ. philoſ. Lib. II. prop. 23.
[d] Book I. Ch. 2. § 30
[e] Princ. philoſ. Lib. II. prop. 23. in ſchol.

to future examination, and to be difcuffed by philofophers. Only he takes occafion from hence to confider the refiftance in elaftic fluids, under this notion; making remarks, as he paffes along, upon the differences, which will arife, if their elafticity be derived from any other fountain [a].   And this, I think, muft be confeffed to be done by him with great judgment; for this is far the moft reafonable account, which has been given of this furprizing power, as muft without doubt be freely acknowledged by any one, who in the leaft confiders the infufficiency of all the other conjectures, which have been framed; and alfo how little reafon there is to deny to bodies other powers, by which they may act upon each other at a diftance, as well as that of gravity; which we fhall hereafter fhew to be a property univerfally belonging to all the bodies of the univerfe, and to all their parts [b].   Nay we actually find in the loadftone a very apparent repelling, as well as an attractive power.   But of this more in the conclufion of this difcourfe.

14. By thefe fteps our author leads the way to explain the refiftance, which the air and fuch like fluids will give to bodies by their elafticity; which refiftance he explains thus.   If the elaftic power of the fluid were to be varied fo, as to be always in the duplicate proportion of the velocity of the refifted body, it is fhewn that then the refiftance derived from the elafticity, would increafe in the duplicate proportion of the velocity; in fo much that the

a Princ. philof. Lib. II. prop. 33. coroll. | b Lib. II. Ch. 5.

whole

whole refiftance would be in that proportion, excepting on-
ly that fmall part, which arifes from the friction between the
body and the parts of the fluid. From whence it follows,
that becaufe the elaftic power of the fame fluid does in
truth continue the fame, if the velocity of the moving body be
diminifhed, the refiftance from the elafticity, and therefore
the whole refiftance, will decreafe in a lefs proportion, than the
duplicate of the velocity ; and if the velocity be increafed, the
refiftance from the elafticity will increafe in a lefs proportion,
than the duplicate of the velocity, that is in a lefs proportion,
than the refiftance made by the power of inactivity of the
parts of the fluid. And from this foundation is raifed the proof
of a property of this refiftance, given by the elafticity in com-
mon with the others from the tenacity and friction of the
parts of the fluid ; that the velocity may be increafed, till this
refiftance from the fluid's elafticity fhall bear no confiderable
proportion to that, which is produced by the power of inacti-
vity thereof [a].   From whence our author draws this conclu-
fion ; that the refiftance of a body, which moves very fwift-
ly in an elaftic fluid, is near the fame, as if the fluid were
not elaftic ; provided the elafticity arifes from the centrifugal
power of the parts of the medium, as before explained, efpeci-
ally if the velocity be fo great, that this centrifugal power
fhall want time to exert it felf [b].   But it is to be obferved,
that in the proof of all this our author proceeds upon the fup-
pofition of this centrifugal power in the parts of the fluid ; but
if the elafticity be caufed by the expanfion of the parts in the

[a] Ibid. Prop. 33. coroll. 2. | [b] Ibid. coroll. 3.

manner

manner of wool compreffed, and fuch like bodies, by which the parts of the fluid will be in fome meafure entangled together, and their motion be obftructed, the fluid will be in a manner tenacious, and give a refiftance upon that account over and above what depends upon its elafticity only [a]; and the refiftance derived from that caufe is to be judged of in the manner before fet down.

15. It is now time to pafs to the fecond part of this theory; which is to affign the meafure of refiftance, according to the proportion between the denfity of the body and the denfity of the fluid. What is here to be underftood by the word denfity has been explained above [b]. For this purpofe as our author before confidered two diftinct cafes of bodies moving in mediums; one when they oppofed themfelves to the fluid by their power of inactivity only, and another when by afcending or defcending their weight was combined with that other power: fo likewife, the fluids themfelves are to be regarded under a double capacity; either as having their parts at reft, and difpofed freely without reftraint, or as being compreffed together by their own weight, or any other caufe.

16. In the firft cafe, if the parts of the fluid be wholly difingaged from one another, fo that each particle is at liberty to move all ways without any impediment, it is fhewn, that if a globe move in fuch a fluid, and the globe and par-

[a] Vid. ibid. coroll. 6. | [b] In § 2.

ticles

ticles of the fluid are endued with perfect elasticity; so that as the globe impinges upon the particles of it, they shall bound off and separate themselves from the globe, with the same velocity, with which the globe strikes upon them; then the resistance, which the globe moving with any known velocity suffers, is to be thus determined. From the velocity of the globe, the time, wherein it would move over two third parts of its own diameter with that velocity, will be known. And such proportion as the density of the fluid bears to the density of the globe, the same the resistance given to the globe will bear to the force, which acting, like the power of gravity, on the globe without intermission during the space of time now mentioned, would generate in the globe the same degre of motion, as that wherewith it moves in the fluid [a]. But if neither the globe nor the particles of the fluid be elastic, so that the particles, when the globe strikes against them, do not rebound from it, then the resistance will be but half so much [b]. Again, if the particles of the fluid and the globe are imperfectly elastic, so that the particles will spring from the globe with part only of that velocity wherewith the globe impinges upon them; then the resistance will be a mean between the two preceding cases, approaching nearer to the first or second, according as the elasticity is more or less [c].

17. The elasticity, which is here ascribed to the particles of the fluid, is not that power of repelling one another,

[a] Princ. philos. Lib. II. Prop. 35.   [c] Id.
[b] Ibid.

when

when out of contact, by which, as has before been mentioned, the whole fluid may be rendred elastic ; but such an elasticity only, as many solid bodies have of recovering their figure, whenever any forcible change is made in it, by the impulse of another body or otherwise. Which elasticity has been explained above at large [a].

18. This is the case of discontinued fluids, where the body, by pressing against their particles, drives them before itself, while the space behind the body is left empty. But in fluids which are compressed, so that the parts of them removed out of place by the body resisted immediately retire behind the body, and fill that space, which in the other case is left vacant, the resistance is still less ; for a globe in such a fluid which shall be free from all elasticity, will be resisted but half as much as the least resistance in the former case [b]. But by elasticity I now mean that power, which renders the whole fluid so ; of which if the compressed fluid be possessed, in the manner of the air, then the resistance will be greater than by the foregoing rule ; for the fluid being capable in some degree of condensation, it will resemble so far the case of uncompressed fluids [c]. But, as has been before related, this difference is most considerable in slow motions.

19. In the next place our author is particular in determining the degrees of resistance accompanying bodies of different figures ; which is the last of the three heads, we

[a] h. 1. § 29.
[b] Princ. philof. Lib. II. Prop. 38, compared with { [c] coroll. 1. of prop. 35.   [c] L. II. Lem. 7. schol. pag. 341.

divided

divided the whole difcourfe of refiftance into.   And in this difquifition he finds a very furprizing and unthought of difference, between free and compreffed fluids.   He proves, that in the former kind, a globe fuffers but half the refiftance, which the cylinder, that circumfcribes the globe, will do, if it move in the direction of its axis [a].   But in the latter he proves, that the globe and cylinder are refifted alike [b].   And in general, that let the fhape of bodies be ever fo different, yet if the greateft fections of the bodies perpendicular to the axis of their motion be equal, the bodies will be refifted equally [c].

20. Pursuant to the difference found between the refiftance of the globe and cylinder in rare and uncompreffed fluids, our author gives us the refult of fome other inquiries of the fame nature.   Thus of all the fruftums of a cone, that can be defcribed upon the fame bafe and with the fame altitude, he fhews how to find that, which of all others will be the leaft refifted, when moving in the direction of its axis [d].   And from hence he draws an eafy method of altering the figure of any fpheroidical folid, fo that its capacity may be enlarged, and yet the refiftance of it diminifhed [e]: a note which he thinks may not be ufelefs to fhipwrights.   He concludes with determining the folid, which will be refifted the leaft that is poffible, in thefe difcontinued fluids [f].

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[a] Lib. II. Prop. 34.
[b] Lib. II. Lem. 7. p. 341.
[c] Schol. to Lem. 7.

[d] Prop. 34. fchol.
[e] Ibid.
[f] Ibid.

21. That

21. That I may here be underſtood by readers unac-
quainted with mathematical terms, I ſhall explain what I
mean by a fruſtum of a cone, and a ſpheroidical ſolid. A
cone has been defined above. A fruſtum is what remains,
when part of the cone next the vertex is cut away by a ſec-
tion parallel to the baſe of the cone, as in fig. 86. A ſpheroid
is produced from an ellipſis, as a ſphere or globe is made
from a circle. If a circle turn round on its diameter, it de-
ſcribes by its motion a ſphere; ſo if an ellipſis (which figure
has been defined above, and will be more fully explained
hereafter [a]) be turned round either upon the longeſt or
ſhorteſt line, that can be drawn through the middle of it,
there will be deſcribed a kind of oblong or flat ſphere, as
in fig. 87. Both theſe figures are called ſpheroids, and any
ſolid reſembling theſe I here call ſpheroidical.

22. If it ſhould be asked, how the method of altering
ſpheroidical bodies, here mentioned, can contribute to the
facilitating a ſhip's motion, when I juſt above affirmed,
that the figure of bodies, which move in a compreſſed
fluid not elaſtic, has no relation to the augmentation or di-
minution of the reſiſtance; the reply is, that what was
there ſpoken relates to bodies deep immerged into ſuch flu-
ids, but not of thoſe, which ſwim upon the ſurface of them;
for in this latter caſe the fluid, by the appulſe of the an-
terior parts of the body, is raiſed above the level of the
ſurface, and behind the body is ſunk ſomewhat below; ſo

[a] Book II. Ch. 1. § 6.

that

that by this inequality in the fuperficies of the fluid, that part of it, which at the head of the body is higher than the fluid behind, will refift in fome meafure after the manner of difcontinued fluids [a], analogous to what was before obferved to happen in the air through its elafticity, though the body be furrounded on every fide by it [b]. And as far as the power of thefe caufes extends, the figure of the moving body affects its refiftance; for it is evident, that the figure, which preffes leaft directly againft the parts of the fluid, and fo raifes leaft the furface of a fluid not elaftic, and leaft compreffes one that is elaftic, will be leaft refifted.

23. The way of collecting the difference of the refiftance in rare fluids, which arifes from the diverfity of figure, is by confidering the different effect of the particles of the fluid upon the body moving againft them, according to the different obliquity of the feveral parts of the body upon which they refpectively ftrike; as it is known, that any body impinging againft a plane obliquely, ftrikes with a lefs force, than if it fell upon it perpendicularly; and the greater the obliquity is, the weaker is the force. And it is the fame thing, if the body be at reft, and the plane move againft it [c].

24. That there is no connexion between the figure of a body and its refiftance in compreffed fluids, is proved thus. Suppofe A B C D (in fig. 88.) to be a canal, having fuch a fluid, water for inftance, running through it with an equable

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[a] Vid. Newt. princ. in fchol. to Lem. 7, of Lib. II. pag. 341.

[b] Sect. 17. of this chapter.
[c] See Princ. philof. Lib. II. prop. 34.

velocity;
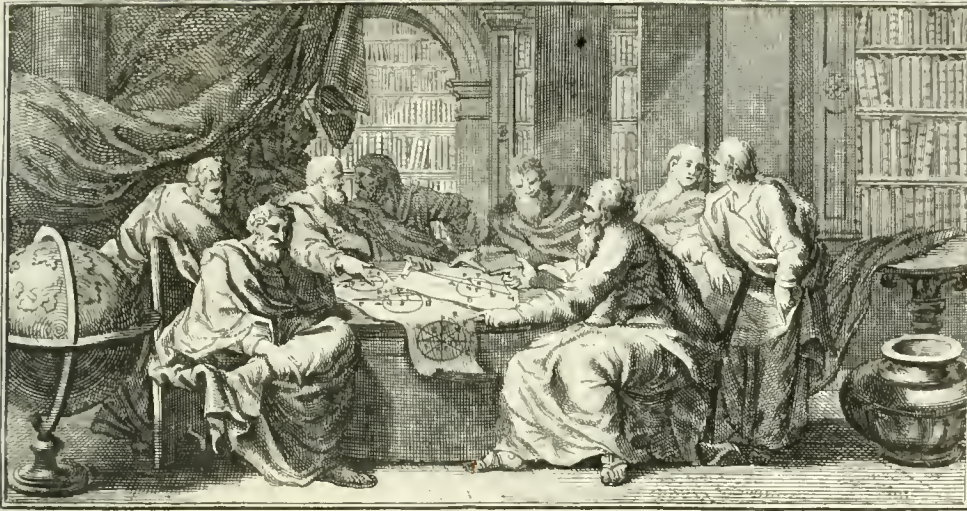
BOOK II.

CONCERNING THE

SYSTEM of the WORLD.

CHAP. I.

That the Planets move in a space empty of all senfible matter.

I HAVE now gone through the firft part of my defign, and have explained, as far as the nature of my undertaking would permit, what Sir ISAAC NEWTON has delivered in general concerning the motion of bodies. It follows now to fpeak of the difcoveries, he has made in the fyftem of the world;

Y   and

and to shew from him what cause keeps the heavenly bo-
dies in their courses.   But it will be necessary for the use of
such, as are not skilled in astronomy, to premise a brief de-
scription of the planetary system.

2. This system is disposed in the following manner.   In
the middle is placed the sun.   About him six globes con-
tinually roll.   These are the primary planets; that which
is nearest to the sun is called Mercury, the next Venus,
next to this is our earth, the next beyond is Mars, after
him Jupiter, and the outermost of all Saturn.   Besides these
there are discovered in this system ten other bodies, which
move about some of these primary planets in the same
manner, as they move round the sun.   These are called
secondary planets.   The most conspicuous of them is the
moon, which moves round our earth; four bodies move in
like manner round Jupiter; and five round Saturn.   Those
which move about Jupiter and Saturn, are usually called
satellites; and cannot any of them be seen without a te-
lescope.   It is not impossible, but there may be more se-
condary planets, beside these ; though our instruments
have not yet discovered any other.   This disposition of
the planetary or solar system is represented in fig. 89.

3. The same planet is not always equally distant from
the sun.   But the middle distance of Mercury is between
$\frac{1}{3}$ and $\frac{2}{5}$ of the distance of the earth from the sun; Venus
is distant from the sun almost $\frac{3}{4}$ of the distance of the
earth; the middle distance of Mars is something more than
half

Fig. 79.

B

F D

A

E

Fig. 80.

B

I A E

A

G

Fig. 81.

L D K H C
M
E F
H G
B
N O Q
A P

Fig. 85.

K I H
G B
E F
D
C A

Fig. 86.

Fig. 83.

A D
B C
E F

Fig. 82.

C B

A

D

A

Fig. 84.

D
C
E I
A B

Fig. 87.

Fig. 88.

A B

E

C D

Fig. 89.

The orbit of Saturn.

The orbit of Jupiter

The orbit of Mars

The orbit of the Earth

The orbit of Venus

Mercury
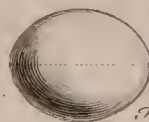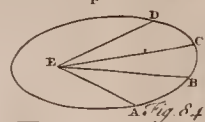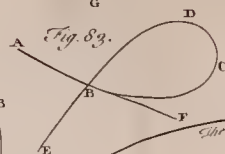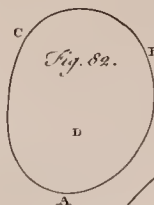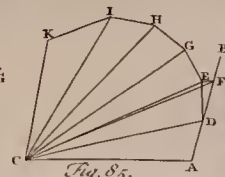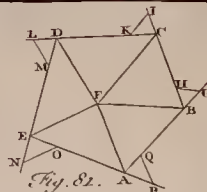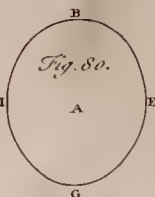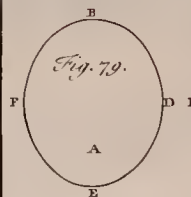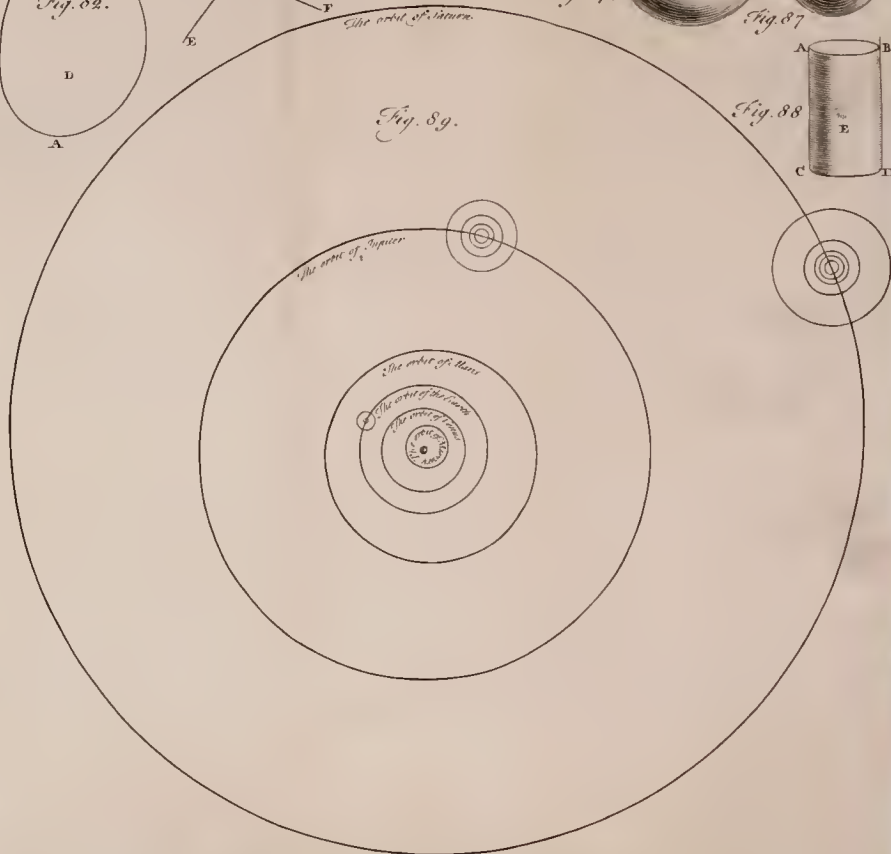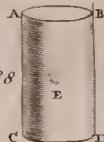
half as much again, as the diftance of the earth; Jupiter's middle diftance exceeds five times the diftance of the earth, by between $\frac{1}{5}$ and $\frac{1}{6}$ part of this diftance; Saturn's middle diftance is fcarce more than $9\frac{1}{2}$ times the diftance between the earth and fun; but the middle diftance between the earth and fun is about $217\frac{1}{8}$ times the fun's femidiameter.

4. All thefe planets move one way, from weft to eaft; and of the primary planets the moft remote is longeft in finifhing its courfe round the fun. The period of Saturn falls fhort only fixteen days of 29 years and a half. The period of Jupiter is twelve years wanting about 50 days. The period of Mars falls fhort of two years by about 43 days. The revolution of the earth conftitutes the year. Venus performs her period in about $224\frac{1}{2}$ days, and mercury in about 88 days.

5. The courfe of each planet lies throughout in one plane or flat furface, in which the fun is placed; but they do not all move in the fame plane, though the different planes, in which they move, crofs each other in very fmall angles. They all crofs each other in lines, which pafs through the fun; becaufe the fun lies in the plane of each orbit. This inclination of the feveral orbits to each other is reprefented in fig. 90. The line, in which the plane of any orbit croffes the plane of the earth's motion, is called the line of the nodes of that orbit.

Y 2

6. Each

6. Each planet moves round the fun in the line, which we have mentioned above [a] under the name of ellipfis; which I fhall here fhew more particularly how to defcribe. I have there faid how it is produced in the cone. I fhall now fhew how to form it upon a plane. Fix upon any plane two pins, as at A and B in fig. 9 1. To thefe tye a ftring A C B of any length. Then apply a third pin D fo to the ftring, as to hold it ftrained; and in that manner carrying this pin about, the point of it will defcribe an ellipfis. If through the points A, B the ftraight line E A B F be drawn, to be terminated at the ellipfis in the points E and F, this is the longeft line of any, that can be drawn within the figure, and is called the greater axis of the ellipfis. The line G H, drawn perpendicular to this axis E F, fo as to pafs through the middle of it, is called the leffer axis. The two points A and B are called focus's. Now each planet moves round the fun in a line of this kind, fo that the fun is found in one focus. Suppofe A to be the place of the fun. Then E is the point, wherein the planet will be neareft of all to the fun, and at F it will be moft remote. The point E is called the perihelion of the planet, and F the aphelion. In G and H the planet is faid to be in its middle or mean diftance; becaufe the diftance A G or A H is truly the middle between A E the leaft, and A F the greateft diftance. In fig. 9 2. is reprefented how the greater axis of each orbit is fituated in refpect of the reft. The proportion between the greateft and leaft diftances of the planet from the fun is very different in the different planets. In Saturn the proportion of the

greateft

[a] Book I. ch. 2. § 82.

Fig. 90

orb of Saturn.

orb of Jupiter.

orb of Mars.

orb of the Earth.

orb of Venus.

orb of Mercury.

Fig. 90

orb of Saturn

orb of Jupiter

orb of Mars

orb of the Earth

orb of Venus

orb of Mercury
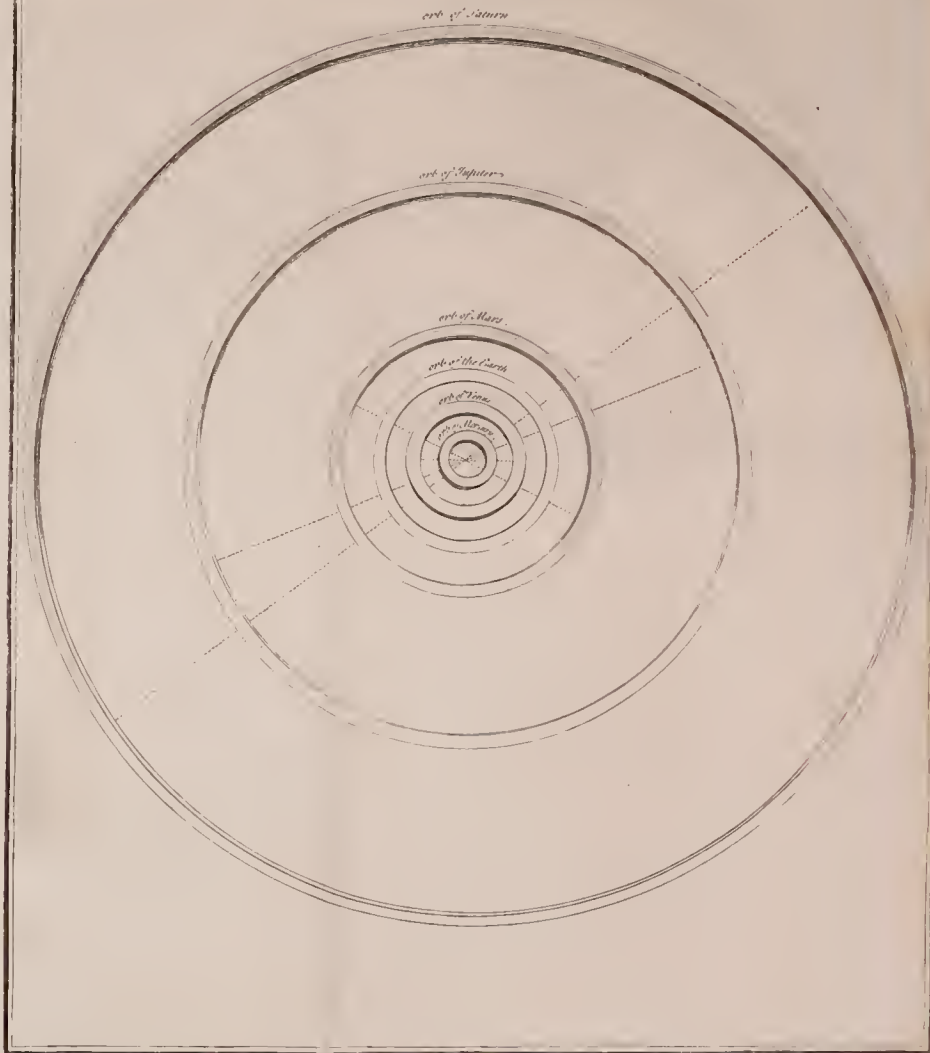
greateſt diſtance to the leaſt is ſomething leſs, than the propor-
tion of 9 to 8; but much nearer to this, than to the propor-
tion of 10 to 9.   In Jupiter this proportion is a little greater,
than that of 11 to 10.   In Mars it exceeds the proportion of
6 to 5.   In the earth it is about the proportion of 30 to 29.
In Venus it is near to that of 70 to 69.   And in Mercury it
comes not a great deal ſhort of the proportion of 3 to 2.

7. E a c h of theſe planets ſo moves through its ellipſis, that
the line drawn from the ſun to the planet, by accompanying
the planet in its motion, will deſcribe about the ſun equal ſpa-
ces in equal times, after the manner ſpoke of in the chapter of
centripetal forces[a].   There is alſo a certain relation between
the greater axis's of theſe ellipſis's, and the times, in which
the planets perform their revolutions through them.   Which
relation may be expreſſed thus. Let the period
of one planet be denoted by the letter A,   the   A   D
greater axis of its orbit by D;   let the period   B   E
of another planet be denoted by B,   and the   C   F
greater axis of this planet's orbit by E.   Then   G
if C be taken to bear the ſame proportion to B,
as B bears to A;   likewiſe if F be taken to bear the ſame pro-
portion to E, as E bears to D; and G taken to bear the ſame
proportion likewiſe to F, as E bears to D;   then A ſhall bear
the ſame proportion to C, as D bears to G.

8. T h e ſecondary planets move round their reſpective
primary, much in the ſame manner as the primary do round

a Book I. Ch. 3. § 29.

the

the fun.   But the motions of thefe fhall be more fully explained hereafter [a].   And there is, befides the planets, another fort of bodies, which in all probability move round the fun ; I mean the comets.   The farther defcription of which bodies I alfo leave to the place, where they are to be particularly treated on [b].

9. Far without this fyftem the fixed ftars are placed. Thefe are all fo remote from us, that we feem almoft incapable of contriving any means to eftimate their diftance.   Their number is exceeding great.   Befides two or three thoufand, which we fee with the naked eye, telefcopes open to our view vaft numbers ; and the farther improved thefe inftruments are, we ftill difcover more and more.   Without doubt thefe are luminous globes, like our fun, and ranged through the wide extent of fpace ; each of which, it is to be fuppofed, perform the fame office, as our fun, affording light and heat to certain planets moving about them.   But thefe conjectures are not to be purfued in this place.

10. I shall therefore now proceed to the particular defign of this chapter, and fhew, that there is no fenfible matter lodged in the fpace where the planets move.

11. That they fuffer no fenfible refiftance from any fuch matter, is evident from the agreement between the obfervations of aftronomers in different ages, with regard to the time, in which the planets have been found to perform their

[a] Ch. 3. of this prefent book.   [b] Ch. 4.

periods.

periods.   But it was the opinion of Des Cartes[a], that the planets might be kept in their courses by the means of a fluid matter, which continually circulating round should carry the planets along with it.   There is one appearance that may seem to favour this opinion; which is, that the sun turns round its own axis the same way, as the planets move.   The earth also turns round its axis the same way, as the moon moves round the earth.   And the planet Jupiter turns upon its axis the same way, as his satellites revolve round him.   It might therefore be supposed, that if the whole planetary region were filled with a fluid matter, the sun, by turning round on its own axis, might communicate motion first to that part of the fluid, which was contiguous, and by degrees propagate the like motion to the parts more remote.   After the same manner the earth might communicate motion to this fluid, to a distance sufficient to carry round the moon, and Jupiter communicate the like to the distance of its satellites.   Sir Isaac Newton has particularly examined what might be the result of such a motion as this [b];   and he finds, that the velocities, with which the parts of this fluid will move in different distances from the center of the motion, will not agree with the motion observed in different planets : for instance, that the time of one intire circulation of the fluid, wherein Jupiter should swim, would bear a greater proportion to the time of one intire circulation of the fluid, where the earth is ; than the period of Jupiter bears to the period of the earth.   But he also proves [c],   that the planet cannot circulate in such a fluid,

[a] In Princ. philof. part. 3.
[b] Philof. princ. mathem. Lib. II. prop. 2.   & fchol.
[c] Ibid. prop. 53.

So

fo as to keep long in the fame courfe, unlefs the planet and the contiguous fluid are of the fame denfity, and the planet be carried along with the fame degree of motion, as the fluid. There is alfo another remark made upon this motion by our author; which is, that fome vivifying force will be continually neceffary at the center of the motion [a]. The fun in particular, by communicating motion to the ambient fluid, will lofe from it felf as much motion, as it imparts to the fluid; unlefs fome acting principle refide in the fun to renew its motion continually. If the fluid be infinite, this gradual lofs of motion would continue till the whole fhould ftop [b]; and if the fluid were limited, this lofs of motion would continue, till there would remain no fwifter a revolution in the fun, than in the utmoft part of the fluid; fo that the whole would turn together about the axis of the fun, like one folid globe [c].

12. It is farther to be obferved, that as the planets do not move in perfect circles round the fun; there is a greater diftance between their orbits in fome places, than in others. For inftance, the diftance between the orbit of Mars and Venus is near half as great again in one part of their orbits, as in the oppofite place. Now here the fluid, in which the earth fhould fwim, muft move with a lefs rapid motion, where there is this greater interval between the contiguous orbits; but on the contrary, where the fpace is ftraiteft, the earth moves more flowly, than where it is wideft [d].

[a] Philof. princ. prop. 52. coroll. 4.
[b] Ibid.
[c] Coroll. 11.
[d] See ibid. fchol. poft prop. 53.

13. Farther

13. FARTHER, if this our globe of earth swam in a fluid of equal density with the earth it self, that is, in a fluid more dense than water; all bodies put in motion here upon the earth's surface must suffer a great resistance from it ; where as, by Sir ISAAC NEWTON's experiments mentioned in the preceding chapter, bodies, that fell perpendicularly down through the air, felt about $\frac{1}{860}$ part only of the resistance, which bodies suffered that fell in like manner through water.

14. Sir ISAAC NEWTON applies these experiments yet farther, and examines by them the general question concerning the absolute plenitude of space. According to the Aristotelians, all space was full without any the least vacuities whatever. DESCARTES embraced the same opinion, and therefore supposed a subtile fluid matter, which should pervade all bodies, and adequately fill up their pores. The Atomical philosophers, who suppose all bodies both fluid and solid to be composed of very minute but solid atoms, assert that no fluid, how subtile soever the particles or atoms whereof it is composed should be, can ever cause an absolute plenitude ; because it is impossible that any body can pass through the fluid without putting the particles of it into such a motion, as to separate them, at least in part, from one another, and so perpetually to cause small vacuities ; by which these Atomists endeavour to prove, that a vacuum, or some space empty of all matter, is absolutely necessary to be in nature. Sir ISAAC NEWTON objects against the filling of space with such a subtile fluid, that all bodies in motion must be unmeasurably re-

Z   sisted

fifted by a fluid fo denfe, as abfolutely to fill up all the fpace, through which it is fpread. And left it fhould be thought, that this objection might be evaded by afcribing to this fluid fuch very minute and fmooth parts, as might remove all adhefion or friction between them, whereby all refiftance would be loft, which this fluid might otherwife give to bodies moving in it; Sir Isaac Newton proves, in the manner above related, that fluids refift from the power of inactivity of their particles; and that water and the air refift almoft entirely on this account : fo that in this fubtile fluid, however minute and lubricated the particles, which compofe it, might be; yet if the whole fluid was as denfe as water, it would refift very near as much as water does; and whereas fuch a fluid, whofe parts are abfolutely clofe together without any intervening fpaces, muft be a great deal more denfe than water, it muft refift more than water in proportion to its greater denfity; unlefs we will fuppofe the matter, of which this fluid is compofed, not to be endued with the fame degree of inactivity as other matter. But if you deprive any fubftance of the property fo univerfally belonging to all other matter, without impropriety of fpeech it can fcarce be called by this name.

15. Sir Isaac Newton made alfo an experiment to try in particular, whether the internal parts of bodies fuffered any refiftance. And the refult did indeed appear to favour fome fmall degree of refiftance; but fo very little, as to leave it doubtful, whether the effect did not arife from fome other latent caufe[a].

[a] Princ. philof. pag. 316, 317.

CHAP.

CHAP. II.

Concerning the caufe, which keeps in motion the primary planets.

SINCE the planets move in a void fpace and are free from refiftance ; they, like all other bodies, when once in motion, would move on in a ftraight line without end, if left to themfelves. And it is now to be explained what kind of action upon them carries them round the fun. Here I fhall treat of the primary planets only, and dif-courfe of the fecondary apart in the next chapter. It has been juft now declared, that thefe primary planets move fo about the fun, that a line extended from the fun to the planet, will, by accompanying the planet in its motion, pafs over equal fpaces in equal portions of time [a]. And this one property in the motion of the planets proves, that they are continually acted on by a power directed perpetually to the fun as a center. This therefore is one property of the caufe, which keeps the planets in their courfes, that it is a centripetal power, whofe center is the fun.

2. AGAIN, in the chapter upon centripetal forces [b] it was obferv'd, that if the ftrength of the centripetal power was fuitably accommodated every where to the motion of any body round a center, the body might be carried in

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[a] Ch. 1. § 7.   [b] Book I. Ch. 3.

any

any bent line whatever, whose concavity should be every where turned towards the center of the force. It was farther remarked, that the strength of the centripetal force, in each place, was to be collected from the nature of the line, wherein the body moved[a]. Now since each planet moves in an ellipsis, and the sun is placed in one focus; Sir Isaac Newton deduces from hence, that the strength of this power is reciprocally in the duplicate proportion of the distance from the sun. This is deduced from the properties, which the geometers have discovered in the ellipsis. The process of the reasoning is not proper to be enlarged upon here; but I shall endeavour to explain what is meant by the reciprocal duplicate proportion. Each of the terms reciprocal proportion, and duplicate proportion, has been already defined[b]. Their sense when thus united is as follows. Suppose the planet moved in the orbit A B C (in fig. 93.) about the sun in S. Then, when it is said, that the centripetal power, which acts on the planet in A, bears to the power acting on it in B a proportion, which is the reciprocal of the duplicate proportion of the distance S A to the distance S B; it is meant that the power in A bears to the power in B the duplicate of the proportion of the distance S B to the distance S A. The reciprocal duplicate proportion may be explained also by numbers as follows. Suppose several distances to bear to each other proportions expressed by the numbers 1, 2, 3, 4, 5; that is, let the second distance be double the first, the third be three times, the fourth four times, and the fifth five times as great as the

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[a] Book I. Ch. 3. § 29.   [b] Ibid. Ch. 2. § 30, 17.

first,

firſt. Multiply each of theſe numbers by it ſelf, and 1 multi-
plied by 1 produces ſtill 1, 2 multiplied by 2 produces 4, 3
by 3 makes 9, 4 by 4 makes 16, and 5 by 5 gives 25. This
being done, the fractions $\frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$, will reſpectively expreſs
the proportion, which the centripetal power in each of the
following diſtances bears to the power at the firſt diſtance: for
in the ſecond diſtance, which is double the firſt, the centri-
petal power will be one fourth part only of the power at the
firſt diſtance ; at the third diſtance the power will be one
ninth part only of the firſt power ; at the fourth diſtance,
the power will be but one ſixteenth part of the firſt ; and at
the fifth diſtance, one twenty fifth part of the firſt power.

3. Thus is found the proportion, in which this centripetal
power decreaſes, as the diſtance from the ſun increaſes, within
the compaſs of one planet's motion. How it comes to paſs,
that the planet can be carried about the ſun by this centripetal
power in a continual round, ſometimes riſing from the ſun,
then deſcending again as low, and from thence be carried
up again as far remote as before, alternately riſing and falling
without end ; appears from what has been written above con-
cerning centripetal forces: for the orbits of the planets re-
ſemble in ſhape the curve line propoſed in § 17 of the chapter
on theſe forces [a].

4. But farther, in order to know whether this centripetal
force extends in the ſame proportion throughout, and conſe-
quently whether all the planets are influenced by the very ſame

[a] Book I. Ch. 3.

power,

power, our author proceeds thus. He inquires what relation there ought to be between the periods of the different planets, provided they were acted upon by the fame power decreafing throughout in the forementioned proportion; and he finds, that the period of each in this cafe would have that very relation to the greater axis of its orbit, as I have declared above [a] to be found in the planets by the obfervations of aftronomers. And this puts it beyond queftion, that the different planets are preffed towards the fun, in the fame proportion to their diftances, as one planet is in its feveral diftances. And thence in the laft place it is juftly concluded, that there is fuch a power acting towards the fun in the forefaid proportion at all diftances from it.

5. This power, when referred to the planets, our author calls centripetal, when to the fun attractive; he gives it likewife the name of gravity, becaufe he finds it to be of the fame nature with that power of gravity, which is obferved in our earth, as will appear hereafter [b]. By all thefe names he defigns only to fignify a power endued with the properties before mentioned; but by no means would he have it underftood, as if thefe names referred any way to the caufe of it. In particular in one place where he ufes the name of attraction, he cautions us exprefsly againft implying any thing but a power directing a body to a center without any reference to the caufe of it, whether refiding in that center, or arifing from any external impulfe [c].

[a] Ch. 1. § 7.   [b] Chap. 5. § 8.   [c] Princ. pag. 60.

6. But

6. But now, in thefe demonftrations fome very minute inequalities in the motion of the planets are neglected; which is done with a great deal of judgment; for whatever be their caufe, the effects are very inconfiderable, they being fo exceeding fmall, that fome aftronomers have thought fit wholly to pafs them by [a]. However the excellency of this philofophy, when in the hands of fo great a geometer as our author, is fuch, that it is able to trace the leaft variations of things up to their caufes. The only inequalities, which have been obferved common to all the planets, are the motion of the aphelion and the nodes The tranfverfe axis of each orbit does not always remain fixed, but moves about the fun with a very flow progreffive motion: nor do the planets keep conftantly the fame plane, but change them, and the lines in which thofe planes interfect each other by infenfible degrees. The firft of thefe inequalities, which is the motion of the aphelion, may be accounted for, by fuppofing the gravitation of the planets towards the fun to differ a little from the forementioned reciprocal duplicate proportion of the diftances; but the fecond, which is the motion of the nodes, cannot be accounted for by any power directed towards the fun; for no fuch can give the planet any lateral impulfe to divert it from the plane of its motion into any new plane, but of neceffity muft be derived from fome other center. Where that power is lodged, remains to be difcovered. Now it is proved, as fhall be explained in the following chapter, that the three primary planets Saturn, Jupiter, and the earth, which have fatellites revolving about them, are endued with a power of

[a] Street, in Aftron. Carolin.

caufing

caufing bodies, in particular thofe fatellites, to gravitate towards them with a force, which is reciprocally in the duplicate proportion of their diftances; and the planets are in all refpects, in which they come under our examination, fo fimilar and alike, that there is no reafon to queftion, but they have all the fame property. Though it be fufficient for the prefent purpofe to have it proved of Jupiter and Saturn only; for thefe planets contain much greater quantities of matter than the reft, and proportionally exceed the others in power [a]. But the influence of thefe two planets being allowed, it is evident how the planets come to fhift continually their planes: for each of the planets moving in a different plane, the action of Jupiter and Saturn upon the reft will be oblique to the planes of their motion; and therefore will gradually draw them into new ones. The fame action of thefe two planets upon the reft will caufe likewife a progreffive motion of the aphelion; fo that there will be no neceffity of having recourfe to the other caufe for this motion, which was before hinted at [b]; viz, the gravitation of the planets towards the fun differing from the exact reciprocal duplicate proportion of the diftances. And in the laft place, the action of Jupiter and Saturn upon each other will produce in their motions the fame inequalities, as their joint action produces in the reft. All this is effected in the fame manner, as the fun produces the fame kind of inequalities and many others in the motion of the moon and the other fecondary planets; and therefore will be beft apprehended by what fhall be faid in the next chapter.

[a] See Chap. 5. § 9, &c.　　　　　[b] In the foregoing page.

Thofe

Thofe other irregularities in the motion of the fecondary planets have place likewife here ; but are too minute to be obfervable: becaufe they are produced and rectified alternately, for the moft part in the time of a fingle revolution ; whereas the motion of the aphelion and nodes, which continually increafe, become fenfible in a long feries of years. Yet fome of thefe other inequalities are difcernible in Jupiter and Saturn, in Saturn chiefly ; for when Jupiter, who moves fafter than Saturn, approaches near to a conjunction with him, his action upon Saturn will a little retard the motion of that planet, and by the reciprocal action of Saturn he will himfelf be accelerated. After conjunction, Jupiter will again accelerate Saturn, and be likewife retarded in the fame degree, as before the firft was retarded and the latter accelerated. Whatever inequalities befides are produced in the motion of Saturn by the action of Jupiter upon that planet, will be fufficiently rectified, by placing the focus of Saturn's ellipfis, which fhould otherwife be in the fun, in the common center of gravity of the fun and Jupiter. And all the inequalities in the motion of Jupiter, caufed by Saturn's action upon him, are much lefs confiderable than the irregularities of Saturn's motion[a].

7. THIS one principle therefore of the planets having a power, as well as the fun, to caufe bodies to gravitate towards them, which is proved by the motion of the fecondary planets to obtain in fact, explains all the irregularities relating to the planets ever obferved by aftronomers.

[a] See Newton. Princ. Lib. III. prop. 13.

A a   8. Sir

8. Sir Isaac Newton after this proceeds to make an improvement in aftronomy by applying this theory to the farther correction of their motions. For as we have here obferved the planets to poffefs a principle of gravitation, as well as the fun ; fo it will be explained at large hereafter, that the third law of motion, which makes action and reaction equal, is to be applied in this cafe [a] ; and that the fun does not only attract each planet, but is it felf alfo attracted by them ; the force, wherewith the planet is acted on, bearing to the force, wherewith the fun it felf is acted on at the fame time, the proportion, which the quantity of matter in the fun bears to the quantity of matter in the planet. From the action between the fun and planet being thus mutual Sir Isaac Newton proves that the fun and planet will defcribe about their common center of gravity fimilar ellipfis's; and then that the tranfverfe axis of the ellipfis defcribed thus about the moveable fun, will bear to the tranfverfe axis of the ellipfis, which would be defcribed about the fun at reft in the fame time, the fame proportion as the quantity of folid matter in the fun and planet together bears to the firft of two mean proportionals between this quantity and the quantity of matter in the fun only [b].

9. Above, where I fhewed how to find a cube, that fhould bear any proportion to another cube [c], the lines F T and T S are two mean proportionals between E F and F G ; and counting from E F, F T is called the firft, and F S the fecond of thofe means. In numbers thefe mean proportionals

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[a] Chap. 5. § 10.   [b] Princ. Lib. I. prop. 60.   [c] Book I. Chap. 2. § 80.

are

are thus found. Suppose A and B two numbers, and it be required to find C the first, and D the second of the two mean proportionals between them. First   A C
multiply A by it self, and the product multiply   B D ·
by B; then C will be the number which in arithmetic is called the cubic root of this last product; that is, the number C being multiplied by it self, and the product again multiplied by the same number C, will produce the product above mentioned. In like manner D is the cubic root of the product of B multiplied by it self, and the produce of that multiplication multiplied again by A.

10. It will be asked, perhaps, how this correction can be admitted, when the cause of the motions of the planets was before found by supposing the sun the center of the power, which acted upon them: for according to the present correction this power appears rather to be directed to their common center of gravity. But whereas the sun was at first concluded to be the center, to which the power acting on the planets was directed, because the spaces described round the sun in equal times were found to be equal; so Sir Isaac Newton proves, that if the sun and planet move round their common center of gravity, yet to an eye placed in the planet, the spaces, which will appear to be described about the sun, will have the same relation to the times of their description, as the real spaces would have, if the sun were at rest [a]. I farther asserted, that, supposing the planets to move round the sun at rest,

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[a] Princ. philof. Lib. I. prop. 58. coroll. 3.

A a 2   and

and to be attracted by a power, which every where should act with degrees of strength reciprocally in the duplicate proportion of the distances ; then the periods of the planets must observe the same relation to their distances, as astronomers find them to do. But here it must not be supposed, that the observations of astronomers absolutely agree without any the least difference ; and the present correction will not cause a deviation from any one astronomer's observations, so much as they differ from one another. For in Jupiter, where this correction is greatest, it hardly amounts to the 3000th part of the whole axis.

11. Upon this head I think it not improper to mention a reflection made by our excellent author upon these small inequalities in the planets motions; which contains under it a very strong philosophical argument against the eternity of the world. It is this, that these inequalities must continually increase by slow degrees, till they render at length the present frame of nature unfit for the purposes, it now serves [a]. And a more convincing proof cannot be desired against the present constitution's having existed from eternity than this, that a certain period of years will bring it to an end. I am aware this thought of our author has been represented even as impious, and as no less than casting a reflection upon the wisdom of the author of nature, for framing a perishable work. But I think so bold an assertion ought to have been made with singular caution. For if this remark upon the increasing irregularities of the heavenly motions

[a] Newt. Optics, pag. 378.

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be

be true in fact, as it really is, the imputation must return upon the asserter, that this does detract from the divine wisdom. Certainly we cannot pretend to know all the omniscient Creator's purposes in making this world, and therefore cannot undertake to determine how long he designed it should last. And it is sufficient, if it endure the time intended by the author. The body of every animal shews the unlimited wisdom of its author no less, nay, in many respects more, than the larger frame of nature ; and yet we see, they are all designed to last but a small space of time.

12. There need nothing more be said of the primary planets ; the motions of the secondary shall be next considered.

Chap. III.

Of the motion of the MOON and the other SECONDARY PLANETS.

THE excellency of this philosophy sufficiently appears from its extending in the manner, which has been related, to the minutest circumstances of the primary planets motions ; which nevertheless bears no proportion to the vast success of it in the motions of the secondary ; for it not only accounts for all the irregularities, by which their motions were known to be disturbed, but has discovered others so complicated, that astronomers were never able to distinguish them, and reduce them under proper heads ; but these were only to be

found

found out from their caufes, which this philofophy has brought to light, and has fhewn the dependence of thefe inequalities upon fuch caufes in fo perfect a manner, that we not only learn from thence in general, what thofe inequalities are, but are able to compute the degree of them. Of this Sir Is. Newton has given feveral fpecimens, and has moreover found means to reduce the moon's motion fo completely to rule, that he has framed a theory, from which the place of that planet may at all times be computed, very nearly or altogether as exactly, as the places of the primary planets themfelves, which is much beyond what the greateft aftronomers could ever effect.

2. The firft thing demonftrated of thefe fecondary planets is, that they are drawn towards their refpective primary in the fame manner as the primary planets are attracted by the fun. That each fecondary planet is kept in its orbit by a power pointed towards the center of the primary planet, about which the fecondary revolves; and that the power, by which the fecondaries of the fame primary are influenced, bears the fame relation to the diftance from the primary, as the power, by which the primary planets are guided, does in regard to the diftance from the fun [a]. This is proved in the fatellites of Jupiter and Saturn, becaufe they move in circles, as far as we can obferve, about their refpective primary with an equable courfe, the refpective primary being the center of each orbit: and by comparing the times, in which the different fatellites of the fame primary perform their periods, they are

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[a] Newton. Princ. Lib. III. prop. 1.

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found to obferve the fame relation to the diftances from their primary, as the primary planets obferve in refpect of their mean diftances from the fun[a]. Here thefe bodies moving in circles with an equable motion, each fatellite paffes over equal parts of its orbit in equal portions of time ; confequently the line drawn from the center of the orbit, that is, from the primary planet, to the fatellite, will pafs over equal fpaces along with the fatellite in equal portions of time; which proves the power, by which each fatellite is held in its orbit, to be pointed towards the primary as a center[b]. It is alfo manifeft that the centripetal power, which carries a body in a circle concentrical with the power, acts upon the body at all times with the fame ftrength. But Sir Isaac Newton demonftrates that, when bodies are carried in different circles by centripetal powers directed to the centers of thofe circles, then the degrees of ftrength of thofe powers are to be compared by confidering the relation between the times, in which the bodies perform their periods through thofe circles[c]; and in particular he fhews, that if the periodical times bear that relation, which I have juft now afferted the fatellites of the fame primary to obferve; then the centripetal powers are reciprocally in the duplicate proportion of the femidiameters of the circles, or in that proportion to the diftances of the bodies from the centers[d]. Hence it follows that in the planets Jupiter and Saturn, the centripetal power in each decreafes with the increafe of diftance, in the fame proportion as the centripetal

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[a] Newton. Princ. Lib. III. pag. 390, 391. compared with pag. 393.
[b] Book I. Ch. 2. § 29.

[c] Princ. philof. Lib. I. prop. 4.
[d] Ibid. coroll.

power

power appertaining to the fun decreafes with the increafe of diftance. I do not here mean that this proportion of the centripetal powers holds between the power of Jupiter at any diftance compared with the power of Saturn at any other diftance; but only in the change of ftrength of the power belonging to the fame planet at different diftances from him. Moreover what is here difcovered of the planets Jupiter and Saturn by means of the different fatellites, which revolve round each of them, appears in the earth by the moon alone; becaufe fhe is found to move round the earth in an ellipfis after the fame manner as the primary planets do about the fun; excepting only fome fmall irregularities in her motion, the caufe of which will be particularly explained in what follows, whereby it will appear, that they are no objection againft the earth's acting on the moon in the fame manner as the fun acts on the primary planets; that is, as the other primary planets Jupiter and Saturn act upon their fatellites. Certainly fince thefe irregularities can be otherwife accounted for, we ought not to depart from that rule of induction fo neceffary in philofophy, that to like bodies like properties are to be attributed, where no reafon to the contrary appears. We cannot therefore but afcribe to the earth the fame kind of action upon the moon, as the other primary planets Jupiter and Saturn have upon their fatellites; which is known to be very exactly in the proportion affigned by the method of comparing the periodical times and diftances of all the fatellites, which move about the fame planet; this abundantly compenfating our not being near enough to obferve the exact figure of their orbits. For if the little deviation of the moon's orbit

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orbit from a true permanent ellipfis arofe from the action of the earth upon the moon not being in the exact reciprocal dupli- cate proportion of the diftance, were another moon to revolve about the earth, the proportion between the periodical times of this new moon, and the prefent, would difcover the devia- tion from the mentioned proportion much more manifeftly.

3. B y the number of fatellites, which move round Jupiter and Saturn, the power of each of thefe planets is meafured in a great diverfity of diftance; for the diftance of the outermoft fatellite in each of thefe planets exceeds feveral times the dift- ance of the innermoft. In Jupiter the aftronomers have ufually placed the innermoft fatellite at a diftance from the center of that planet equal to about $5\frac{2}{3}$ of the femidiameters of Jupiter's body, and this fatellite performs its revolution in about 1 day $18\frac{1}{2}$ hours. The next fatellite, which revolves round Jupiter in about 3 days $13\frac{1}{3}$ hours, they place at the diftance from Jupiter of about 9 of that planet's femidiameters.   To the third fa- tellite, which performs its period nearly in 7 days $3\frac{3}{4}$ hours, they affign the diftance of about $14\frac{2}{3}$ femidiameters.   But the outermoft fatellite they remove to $25\frac{1}{3}$ femidiameters, and this fatellite makes its period in about 16 days $16\frac{1}{2}$ hours [a]. In Saturn there is ftill a greater diverfity in the diftance of the feveral fatellites.   By the obfervations of the late C a s s i n i, a celebrated aftronomer in France, who firft difcovered all thefe fatellites, except one known before, the innermoft is diftant about $4\frac{1}{2}$ of Saturn's femidiameters from his center,   and re-

[a] Newt. Princ. philof. Lib. III. pag 390.

B b   rolves

volves round in about 1 day 21⅓ hours. The next satellite is distant about 5¾ semidiameters, and makes its period in about 2 days 17⅔ hours. The third is removed to the distance of about 8 semidiameters, and performs its revolution in near 4 days 12½ hours. The fourth satellite discovered first by the great Huygens, is near 18⅔ semidiameters, and moves round Saturn in about 15 days 22⅔ hours. The outermost is distant 56 semidiameters, and makes its revolution in about 79 days 7¼ hours [a]. Besides these satellites, there belongs to the planet Saturn another body of a very singular kind. This is a shining, broad, and flat ring, which encompasses the planet round. The diameter of the outermost verge of this ring is more than double the diameter of Saturn. Huygens, who first described this ring, makes the whole diameter thereof to bear to the diameter of Saturn the proportion of 9 to 4. The late reverend Mr. Pound makes the proportion something greater, viz. that of 7 to 3. The distances of the satellites of this planet Saturn are compared by Cassini to the diameter of the ring. His numbers I have reduced to those above, according to Mr. Pound's proportion between the diameters of Saturn and of his ring. As this ring appears to adhere no where to Saturn, so the distance of Saturn from the inner edge of the ring seems rather greater than the breadth of the ring. The distances, which have here been given, of the several satellites, both for Jupiter and Saturn, may be more depended on in relation to the proportion, which those belonging to the same primary planet

[a] Newt. Princ. philos. Lib. III. pag. 391, 392.

bear.

bear one to another, than in refpect to the very numbers, that have been here fet down, by reafon of the difficulty there is in meafuring to the greateft exactnefs the diameters of the primary planets; as will be explained hereafter, when we come to treat of telefcopes [a]. By the obfervations of the forementioned Mr. P o u n d, in Jupiter the diftance of the innermoft fatellite fhould rather be about 6 femidiameters, of the fecond $9\frac{1}{2}$, of the third 15, and of the outermoft $26\frac{2}{3}$ [b]; and in Saturn the diftance of the innermoft fatellite 4 femidiameters, of the next $6\frac{1}{4}$, of the third $8\frac{3}{4}$, of the fourth $20\frac{1}{3}$, and of the fifth 59 [c]. However the proportion between the diftances of the fatellites in the fame primary is the only thing necefary to the point we are here upon.

4. But moreover the force, wherewith the earth acts in different diftances, is confirmed from the following confideration, yet more exprefly than by the preceding analogical reafoning. It will appear, that if the power of the earth, by which it retains the moon in her orbit, be fuppofed to act at all diftances between the earth and moon, according to the forementioned rule; this power will be fufficient to produce upon bodies, near the furface of the earth, all the effects afcribed to the principle of gravity. This is difcovered by the following method. Let A (in fig. 94.) reprefent the earth, B the moon, B C D the moon's orbit, which differs little from a circle, of which A is the center. If the moon in B were left to it felf to move with the velocity, it has in the point B, it

[a] Book III. Ch. 4.   [b] Newt, Princ. philof. Lib. III, pag. 391.   [c] Ibid, pag. 392.

Bb 2   would

would leave the orbit, and proceed right forward in the line B E, which touches the orbit in B. Suppose the moon would upon this condition move from B to E in the space of one minute of time. By the action of the earth upon the moon, whereby it is retained in its orbit, the moon will really be found at the end of this minute in the point F, from whence a straight line drawn to A shall make the space B F A in the circle equal to the triangular space B E A ; so that the moon in the time wherein it would have moved from B to E, if left to it self, has been impelled towards the earth from E to F. And when the time of the moon's passing from B to F is small, as here it is only one minute, the distance between E and F scarce differs from the space, through which the moon would descend in the same time, if it were to fall directly down from B toward A without any other motion. A B the distance of the earth and moon is about 60 of the earth's semidiameters, and the moon completes her revolution round the earth in about 27 days 7 hours and 43 minutes: therefore the space E F will here be found by computation to be about $16\frac{1}{8}$ feet. Consequently, if the power, by which the moon is retained in its orbit, be near the surface of the earth greater, than at the distance of the moon in the duplicate proportion of that distance; the number of feet, a body would descend near the surface of the earth by the action of this power upon it in one minute of time, would be equal to $16\frac{1}{8}$ multiplied twice into the number 60, that is, equal to 58050. But how fast bodies fall near the surface of the earth may be known by the pendulum[a]; and

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[a] See Book I. Ch. 2. § 60, 64.

by the exacteſt experiments they are found to deſcend the ſpace of $16\frac{1}{3}$ feet in a ſecond of time; and the ſpaces deſcribed by falling bodies being in the duplicate proportion of the times of their fall [a], the number of feet, a body would deſcribe in its fall near the ſurface of the earth in one minute of time, will be equal to $16\frac{1}{3}$ twice multiplied by 60, the ſame as would be cauſed by the power which acts upon the moon.

5. In this computation the earth is ſuppoſed to be at reſt, whereas it would have been more exact to have ſuppoſed it to move, as well as the moon, about their common center of gravity; as will eaſily be underſtood, by what has been ſaid in the preceding chapter, where it was ſhewn, that the ſun is ſubjected to the like motion about the common center of gravity of it ſelf and the planets. The action of the ſun upon the moon, which is to be explain'd in what follows, is likewiſe here neglected: and Sir Isaac Newton ſhews, if you take in both theſe conſiderations, the preſent computation will beſt agree to a ſomewhat greater diſtance of the moon and earth, viz. to $60\frac{1}{2}$ ſemidiameters of the earth, which diſtance is more conformable to aſtronomical obſervations.

6. These computations afford an additional proof, that the action of the earth obſerves the ſame proportion to the diſtance, which is here contended for. Before I ſaid, it was reaſonable to conclude ſo by induction from the pla-

[a] Book I. Ch. 2. § 17.

nets

nets Jupiter and Saturn; becaufe they act in that manner. But now the fame thing will be evident by drawing no other confequence from what is feen in thofe planets, than that the power, by which the primary planets act on their fecondary, is extended from the primary through the whole interval between, fo that it would act in every part of the intermediate fpace. In Jupiter and Saturn this power is fo far from being confined to a fmall extent of diftance, that it not only reaches to feveral fatellites at very different diftances, but alfo from one planet to the other, nay even through the whole planetary fyftem [a]. Confequently there is no appearance of reafon, why this power fhould not act at all diftances, even at the very furfaces of thefe planets as well as farther off. But from hence it follows, that the power, which retains the moon in her orbit, is the fame, as caufes bodies near the furface of the earth to gravitate. For fince the power, by which the earth acts on the moon, will caufe bodies near the furface of the earth to defcend with all the velocity they are found to do, it is certain no other power can act upon them befides; becaufe if it did, they muft of neceffity defcend fwifter. Now from all this it is at length very evident, that the power in the earth, which we call gravity, extends up to the moon, and decreafes in the duplicate proportion of the increafe of the diftance from the earth.

7. This finifhes the difcoveries made in the action of the primary planets upon their fecondary. The next thing

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[a] See Ch. II. § 6.

to be fhewn is, that the fun acts upon them likewife: for this purpofe it is to be obferved, that if to the motion of the fatellite, whereby it would be carried round its primary at reft, be fuperadded the fame motion both in regard to velocity and direction, as the primary it felf has, it will defcribe about the primary the fame orbit, with as great regularity, as if the primary was indeed at reft. The caufe of this is that law of motion, which makes a body near the furface of the earth, when let fall, to defcend perpendicularly, though the earth be in fo fwift a motion, that if the falling body did not partake of it, its defcent would be remarkably oblique; and that a body projected defcribes in the moft regular manner the fame parabola, whether projected in the direction, in which the earth moves, or in the oppofite direction, if the projecting force be the fame [a]. From this we learn, that if the fatellite moved about its primary with perfect regularity, befides its motion about the primary, it would participate of all the motion of its primary; have the fame progreffive velocity, with which the primary is carried about the fun; and be impelled with the fame velocity as the primary towards the fun, in a direction parallel to that impulfe of its primary. And on the contrary, the want of either of thefe, in particular of the impulfe towards the fun, will occafion great inequalities in the motion of the fecondary planet. The inequalities, which would arife from the abfence of this impulfe towards the fun are

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[a] The fecond of the laws of motion laid down in Book I. Ch. 3.

fo great, that by the regularity, which appears in the motion of the fecondary planets, it is proved, that the fun communicates the fame velocity to them by its action, as it gives to their primary at the fame diftance. For Sir Isaac Newton informs us, that upon examination he found, that if any of the fatellites of Jupiter were attracted by the fun more or lefs, than Jupiter himfelf at the fame diftance, the orbit of that fatellite, inftead of being concentrical to Jupiter, muft have its center at a greater or lefs diftance, than the center of Jupiter from the fun, nearly in the fubduplicate proportion of the difference between the fun's action upon the fatellite, and upon Jupiter; and therefore if any fatellite were attracted by the fun but $\frac{1}{1000}$ part more or lefs, than Jupiter is at the fame diftance, the center of the orbit of that fatellite would be diftant from the center of Jupiter no lefs than a fifth part of the diftance of the outermoft fatellite from Jupiter [a]; which is almoft the whole diftance of the innermoft fatellite. By the like argument the fatellites of Saturn gravitate towards the fun, as much as Saturn it felf at the fame diftance; and the moon as much as the earth.

8. Thus is proved, that the fun acts upon the fecondary planets, as much as upon the primary at the fame diftance: but it was found in the laft chapter, that the action of the fun upon bodies is reciprocally in the duplicate proportion of the diftance; therefore the fecondary

[a] Newton. Princ. philof. Lib. III. prop. 6. pag. 401.

planets

planets being fometimes nearer to the fun than the pri-
mary, and fometimes more remote, they are not alway
acted upon in the fame degree with their primary, but
when nearer to the fun, are attracted more, and when far-
ther diftant, are attracted lefs.   Hence arife various inequali-
ties in the motion of the fecondary planets [a].

9. S O M E of thefe inequalities would take place, though
the moon, if undifturbed by the fun, would have moved in
a circle concentrical to the earth, and in the plane of the earth's
motion ; others depend on the elliptical figure, and the ob-
lique fituation of the moon's orbit.   One of the firft kind is,
that the moon is caufed fo to move, as not to defcribe equal
fpaces in equal times, but is continually accelerated, as fhe
paffes from the quarter to the new or full, and is retarded
again by the like degrees in returning from the new and full
to the next quarter.   Here we confider not fo much the ab-
folute, as the apparent motion of the moon in refpect to us.

10. T H E principles of aftronomy teach how to diftin-
guifh thefe two motions.   Let S (in fig. 95.) reprefent the
fun, A the earth moving in its orbit B C, D E F G the moon's
orbit, the place of the moon H.   Suppofe the earth to have
moved from A to I.   Becaufe it has been fhewn, that the
moon partakes of all the progreffive motion of the earth ; and
likewife that the fun attracts both the earth and moon equal-
ly, when they are at the fame diftance from it, or that the
mean action of the fun upon the moon is equal to its action

[a] Newton's Princ. philof. Lib. III. prop. 22, 23.

C c   upon

upon the earth : we muſt therefore conſider the earth as car-rying about with it the moon's orbit ; ſo that when the earth is removed from A to I, the moon's orbit ſhall like-wiſe be removed from its former ſituation into that denoted by KLMN. But now the earth being in I, if the moon were found in O, ſo that OI ſhould be parallel to HA, though the moon would really have moved from H to O, yet it would not have appeared to a ſpectator upon the earth to have moved at all, becauſe the earth has moved as much it ſelf ; ſo that the moon would ſtill appear in the ſame place with reſpect to the fixed ſtars. But if the moon be obſerved in P, it will then appear to have moved, its apparent motion being meaſured by the angle under OIP. And if the angle under PIS be leſs than the angle under HAS, the moon will have approached nearer to its conjunction with the ſun.

11. To come now to the explication of the mentioned inequality in the moon's motion : let S (in fig. 96.) repreſent the ſun, A the earth, BCDE the moon's orbit, C the place of the moon, when in the latter quarter. Here it will be nearly at the ſame diſtance from the ſun, as the earth is. In this caſe there-fore they will both be equally attracted, the earth in the dire-ction AS, and the moon in the direction CS. Whence as the earth in moving round the ſun is continually deſcending to-ward it, ſo the moon in this ſituation muſt in any equal por-tion of time deſcend as much ; and therefore the poſition of the line AC in reſpect of AS, and the change, which the moon's motion produces in the angle under CAS, will not be altered by the ſun.

12. But

12. But now as soon as ever the moon is advanced from the quarter toward the new or conjunction, suppose to G, the action of the fun upon it will have a different effect. Here, were the fun's action upon the moon to be applied in the direction G H parallel to A S, if its action on the moon were equal to its action on the earth, no change would be wrought by the fun on the apparent motion of the moon round the earth. But the moon receiving a greater impulse in G than the earth receives in A, were the fun to act in the direction G H, yet it would accelerate the description of the space D A G, and caufe the angle under G A D to decreafe fafter, than otherwife it would. The fun's action will have this effect upon account of the obliquity of its direction to that, in which the earth attracts the moon. For the moon by this means is drawn by two forces oblique to each other, one drawing from G toward A, the other from G toward H, therefore the moon muft necefsarily be impelled toward D. Again, becaufe the fun does not act in the direction G H parallel to S A, but in the direction G S oblique to it, the fun's action on the moon will by reafon of this obliquity farther contribute to the moon's acceleration. Suppofe the earth in any fhort fpace of time would have moved from A to I, if not attracted by the fun; the point I being in the ftraight line C E, which touches the earth's orbit in A. Suppofe the moon in the fame time would have moved in her orbit from G to K, and befides have partook of all the progrefsive motion of the earth. Then if K L be drawn parallel to A I, and taken equal to it, the moon, if not attracted by the fun, would be found

in L. But the earth by the fun's action is removed from I. Suppofe it were moved down to M in the line I M N parallel to S A, and if the moon were attracted but as much, and in the fame direction, as the earth is here fuppofed to be attracted, fo as to have defcended during the fame time in the line L O, parallel alfo to A S, down as far as P, till L P were equal to I M, the angle under P M N would be equal to that under L I N, that is, the moon will appear advanced no farther forward, than if neither it nor the earth had been fubject to the fun's action. But this is upon the fuppofition, that the action of the fun upon the moon and earth were equal; whereas the moon being acted upon more than the earth, did the fun's action draw the moon in the line L O parallel to A S, it would draw it down fo far as to make L P greater than I M; whereby the angle under P M N will be rendred lefs, than that under L I N. But moreover, as the fun draws the earth in a direction oblique to I N, the earth will be found in its orbit fomewhat fhort of the point M; however the moon is attracted by the fun ftill more out of the line L O, than the earth is out of the line I N; therefore this obliquity of the fun's action will yet farther diminifh the angle under P M N.

13. Thus the moon at the point G receives an impulfe from the fun, whereby her motion is accelerated. And the fun producing this effect in every place between the quarter and the conjunction, the moon will move from the quarter with a motion continually more and more accelerated; and therefore by acquiring from time to time additional degrees

of

of velocity in its orbit, the spaces, which are described in equal times by the line drawn from the earth to the moon, will not be every where equal, but those toward the conjunction will be greater, than those toward the quarter. But now in the moon's passage from the conjunction D to the next quarter the sun's action will again retard the moon, till at the next quarter in E it be restored to the first velocity, which it had in C.

14. Again as the moon moves from E to the full or opposition to the sun in B, it is again accelerated, the deficiency of the sun's action upon the moon, from what it has upon the earth, producing here the same effect as before the excess of its action. Consider the moon in Q moving from E towards B. Here if the moon were attracted by the sun in a direction parallel to A S, yet being acted on less than the earth, as the earth descends toward the sun, the moon will in some measure be left behind. Therefore Q F being drawn parallel to S B, a spectator on the earth would see the moon move, as if attracted from the point Q in the direction Q F with a degree of force equal to that, whereby the sun's action on the moon falls short of its action on the earth. But the obliquity of the sun's action has also here an effect. In the time the earth would have moved from A to I without the influence of the sun, let the moon have moved in its orbit from Q to R. Drawing therefore R T parallel to A I, and equal to the same, for the like reason as before, the moon by the motion of its orbit, if not at all attracted by the sun, must be found in T; and therefore, if attracted in a direction parallel to S A, would

be

be in the line T V parallel to A S ; fuppofe in W.   But the
moon in Q being farther off the fun than the earth, it will be
lefs attracted, that is, T W will be lefs than I M, and if the
line S M be prolonged toward X, the angle under X M W
will be lefs than that under X I T.   Thus by the fun's action
the moon's paffage from the quarter to the full would be ac-
celerated, if the fun were to act on the earth and moon in a
direction parallel to A S : and the obliquity of the fun's ac-
tion will ftill more increafe this acceleration.   For the action
of the fun on the moon is oblique to the line S A the whole
time of the moon's paffage from Q to T, and will carry
the moon out of the line T V toward the earth. Here I fup-
pofe the time of the moon's paffage from Q to T fo fhort, that
it fhall not pafs beyond the line S A. The earth alfo will come
a little fhort of the line I N, as was faid before.   From thefe
caufes the angle under X M W will be ftill farther leffened.

15. The moon in paffing from the oppofition B to the
next quarter will be retarded again by the fame degrees, as
it is accelerated before its appulfe to the oppofition.   Becaufe
this action of the fun, which in the moon's paffage from the
quarter to the oppofition caufes it to be extraordinarily accele-
rated, and diminifhes the angle, which meafures its diftance
from the oppofition ; will make the moon flacken its pace af-
terwards, and retard the augmentation of the fame angle in
its paffage from the oppofition to the following quarter ; that
is, will prevent that angle from increafing fo faft, as otherwife
it would. And thus the moon, by the fun's action upon it, is
twice accelerated and twice reftored to its firft velocity, every
circuit

circuit it makes round the earth. This inequality of the moon's motion about the earth is called by aftronomers its variation.

16. The next effect of the fun upon the moon is, that it gives the orbit of the moon in the quarters a greater degree of curvature, than it would receive from the action of the earth alone ; and on the contrary in the conjunction and oppofition the orbit is lefs inflected.

17. When the moon is in conjunction with the fun in the point D, the fun attracting the moon more forcibly than it does the earth, the moon by that means is impelled lefs toward the earth, than otherwife it would be, and fo the orbit is lefs incurvated ; for the power, by which the moon is impelled toward the earth, being that, by which it is inflected from a rectilinear courfe, the lefs that power is, the lefs it will be inflected. Again, when the moon is in the oppofition in B, farther removed from the fun than the earth is ; it follows then, though the earth and moon are both continually defcending to the fun, that is, are drawn by the fun toward it felf out of the place they would otherwife move into, yet the moon defcends with lefs velocity than the earth ; infomuch that the moon in any given fpace of time from its paffing the point of oppofition will have lefs approached the earth, than otherwife it would have done, that is, its orbit in refpect of the earth will approach nearer to a ftraight line. In the laft place, when the moon is in the quarter in F, and equally diftant from the fun as the earth, we obferved before, that
the

the earth and moon would descend with equal pace to-ward the sun, so as to make no change by that descent in the angle under F A S ; but the length of the line F A must of necessity be shortned. Therefore the moon in moving from F toward the conjunction with the sun will be impelled more toward the earth by the sun's action, than it would have been by the earth alone, if neither the earth nor moon had been acted on by the sun ; so that by this additional impulse the orbit is rendred more curve, than it would otherwise be. The same effect will also be produced in the other quarter.

18. Another effect of the sun's action, consequent upon this we have now explained, is, that though the moon un-disturbed by the sun might move in a circle having the earth for its center ; by the sun's action, if the earth were to be in the very middle or center of the moon's orbit, yet the moon would be nearer the earth at the new and full, than in the quarters. In this probably will at first appear some difficulty, that the moon should come nearest to the earth, where it is least attracted to it, and be farthest off when most attracted. Which yet will appear evidently to follow from that very cause, by considering what was last shewn, that the orbit of the moon in the conjunction and opposition is ren-dred less curve ; for the less curve the orbit of the moon is, the less will the moon have descended from the place it would move into, without the action of the earth. Now if the moon were to move from any place without farther disturbance from that action, since it would proceed in the line, which would touch its orbit in that place, it would

recede

recede continually from the earth; and therefore if the power of the earth upon the moon, be sufficient to retain it at the same distance, this diminution of that power will cause the distance to increase, though in a less degree. But on the other hand in the quarters, the moon, being pressed more towards the earth than by the earth's single action, will be made to approach it; so that in passing from the conjunction or opposition to the quarters the moon ascends from the earth, and in passing from the quarters to the conjunction and opposition it descends again, becoming nearer in these last mentioned places than in the other.

19. ALL these forementioned inequalities are of different degrees, according as the sun is more or less distant from the earth; greater when the earth is nearest the sun, and less when it is farthest off. For in the quarters, the nearer the moon is to the sun, the greater is the addition to the earth's action upon it by the power of the sun; and in the conjunction and opposition, the difference between the sun's action upon the earth and upon the moon is likewise so much the greater.

20. THIS difference in the distance between the earth and the sun produces a farther effect upon the moon's motion; causing the orbit to dilate when less remote from the sun, and become greater, than when at a farther distance. For it is proved by Sir ISAAC NEWTON, that the action of the sun, by which it diminishes the earth's power over the moon, in the conjunction or opposition, is about twice as

great

great, as the addition to the earth's action by the fun in the quarters [a] ; fo that upon the whole, the power of the earth upon the moon is diminifhed by the fun, and therefore is moft diminifhed, when the action of the fun is ftrongeft : but as the earth by its approach to the fun has its influence leffened, the moon being lefs attracted will gradually recede from the earth ; and as the earth in its recefs from the fun recovers by degrees its former power, the orbit of the moon muft again contract. Two confequences follow from hence : the moon will be moft remote from the earth, when the earth is neareft the fun ; and alfo will take up a longer time in performing its revolution through the dilated orbit, than through the more contracted.

21. These irregularities the fun would produce in the moon, if the moon, without being acted on unequally by the fun, would defcribe a perfect circle about the earth, and in the plane of the earth's motion ; but though neither of thefe fuppofitions obtain in the motion of the moon, yet the forementioned inequalities will take place, only with fome difference in refpect to the degree of them; but the moon by not moving in this manner is fubject to fome other inequalities alfo. For as the moon defcribes, inftead of a circle concentrical to the earth, an ellipfis, with the earth in one focus, that ellipfis will be fubjected to various changes. It can neither preferve conftantly the fame pofition, nor yet the fame figure ; and becaufe the plane of this ellipfis is not the fame

[a] Newton. Princ. Lib. I. prop. 66. coroll. 7.

with

with that of the earth's orbit, the situation of the plane, where-in the moon moves, will continually change ; neither the line in which it intersects the plane of the earth's orbit, nor the inclination of the planes to each other, will remain for any time the same.   All these alterations offer themselves now to be explained.

22. I shall first consider the changes which are made in the plane of the moon's orbit.   The moon not moving in the same plane with the earth,   the sun is seldom in the plane of the moon's orbit, viz.   only when the line made by the common intersection of the two planes, if produced, will pass through the sun,   as is represented in fig. 97. where S denotes the sun ;   T the earth ;   A T B the earth's orbit de-scribed upon the plane of this scheme ;   C D E F the moon's orbit,   the part C D E being raised above,   and the part C F E depressed under the plane of this scheme.   Here the line C E, in which the plane of this scheme,   that is,   the plane of the earth's orbit and the plane of the moon's orbit intersect each other,   being continued passes through the sun in S.   When this happens, the action of the sun is directed in the plane of the moon's orbit,   and cannot draw the moon out of this plane, as will evidently appear to any one that shall consider the present scheme :   for suppose the moon in G,   and let a straight line be drawn from G to S,   the sun draws the moon in the direction of this line from G toward S: but this line lies in the plane of the orbit ; and if it be prolonged from S beyond G,   the continuation of it will lie on the plane C D E ;   for the plane itself, if sufficiently extended, will pass through the sun.

<div align="center">D d 2</div>

<div align="right">But</div>

But in other cafes the obliquity of the fun's action to the plane of the orbit will caufe this plane continually to change.

23. Suppose in the firft place, the line, in which the two planes interfect each other, to be perpendicular to the line which joins the earth and fun. Let T (in fig. 98, 99, 100, 101.) reprefent the earth; S the fun; the plane of this fcheme the plane of the earth's motion, in which both the fun and earth are placed. Let A C be perpendicular to S T, which joins the earth and fun; and let the line A C be that, in which the plane of the moon's orbit interfects the plane of the earth's motion. To the center T defcribe in the plane of the earth's motion the circle A B C D. And in the plane of the moon's orbit defcribe the circle A E C F, one half of which A E C will be elevated above the plane of this fcheme, the other half A F C as much depreffed below it.

24. Now fuppofe the moon to fet forth from the point A (in fig. 98.) in the direction of the plane A E C. Here fhe will be continually drawn out of this plane by the action of the fun: for this plane A E C, if extended, will not pafs through the fun, but above it; fo that the fun, by drawing the moon directly toward it felf, will force it continually more and more from that plane towards the plane of the earth's motion, in which it felf is; caufing it to defcribe the line A K G H I, which will be convex to the plane A E C, and concave to the plane of the earth's motion. But here this power of the fun, which is faid to draw the moon toward the plane of the earth's motion, muft be underftood principally of fo much only of the

Fig. 92.

Anhelion Saturn

Anhelion Jupiter

Fig. 96.

Fig. 97.

Fig. 91.

Fig. 92.

Fig. 93.

Fig. 94.

Fig. 95.
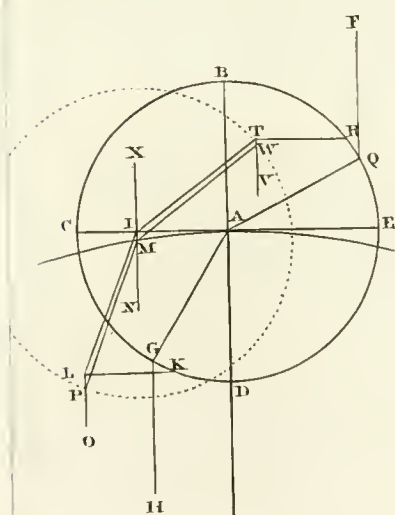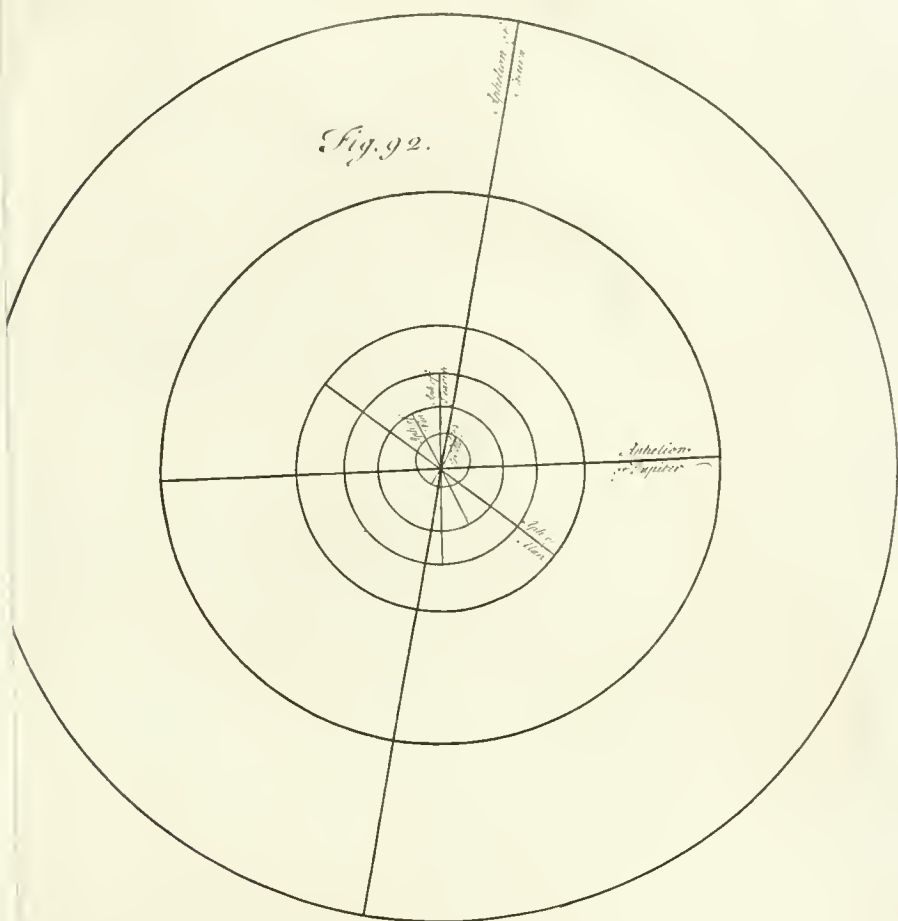
Fig. 96.
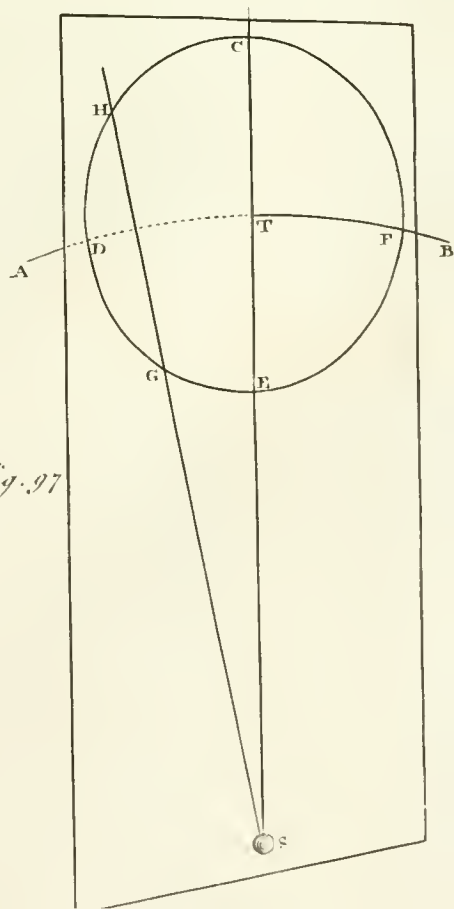
Fig. 97.
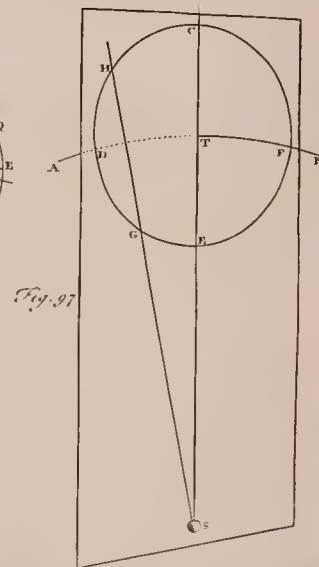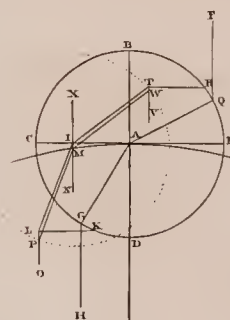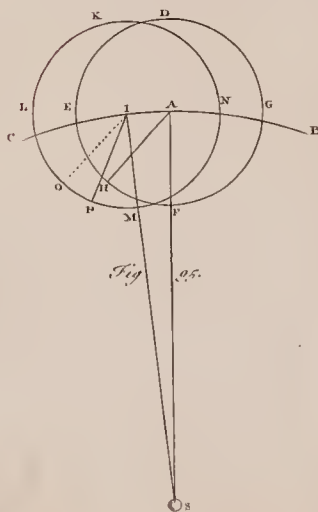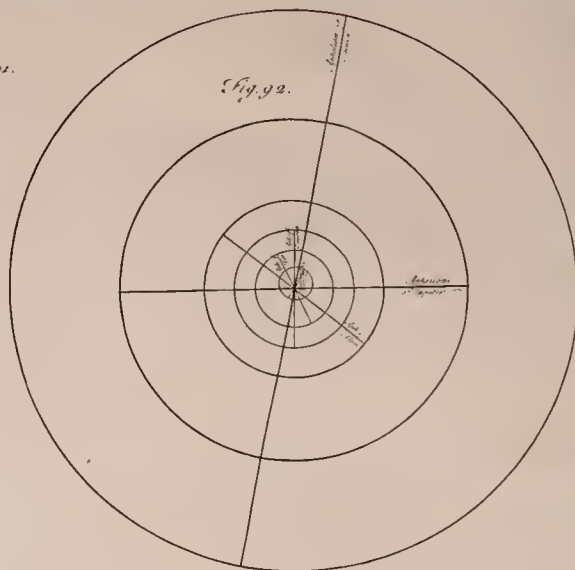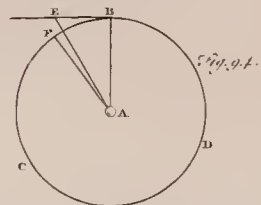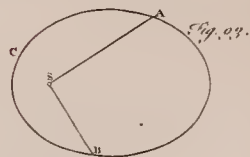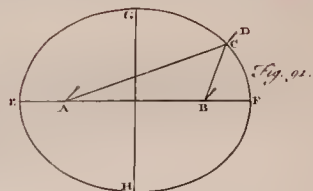
the fun's action upon the moon, as it exceeds the action of the
fame upon the earth.   For fuppofe the preceding figure to be
viewed by the eye, placed in the plane of that fcheme, and in
the line C T A on the fide of A, the plane A B C D will appear as
the ftraight line D T B, (in fig. 102.) and the plane A E C F as an-
other ftraight line F E; and the curve line A K G H I under the
form of the line T K G H I.   Now it is plain, that the earth and
moon being both attracted by the fun, if the fun's action up-
on both was equally ftrong, the earth T, and with it the plane
A E C F or line F T E in this fcheme, would be carried toward
the fun with as great a pace as the moon, and therefore the
moon not drawn out of it by the fun's action,   excepting
only from the fmall obliquity of the direction of this action
upon the moon to that of the fun's action upon the earth,
which arifes from the moon's being out of the plane of the
earth's motion, and is not very confiderable ;   but the action
of the fun upon the moon being greater than upon the earth,
all the time the moon is nearer to the fun than the earth is,
it will be drawn from the plane A E C or the line T E by
that excefs, and made to defcribe the curve line A G I or
T G I.   But it is the cuftom of aftronomers, inftead of con-
fidering the moon as moving in fuch a curve line,   to refer
its motion continually to the plane, which touches the true
line wherein it moves, at the point where at any time the
moon is.   Thus when the moon is in the point A, its motion
is confidered as being in the plane A E C, in whofe direction it
then effaies to move ; and when in the point K (in fig. 99.)
its motion is referred to the plane, which paffes through the
earth, and touches the line A K G H I in the point K.   Thus
the

the moon in paffing from A to I will continually change the plane of her motion. In what manner this change proceeds, I fhall now particularly explain.

2 5. Let the plane, which touches the line AKI in the point K (in fig. 99.) interfect the plane of the earth's orbit in the line LTM. Then, becaufe the line AKI is concave to the plane ABC, it falls wholly between that plane, and the plane which touches it in K; fo that the plane MKL will cut the plane AEC, before it meets with the plane of the earth's motion; fuppofe in the line YT, and the point A will fall between K and L. With a femidiameter equal to TY or TL defcribe the femi-circle LYM. Now to a fpectator on the earth the moon, when in A, will appear to move in the circle AECF, and, when in K, will appear to be moving in the femicircle LYM. The earth's motion is performed in the plane of this fcheme, and to a fpectator on the earth the fun will appear always moving in that plane. We may therefore refer the apparent motion of the fun to the circle ABCD, defcribed in this plane about the earth. But the points where this circle, in which the fun feems to move, interfects the circle in which the moon is feen at any time to move, are called the nodes of the moon's orbit at that time. When the moon is feen moving in the circle AECD, the points A and C are the nodes of the orbit; when fhe appears in the femicircle LYM, then L and M are the nodes. Now here it appears, from what has been faid, that while the moon has moved from A to K, one of the nodes has been carried from A to L, and the other as much from C to M. But the motion from A to L, and from C to

4

M, is

M, is backward in regard to the motion of the moon, which is the other way from A to K, and from thence toward C.

26. FARTHER the angle, which the plane, wherein the moon at any time appears, makes with the plane of the earth's motion, is called the inclination of the moon's orbit at that time. And I shall now proceed to shew, that this inclination of the orbit, when the moon is in K, is less than when she was in A; or, that the plane L Y M, which touches the line of the moon's motion in K, makes a less angle with the plane of the earth's motion or with the circle A B C D, than the plane A E C makes with the same. The semicircle L Y M intersects the semicircle A E C in Y; and the arch A Y is less than L Y, and both together less than half a circle. But it is demonstrated by the writers on that part of astronomy, which is called the doctrine of the sphere, that when a triangle is made, as here, by three arches of circles A L, A Y, and Y L, the angle under YAB without the triangle is greater than the angle under Y L A within, if the two arches A Y, Y L taken together do not amount to a semicircle; if the two arches make a complete semicircle, the two angles will be equal; but if the two arches taken together exceed a semicircle, the inner angle under Y L A is greater than the other [a]. Here therefore the two arches A Y and L Y together being less than a semicircle, the angle under A L Y is less, than the angle under B A E. But from the doctrine of the sphere it is also evident, that the angle under A L Y is equal to that, in which the plane of the

[a] Menelai Sphaeric. Lib. I. prop. 10.

circle

circle L Y K M, that is, the plane which touches the line A K G H I in K, is inclined to the plane of the earth's motion A B C; and the angle under B A E is equal to that, in which the plane A E C is inclined to the fame plane. Therefore the inclination of the former plane is lefs than the inclination of the latter.

27. Suppose now the moon to be advanced to the point G (in fig. 100.) and in this point to be diftant from its node a quarter part of the whole circle; or in other words, to be in the midway between its two nodes. And in this cafe the nodes will have receded yet more, and the inclination of the orbit be ftill more diminifhed : for fuppofe the line A K G H I to be touched in the point G by a plane paffing through the earth T : let the interfection of this plane with the plane of the earth's motion be the line W T O, and the line T P its interfection with the plane L K M. In this plane let the circle N G O be defcribed with the femidiameter T P or N T cutting the other circle L K M in P. Now the line A K G I is convex to the plane L K M, which touches it in K; and therefore the plane N G O, which touches it in G, will interfect the other touching plane between G and K; that is, the point P will fall between thofe two points, and the plane continued to the plane of the earth's motion will pafs beyond L; fo that the points N and O, or the places of the nodes, when the moon is in G, will be farther from A and C than L and M, that is, will have moved farther backward. Befides, the inclination of the plane N G O to the plane of the earth's motion A B C is lefs, than the inclination of the plane L K M to the fame; for here alfo the two arches L P and N P taken together are lefs

than

than a femicircle, each of thefe arches being lefs than a quar-
ter of a circle ; as appears, becaufe G N, the diftance of the
moon in G from its node N,   is here fuppofed to be a quarter
part of a circle.

28. After the moon is paffed beyond G, the cafe is altered ;
for then thefe arches will be greater than quarters of the circle,
by which means the inclination will be again increafed,   tho'
the nodes ftill go on to move the fame way.   Suppofe the
moon in H, (in fig. 101.) and that the plane, which touches
the line A K G I in H, interfects the plane of the earth's mo-
tion in the line Q T R,   and the plane N G O in the line T V,
and befides that the circle Q H R be defcribed in that plane ;
then, for the fame reafon as before, the point V will fall be-
tween H and G, and the plane R V Q will pafs beyond the
laft plane O V N, caufing the points Q and R to fall farther
from A and C than N and O.   But the arches N V, V Q are
each greater than a quarter of a circle, N V the leaft of them
being greater than G N,   which is a quarter of a circle ;   and
therefore the two arches N V and V Q together exceed a fe-
micircle ; confequently the angle under B Q V will be greater,
than that under B N V.

29. In the laft place, when the moon is by this attra-
ction of the fun, drawn at length into the plane of the earth's
motion, the node will have receded yet more,   and the incli-
nation be fo much increafed, as to become fomewhat more
than at firft : for the line A K G H I being convex to all the
planes, which touch it, the part H I will wholly fall between

E e   the

the plane Q V R and the plane A B C ; fo that the point I will fall between B and R ; and drawing I T W, the point W will be farther remov'd from A than Q. But it is evident, that the plane, which paffes through the earth T, and touches the line A G I in the point I, will cut the plane of the earth's motion ABCD in the line I T W, and be inclined to the fame in the angle under H I B ; fo that the node, which was firft in A, after having paffed into L, N and Q, comes at laft into the point W ; as the node which was at firft in C has paffed fucceffively from thence through the points M, O and R to I : but the angle under H I B, which is now the inclination of the orbit to the plane of the ecliptic, is manifeftly not lefs than the angle under E C B or E A B, but rather fomething greater.

30. Thus the moon in the cafe before us, while it paffes from the plane of the earth's motion in the quarter, till it comes again into the fame plane, has the nodes of its orbit continually moved backward, and the inclination of its orbit is at firft diminifhed, viz. till it comes to G in fig. 100, which is near to its conjunction with the fun, but afterwards is increafed again almoft by the fame degrees, till upon the moon's arrival again to the plane of the earth's motion, the inclination of the orbit is reftored to fomething more than its firft magnitude, though the difference is not very great, becaufe the points I and C are not far diftant from each other [a].

[a] Vid. Newt. Princ. Lib. I. prop. 66. coroll. 10.

31. After

31. After the fame manner, if the moon had departed from the quarter in C, it fhould have defcribed the curve line C X W (in fig. 98.) between the planes A F C and A D C, which would be convex to the former of thofe planes, and concave to the latter; fo that, here alfo, the nodes fhould continually recede, and the inclination of the orbit gradually diminifh more and more, till the moon arrived near its oppofition to the fun in X; but from that time the inclination fhould again increafe, till it became a little greater than at firft. This will eafily appear, by confidering, that as the action of the fun upon the moon, by exceeding its action upon the earth, drew it out of the plane A E C towards the fun, while the moon paffed from A to I; fo, during its paffage from C to W, the moon being all that time farther from the fun than the earth, it will be attracted lefs; and the earth, together with the plane A E C F, will as it were be drawn from the moon, in fuch fort, that the path the moon defcribes fhall appear from the earth, as it did in the former cafe by the moon's being drawn away.

32. These are the changes, which the nodes and the inclination of the moon's orbit undergo, when the nodes are in the quarters; but when the nodes by their motion, and the motion of the fun together, come to be fituated between the quarter and conjunction or oppofition, their motion and the change made in the inclination of the orbit are fomewhat different.

E e 2   33. Let

33. Let AGCH (in fig. 103.) be a circle defcribed in the plane of the earth's motion, having the earth in T for its center. Let the point oppofite to the fun be A, and the point G a fourth part of the circle diftant from A. Let the nodes of the moon's orbit be fituated in the line B T D, and B the node, falling between A, the place where the moon would be in the full, and G the place where the moon would be in the quarter. Suppofe BEDF to be the plane, in which the moon effays to move, when it proceeds from the point B. Becaufe the moon in B is more diftant from the fun than the earth, it fhall be lefs attracted by the fun, and fhall not defcend towards the fun fo faft as the earth: confequently it fhall quit the plane BEDF, which we fuppofe to accompany the earth, and defcribe the line BIK convex thereto, till fuch time as it comes to the point K, where it will be in the quarter: but from thenceforth being more attracted than the earth, the moon fhall change its courfe, and the following part of the path it defcribes fhall be concave to the plane BED or BGD, and fhall continue concave to the plane BGD, till **it croffes** that plane in L, juft as in the preceding cafe. Now I fay, while the moon is paffing from B to K, the nodes, contrary to what was found in the foregoing cafe, will proceed forward, or move the fame way with the moon [a]; and at the fame time the inclination of the orbit will increafe [b].

34. When the moon is in the point I, let the plane MIN pafs through the earth T, and touch the path of the

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[a] Vid. Newt. Princ. Lib. III. prop. 30. p. 440.   [b] Ibid. Lib. I. prop. 66. coroll. 10.
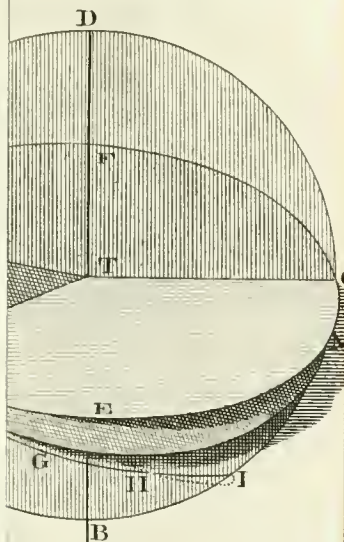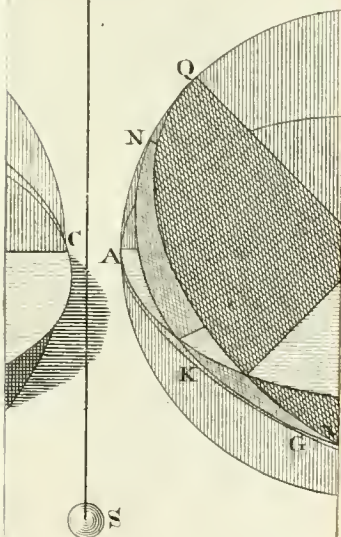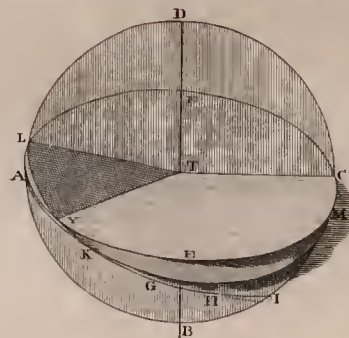
moon

Fig. 99.

Fig. 99.
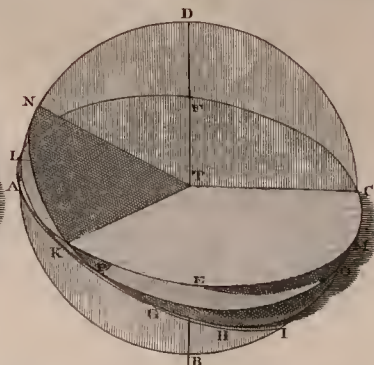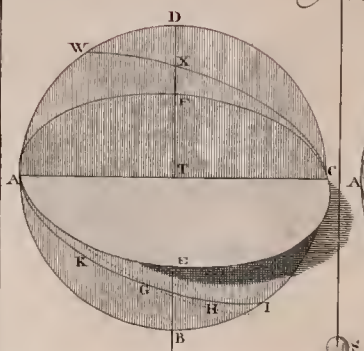
Fig. 100.

Fig. 98.
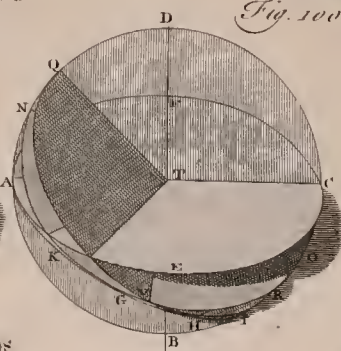
Fig. 101.
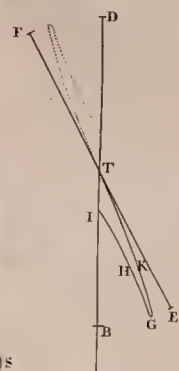
Fig. 102.

moon in I, cutting the plane of the earth's motion in the line M T N, and the plane B E D in the line T O. Becaufe the line B I K is convex to the plane B E D, which touches it in B, the plane N I M muft crofs the plane D E B, before it meets the plane C G B; and therefore the point M will fall from B towards G, and the node of the moon's orbit being tranflated from B to M is moved forward.

35. I say farther, the angle under O M G, which the plane M O N makes with the plane B G C, is greater than the angle under O B G, which the plane B O D makes with the fame. This appears from what has been already explained; becaufe the arches B O, O M are each lefs than the quarter of a circle, and therefore taken both together are lefs than a femicircle.

36. Again, when the moon is come to the point K in its quarter, the nodes will be advanced yet farther forward, and the inclination of the orbit alfo more augmented. Hitherto the moon's motion has been referred to the plane, which paffing through the earth touches the path of the moon in the point, where the moon is, according to what was afferted at the beginning of this difcourfe upon the nodes, that it is the cuftom of aftronomers fo to do. But here in the point K no fuch plane can be found; on the contrary, feeing the line of the moon's motion on one fide the point K is convex to the plane B E D, and on the other fide concave to the fame, no plane can pafs through the points T and K, but will cut the line B K L in that point. Therefore inftead

of

of such a touching plane, we must here make use of what is equivalent, the plane P K Q, with which the line B K L shall make a less angle than with any other plane; for this plane does as it were touch the line B K in the point K, since it so cuts it, that no other plane can be drawn so, as to pass between the line B K and the plane P K Q. But now it is evident, that the point P, or the node, is removed from M towards G, that is, has moved yet farther forward; and it is likewise as manifest, that the angle under K P G, or the inclination of the moon's orbit in the point K, is greater than the angle under I M G, for the reason so often assigned.

37. After the moon has passed the quarter, the path of the moon being concave to the plane A G C H, the nodes, as in the preceding case, shall recede, till the moon arrives at the point L; which shews, that considering the whole time of the moon's passing from B to L, at the end of that time the nodes shall be found to have receded, or to be placed backwarder, when the moon is in L, than when it was in B. For the moon takes a longer time in passing from K to L, than in passing from B to K; and therefore the nodes continue to recede a longer time, than they moved forwards; so that their recess must surmount their advance.

38. In the same manner, while the moon is in its passage from K to L, the inclination of the orbit shall diminish, till the moon comes to the point, in which it is one quarter part of a circle distant from its node; suppose in the point R; and from that time the inclination shall again increase.

Since

Since therefore the inclination of the orbit increases, while the moon is paffing from B to K, and diminifhes itfelf again only, while the moon is paffing from K to R, and then augments again, till the moon arrive in L; while the moon is paffing from B to L, the inclination of the orbit is much more increafed than diminifhed, and will be diftinguifhably greater, when the moon is come to L, than when it fet out from B.

39. In like manner, while the moon is paffing from L on the other fide the plane A G C H, the node fhall advance forward, as long as the moon is between the point L and the next quarter; but afterwards it fhall recede, till the moon come to pafs the plane A G C H again in the point V, between B and A : and becaufe the time between the moon's paffing from L to the next quarter is lefs, than the time between that quarter and the moon's coming to the point V, the node fhall have more receded than advanced ; fo that the point V will be nearer to A, than L is to C. So alfo the inclination of the orbit, when the moon is in V, will be greater, than when the moon was at L ; for this inclination increafes all the time the moon is between L and the next quarter ; it decreafes only while the moon is paffing from this quarter to the mid way between the two nodes, and from thence increafes again during the whole paffage through the other half of the way to the next node.

40. Thus we have traced the moon from her node in the quarter, and fhewn, that at every period of the moon the nodes will have receded, and thereby will have approached toward

toward a conjunction with the fun. But this conjunction will be much forwarded by the visible motion of the fun itself. In the last scheme the fun will appear to move from S toward W. Suppose it appeared to have moved from S to W, while the moon's node has receded from B to V, then drawing the line W T X, the arch V X will represent the distance of the line drawn between the nodes from the fun, when the moon is in V; whereas the arch B A represented that distance, when the moon was in B. This visible motion of the fun is much greater, than that of the node; for the fun appears to revolve quite round each year, and the node is near 19 years in making one revolution. We have also seen, that when the node was in the quadrature, the inclination of the moon's orbit decreased, till the moon came to the conjunction, or opposition, according to which node it set out from; but that afterwards it again increased, till it became at the next node rather greater than at the former. When the node is once removed from the quarter nearer to a conjunction with the fun, the inclination of the moon's orbit, when the moon comes into the node, is more sensibly greater, than it was in the node preceding; the inclination of the orbit by this means more and more increasing till the node comes into conjunction with the fun; at which time it has been shewn above, that the fun has no power to change the plane of the moon's motion; and consequently has no effect either on the nodes, or on the inclination of the orbit.

41. As soon as the nodes, by the action of the fun, are got out of conjunction toward the other quarters, they begin

again

again to recede as before ; but the inclination of the orbit in the appulfe of the moon to each fucceeding node is lefs than at the preceding, till the nodes come again into the quarters.   This will appear as follows.   Let A (in fig. 104.) reprefent one of the moon's nodes placed between the point of oppofition B and the quarter C.   Let the plane A D E pafs through the earth T, and touch the path of the moon in A. Let the line A F G H be the path of the moon in her paffage from A to H, where fhe croffes again the plane of the earth's motion. This line will be convex toward the plane A D E, till the moon comes to G, where fhe is in the quarter ; and after this, between G and H, the fame line will be concave toward this plane.   All the time this line is convex toward the plane A D E, the nodes will recede;   and on the contrary proceed, while it is concave to that plane.   All this will eafily be conceived from what has been before fo largely explained.   But the moon is longer in paffing from A to G, than from G to H ; therefore the nodes recede a longer time, than they proceed ; confequently upon the whole, when the moon is arrived at H, the nodes will have receded, that is, the point H will fall between B and E. The inclination of the orbit will decreafe, till the moon is arrived to the point F, in the middle between A and H.   Through the paffage between F and G the inclination will increafe, but decreafe again in the remaining part of the paffage from G to H, and confequently at H muft be lefs than at A.   The like effects, both in refpect to the nodes and inclination of the orbit, will take place in the following paffage of the moon on the other fide of the plane A B E C, from H, till it comes over that plane again in I.

F f   42. Thus

42. Thus the inclination of the orbit is greatest, when the line drawn between the moon's nodes will pass through the sun; and least, when this line lies in the quarters, especially if the moon at the same time be in conjunction with the sun, or in the opposition. In the first of these cases the nodes have no motion, in all others, the nodes will each month have receded: and this regressive motion will be greatest, when the nodes are in the quarters; for in that case the nodes have no progressive motion during the whole month, but in all other cases the nodes do at some times proceed forward, viz. whenever the moon is between either quarter, and the node which is less distant from that quarter than a fourth part of a circle.

43. It now remains only to explain the irregularities in the moon's motion, which follow from the elliptical figure of the orbit. By what has been said at the beginning of this chapter it appears, that the power of the earth on the moon acts in the reciprocal duplicate proportion of the distance: therefore the moon, if undisturbed by the sun, would move round the earth in a true ellipsis, and the line drawn from the earth to the moon would pass over equal spaces in equal portions of time. That this description of the spaces is altered by the sun, has been already declared. It has also been shewn, that the figure of the orbit is changed each month; that the moon is nearer the earth at the new and full, and more remote in the quarters, than it would be without the sun. Now we must pass by these monthly changes, and consider the effect, which the sun will have in the different

ent

ent fituations of the axis of the orbit in refpect of that luminary.

44. THE action of the fun varies the force, wherewith the moon is drawn toward the earth ; in the quarters the force of the earth is directly increafed by the fun ; at the new and full the fame is diminifhed ; and in the intermediate places the influence of the earth is fometimes aided, and fometimes leffened by the fun. In thefe intermediate places between the quarters and the conjunction or oppofition, the fun's action is fo oblique to the action of the earth on the moon, as to produce that alternate acceleration and retardment of the moon's motion, which I obferved above to be ftiled the variation. But befides this effect, the power, by which the earth attracts the moon toward itfelf, will not be at full liberty to act with the fame force, as if the fun acted not at all on the moon. And this effect of the fun's action, whereby it corroborates or weakens the action of the earth, is here only to be confidered. And by this influence of the fun it comes to pafs, that the power, by which the moon is impelled toward the earth, is not perfectly in the reciprocal duplicate proportion of the diftance. Confequently the moon will not defcribe a perfect ellipfis. One particular, wherein the moon's orbit will differ from an ellipfis, confifts in the places, where the motion of the moon is perpendicular to the line drawn from itfelf to the earth. In an ellipfis, after the moon fhould have fet out in the direction perpendicular to this line drawn from itfelf to the earth, and at its greateft diftance from the earth, its motion would

again

again become perpendicular to this line drawn between it-
felf and the earth, and the moon be at its neareft diftance
from the earth, when it fhould have performed half its pe-
riod; after performing the other half of its period its mo-
tion would again become perpendicular to the foremention-
ed line, and the moon return into the place whence it fet out,
and have recovered again its greateft diftance. But the moon
in its real motion, after fetting out as before, fometimes makes
more than half a revolution, before its motion comes again
to be perpendicular to the line drawn from itfelf to the earth,
and the moon is at its neareft diftance ; and then performs
more than another half of an intire revolution before its mo-
tion can a fecond time recover its perpendicular direction to
the line drawn from the moon to the earth, and the moon
arrive again to its greateft diftance from the earth.   At other
times the moon will defcend to its neareft diftance, before it
has made half a revolution, and recover again its greateft di-
ftance, before it has made an intire revolution.   The place,
where the moon is at its greateft diftance from the earth, is call-
ed the moon's apogeon, and the place of the leaft diftance
the perigeon.   This change of the place, where the moon
fucceffively comes to its greateft diftance from the earth, is
called the motion of the apogeon.   In what manner the fun
caufes the apogeon to move, I fhall now endeavour to explain.

45. Our author fhews, that if the moon were attract-
ed toward the earth by a compofition of two powers, one
of which were reciprocally in the duplicate proportion of
the diftance from the earth, and the other reciprocally

in

in the triplicate proportion of the fame diftance ; then, though the line defcribed by the moon would not be in reality an ellipfis, yet the moon's motion might be perfectly explained by an ellipfis, whofe axis fhould be made to move round the earth; this motion being in confequence, as aftronomers exprefs themfelves, that is, the fame way as the moon itfelf moves, if the moon be attracted by the fum of the two powers; but the axis muft move in antecedence, or the contrary way, if the moon be acted on by the difference of thefe powers.   What is meant by duplicate proportion has been often explained; namely, that if three magnitudes, as A, B, and C, are fo related, that the fecond B bears the fame proportion to the third C, as the firft A bears to the fecond B, then the proportion of the firft A to the third C, is the duplicate of the proportion of the firft A to the fecond B. Now if a fourth magnitude, as D, be affumed, to which C fhall bear the fame proportion as A bears to B, and B to C, then the proportion of A to D is the triplicate of the proportion of A to B.

46. T H E way of reprefenting the moon's motion in this cafe is thus.   T denoting the earth (in fig. 105, 106.) fuppofe the moon in the point A, its apogeon, or greateft diftance from the earth, moving in the direction A F perpendicular to A B, and acted upon from the earth by two fuch forces as have been named.   By that power alone, which is reciprocally in the duplicate proportion of the diftance, if the moon fet out from the point A with a proper degree of velocity, the ellipfis A M B may be defcribed.

fcribed.   But if the moon be acted upon by the fum of the forementioned powers, and the velocity of the moon in the point A be augmented in a certain proportion [a]; or if that velocity be diminifhed in a certain proportion, and the moon be acted upon by the difference of thofe powers; in both thefe cafes the line A E, which fhall be defcribed by the moon, is thus to be determined.   Let the point M be that, into which the moon would have arrived in any given fpace of time, had it moved in the ellipfis A M B.   Draw M T, and likewife C T D in fuch fort, that the angle under A T M fhall bear the fame proportion to the angle under A T C, as the velocity, with which the ellipfis A M B muft have been de-fcribed, bears to the difference between this velocity, and the velocity, with which the moon muft fet out from the point A in order to defcribe the path A E.   Let the angle A T C be ta-ken toward the moon (as in fig. 105.) if the moon be attract-ed by the fum of the powers; but the contrary way (as in fig. 106.) if by their difference.   Then let the line A B be moved into the pofition C D, and the ellipfis A M B into the fituation C N D, fo that the point M be tranflated to L : then the point L fhall fall upon the path of the moon A E.

47. The angular motion of the line A T, wereby it is removed into the fituation C T, reprefents the motion of the apogeon; by the means of which the motion of the moon might be fully explicated by the ellipfis A M B, if the action of the fun upon it was directed to the center of the earth, and

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[a] What this proportion is, may be known from Coroll. 2 prop 44. Lib. I. Princ. philof. Newton.

reci-

reciprocally in the triplicate proportion of the moon's diftance from it. But that not being fo, the apogeon will not move in the regular manner now defcribed. However, it is to be obferved here, that in the firft of the two preceding cafes, where the apogeon moves forward, the whole centripetal power increafes fafter, with the decreafe of diftance, than if the intire power were reciprocally in the duplicate proportion of the diftance; becaufe one part only is in that proportion, and the other part, which is added to this to make up the whole power, increafes fafter with the decreafe of diftance. On the other hand, when the centripetal power is the difference between thefe two, it increafes lefs with the decreafe of the diftance, than if it were fimply in the reciprocal duplicate proportion of the diftance. Therefore if we chufe to explain the moon's motion by an ellipfis (as is moft convenient for aftronomical ufes to be done, and by reafon of the fmall effect of the fun's power, the doing fo will not be attended with any fenfible error;) we may collect in general, that when the power, by which the moon is attracted to the earth, by varying the diftance, increafes in a greater than in the duplicate proportion of the diftance diminifhed, a motion in confequence muft be afcribed to the apogeon; but that when the attraction increafes in a lefs proportion than that named, the apogeon muft have given to it a motion in antecedence[a]. It is then obferved by Sir Is. NEWTON, that the firft of thefe cafes obtains, when the moon is in the conjunction and oppofition; and the latter, when the moon is in the quarters: fo that in the firft the apogeon moves according to the order of the

[a] Princ. Phil. Newt. Lib.I. prop. 45. Coroll. 1.

figns;

figns; in the other, the contrary way [a]. But, as was faid before, the difturbance given to the action of the earth by the fun in the conjunction and oppofition being near twice as great as in the quarters [b], the apogeon will advance with a greater velocity than recede, and in the compafs of a whole revolution of the moon will be carried in confequence [c].

48. It is fhewn in the next place by our author, that when the line A B coincides with that, which joins the earth and the fun, the progreffive motion of the apogeon, when the moon is in the conjunction or oppofition, exceeds the regreffive in the quadratures more than in any other fituation of the line A B [d]. On the contrary, when the line A B makes right angles with that, which joins the earth and fun, the retrograde motion will be more confiderable [e], nay is found fo great as to exceed the progreffive; fo that in this cafe the apogeon in the compafs of an intire revolution of the moon is carried in antecedence. Yet from the confiderations in the laft paragraph the progreffive motion exceeds the other; fo that in the whole the mean motion of the apogeon is in confequence, according as aftronomers find. Moreover, the line A B changes its fituation with that, which joins the earth and fun, by fuch flow degrees, that the inequalities in the motion of the apogeon arifing from this laft confideration, are much greater than what arifes from the other [f].

[a] Pr. Phil. Newt. Lib. I. prop. 66. Coroll. 7.
[b] See § 19. of this chapter.
[c] Phil. Nat. Pr. Math Lib. I. prop. 65 cor. 8.
[d] Ibid. Coroll. 8.
[e] Ibid.
[f] Ibid.

49. FARTHER, this unsteady motion in the apogeon is at-
tended with another inequality in the motion of the moon, that
it cannot be explained at all times by the same ellipsis. The
ellipsis in general is called by astronomers an eccentric orbit.
The point, in which the two axis's cross, is called the center of
the figure ; because all lines drawn through this point within
the ellipsis, from side to side, are divided in the middle by
this point.   But the center, about which the heavenly bodies
revolve, lying out of this center of the figure in one focus,
these orbits are said to be eccentric ; and where the distance of
the focus from this center bears the greatest proportion to the
whole axis, that orbit is called the most eccentric : and in
such an orbit the distance from the focus to the remoter ex-
tremity of the axis bears the greatest proportion to the di-
stance of the nearer extremity.   Now whenever the apo-
geon of the moon moves in consequence, the moon's motion
must be referred to an orbit more eccentric, than what the
moon would describe, if the whole power, by which the
moon was acted on in its passing from the apogeon, changed
according to the reciprocal duplicate proportion of the di-
stance from the earth, and by that means the moon did de-
scribe an immoveable ellipsis ; and when the apogeon moves
in antecedence, the moon's motion must be referred to an
orbit less eccentric.   In the first of the two figures last re-
ferred to, the true place of the moon L falls without the orbit
A M B, to which its motion is referred : whence the orbit A L E,
truly described by the moon, is less incurvated in the point A,
than is the orbit A M B ; therefore the orbit A M B is more ob-
long, and differs farther from a circle, than the ellipsis would,

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whofe curvature in A were equal to that of the line A L B, that is, the proportion of the diftance of the earth T from the center of the ellipfis to its axis will be greater in the ellipfis A M B, than in the other ; but that other is the ellipfis, which the moon would defcribe, if the power acting upon it in the point A were altered in the reciprocal duplicate proportion of the diftance.   In the fecond figure, when the apogeon recedes, the place of the moon L falls within the orbit A M B, and therefore that orbit is lefs eccentric, than the immoveable orbit which the moon fhould defcribe. The truth of this is evident ; for, when the apogeon moves forward, the power, by which the moon is influenced in its defcent from the apogeon, increafes fafter with the decreafe of diftance, than in the duplicate proportion of the diftance ; and confequently the moon being drawn more forcibly toward the earth, it will defcend nearer to it.   On the other hand, when the apogeon recedes, the power acting on the moon increafes with the decreafe of diftance in lefs than the duplicate proportion of the diftance ; and therefore the moon is lefs impelled toward the earth, and will not defcend fo low.

50. Now fuppofe in the firft of thefe figures, that the apogeon A is in the fituation, where it is approaching toward the conjunction or oppofition of the fun. In this cafe the progreffive motion of the apogeon is more and more accelerated. Here fuppofe that the moon, after having defcended from A through the orbit A E as far as F, where it is come to its neareft diftance from the earth, afcends again up the line F G.   Becaufe the motion of the apogeon is here continually more and
more.

more accelerating, the caufe of its motion is conftantly up-
on the increafe ; that is, the power, whereby the moon is
drawn to the earth, will decreafe with the increafe of diftance,
in the moon's afcent from F, in a greater proportion than that
wherewith it increafed with the decreafe of diftance in the
moon's defcent to F. Confequently the moon will afcend high-
er than to the diftance A T, from whence it defcended ; there-
fore the proportion of the greateft diftance of the moon to
the leaft is increafed. And when the moon defcends again, the
power will yet more increafe with the decreafe of diftance,
than in the laft afcent it decreafed with the augmentation
of diftance ; the moon therefore muft defcend nearer to the
earth than it did before, and the proportion of the greateft
diftance to the leaft yet be more increafed.   Thus as long
as the apogeon is advancing toward the conjunction or oppo-
fition, the proportion of the greateft diftance of the moon
from the earth to the leaft will continually increafe ; and
the elliptical orbit, to which the moon's motion is referred,
will be rendered more and more eccentric.

51. As foon as the apogeon is paffed the conjunction
with the fun or the oppofition, the progreffive motion thereof
abates, and with it the proportion of the greateft diftance of
the moon from the earth to the leaft diftance will alfo dimi-
nifh ; and when the apogeon becomes regreffive, the diminu-
tion of this proportion will be ftill farther continued on, till
the apogeon comes into the quarter ; from thence this pro-
portion, and the eccentricity of the orbit will increafe again.
Thus the orbit of the moon is moft eccentric, when the apo-

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geon

geon is in conjunction with the fun, or in oppofition to it, and leaft of all when the apogeon is in the quarters.

52. These changes in the nodes, in the inclination of the orbit to the plane of the earth's motion, in the apogeon, and in the eccentricity, are varied like the other inequalities in the motion of the moon, by the different diftance of the earth from the fun; being greateft, when their caufe is greateft, that is, when the earth is neareft to the fun.

53. I faid at the beginning of this chapter, that Sir Isaac Newton has computed the very quantity of many of the moon's inequalities. That acceleration of the moon's motion, which is called the variation, when greateft, removes the moon out of the place, in which it would otherwife be found, fomething more than half a degree [a]. In the phrafe of aftronomers, a degree is $\frac{1}{360}$ part of the whole circuit of the moon or any planet. If the moon, without difturbance from the fun, would have defcribed a circle concentrical to the earth, the fun will caufe the moon to approach nearer to the earth in the conjunction and oppofition, than in the quarters, nearly in the proportion of 69 to 70 [b]. We had occafion to mention above, that the nodes perform their period in almoft 19 years. This the aftronomers found by obfervation; and our author's computations affign to them the fame period [c]. The inclination of the moon's orbit when leaft, is an angle about $\frac{1}{18}$ part of that angle, which conftitutes

[a] Newt. Princ. Lib. III. prop. 29.   [b] Ibid. prop. 28.   [c] Ibid. prop. 32.

a per-

a perpendicular; and the difference between the greatest and
least inclination of the orbit is determined by our author's
computation to be about $\frac{1}{18}$ of the least inclination [a].   And
this also is agreeable to the observations of astronomers.   The
motion of the apogeon, and the changes in the eccentricity,
Sir Isaac Newton has not computed.   The apogeon
performs its revolution in about eight years and ten months.
When the moon's orbit is most eccentric, the greatest di-
stance of the moon from the earth bears to the least di-
stance nearly the proportion of 8 to 7; when the orbit is
least eccentric, this proportion is hardly so great as that of
12 to 11.

54. Sir Isaac Newton shews farther, how, by com-
paring the periods of the motion of the satellites, which re-
volve round Jupiter and Saturn, with the period of our
moon round the earth, and the periods of those planets
round the sun with the period of our earth's motion, the
inequalities in the motion of those satellites may be derived
from the inequalities in the moon's motion; excepting on-
ly in regard to that motion of the axis of the orbit, which
in the moon makes the motion of the apogeon; for the
orbits of those satellites, as far as can be discerned by us at
this distance, appearing little or nothing eccentric, this
motion, as deduced from the moon, must be diminished.

[a] Newt. Princ. pag. 459.

Chap.

Chap. IV.

Of COMETS.

IN the former of the two preceding chapters the powers have been explained, which keep in motion thofe celeftial bodies, whofe courfes had been well determined by the aftronomers.   In the laft chapter we have fhewn, how thofe powers have been applied by our author to the making a more perfect difcovery of the motion of thofe bodies, the courfes of which were but imperfectly underftood ;   for fome of the inequalities, which we have been defcribing in the moon's motion, were unknown to the aftronomers. In this chapter we are to treat of a third fpecies of the heavenly bodies, the true motion of which was not at all apprehended before our author writ ;   in fo much, that here Sir Isaac Newton has not only explained the caufes of the motion of thefe bodies, but has performed alfo the part of an aftronomer, by difcovering what their motions are.

2. That thefe bodies are not meteors in our air, is manifeft ; becaufe they rife and fet in the fame manner, as the fun and ftars.   The aftronomers had gone fo far in their inquiries concerning them, as to prove by their obfervations, that they moved in the etherial fpaces far beyond the moon ;   but they had no true notion at all of the path, which they defcribed.   The moft prevailing opinion before

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our author was, that they moved in ftraight lines ; but in what part of the heavens was not determined.   D ES C A R-T E S [a] removed them far beyond the fphere of Saturn,   as finding the ftraight motion attributed to them, inconfiftent with the vortical fluid,   by which he explains the motions of the planets, as we have above related [b].   But Sir I S A A C N E W T O N diftinctly proves from aftronomical obfervation, that the comets pafs through the region of the planets,   and are moftly invifible at a lefs diftance, than that of Jupiter [c].

3. A N D from hence finding the comets to be evident-ly within the fphere of the fun's action, he concludes they muft neceffarily move about the fun,   as the planets do [d]. The planets move in ellipfis's ; but it is not neceffary that every body, which is influenced by the fun,   fhould move in that particular kind of line.   However our author proves, that the power of the fun being reciprocally in the duplicate proportion of the diftance, every body acted on by the fun muft either fall directly down, or move in fome conic fe-ction ; of which lines I have above obferved, that there are three fpecies,   the ellipfis, parabola, and hyperbola [e].   If a body, which defcends toward the fun as low as the orbit of any planet,   move with a fwifter motion than the pla-net does, that body will defcribe an orbit of a more oblong figure, than that of the planet,   and have a longer axis at leaft.   The velocity of the body may be fo great, that it

[a] In Princ. philof. part. 3. § 41.
[b] Chap. 1. § 11.
[c] Newton. Princ. philof. Lib. III. Lemm. 4.
pag. 478.
[d] Princ. philof. Lib. III. prop. 40.
[e] Book 1. chap. 2. § 82.

shall move in a parabola, and having once passed about the sun, shall ascend for ever without returning any more: but the sun will be placed in the focus of this parabola. With a velocity still greater the body will move in an hyperbola.   But it is most probable, that the comets move in elliptical orbits, though of a very oblong, or in the phrase of astronomers, of a very eccentric form, such as is represented in fig. 107, where S is the sun, C the comet, and A B D E its orbit, wherein the distance of S and D far exceeds that of S and A.   Whence it is, that they sometimes are found at a moderate distance from the sun, and appear within the planetary regions; at other times they ascend to vast distances, far beyond the very orbit of Saturn, and so become invisible.   That the comets do move in this manner is proved by our author, from computations built upon the observations, which astronomers had made on many comets.   These computations were performed by Sir ISAAC NEWTON himself upon the comet, which appeared toward the latter end of the year 1680, and at the beginning of the year following[a]; but the learned Dr. HALLEY prosecuted the like computations more at large in this, and also in many other comets[b]. Which computations are made upon propositions highly worthy of our author's unparallel'd genius, such as could scarce have been discovered by any one not possessed of the utmost force of invention;

[a] Princ. philof. Lib. III. prg. 499, 500.   [b] Ibid. pag. 500, and 520, &c.

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4. Those computations depend upon this principle, that the eccentricity of the orbits of the comets is so great, that if they are really elliptical, yet they approach so near to parabolas in that part of them, where they come under our view, that they may be taken for such without sensible error [a] : as in the preceding figure the parabola FAG differs in the lower part of it about A very little from the ellipsis DEAB. Upon which ground our great author teaches a method of finding by three observations made upon any comet the parabola, which nearest agrees with its orbit [b].

5. Now what confirms this whole theory beyond the least room for doubt is, that the places of the comets computed in the orbits, which the method here mentioned assigns them, agree to the observations of astronomers with the same degree of exactness, as the computations of the primary planets places usually do; and this in comets, whose motions are very extraordinary [c].

6. Our author afterwards shews how to make use of any small deviation from the parabola, that shall be observed, to determine whether the orbits of the comets are elliptical or not, and so to discover if the same comet returns at certain periods [d]. And upon examining the comet in 1680, by the rule laid down for this purpose, he finds its orbit to agree more exactly to an ellipsis than

[a] Princ. Philof. Lib.III. prop.40.
[b] Ibid. prop. 41.
[c] Ibid. pag 522.
[d] Ibid prop. 42.

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to a parabola, though the ellipfis be fo very eccentric, that the comet cannot perform its period through it in the fpace of 500 years [a]. Upon this Dr. Halley obferved, that mention is made in hiftory of a comet, with the like eminent tail as this, having appeared three feveral times before; the firft of which appearances was at the death of Julius Cesar, and each appearance was at the diftance of 575 years from the next preceding. He therefore computed the motion of this comet in fuch an elliptic orbit, as would require this number of years for the body to revolve through it; and thefe computations agree yet more perfectly with the obfervations made on this comet, than any parabolical orbit will do [b].

7. The comparing together different appearances of the fame comet, is the only way to difcover certainly the true form of the orbit: for it is impoffible to determine with exactnefs the figure of an orbit fo exceedingly eccentric, from fingle obfervations taken in one part of it; and therefore Sir Isaac Newton [c] propofes to compare the orbits, upon the fuppofition that they are parabolical, of fuch comets as appear at different times; for if the fame orbit be found to be defcribed by a comet at different times, in all probability it will be the fame comet which defcribes it. And here he remarks from Dr. Halley, that the fame orbit very nearly agrees to two appearances of a comet about the fpace of 75 years diftance [d]; fo that

[a] Newt. Princ. philof. edit. 2. p. 464, 465.
[b] Ibid. edit. 3. p 501, 502.
[c] Ibid pag. 519.
[d] Ibid. pag. 524.

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if thofe two appearances were really of the fame comet, the tranfverfe axis of the orbit of the comet would be near 18 times the axis of the earth's orbit; and the comet, when at its greateft diftance from the fun, will be removed not lefs than 35 times as far as the middle diftance of the earth.

8. AND this feems to be the fhorteft period of any of the comets. But it will be farther confirmed, if the fame comet fhould return a third time after another period of 75 years. However it is not to be expected, that comets fhould preferve the fame regularity in their periods, as the planets; becaufe the great eccentricity of their orbits makes them liable to fuffer very confiderable alterations from the action of the planets, and other comets, upon them.

9. IT is therefore to prevent too great difturbances in their motions from thefe caufes, as our author obferves, that while the planets revolve all of them nearly in the fame plane, the comets are difpofed in very different ones, and diftributed over all parts of the heavens; that, when in their greateft diftance from the fun, and moving floweft, they might be removed as far as poffible out of the reach of each other's action [a]. The fame end is likewife farther anfwered in thofe comets, which by moving floweft in the aphelion, or remoteft diftance from the fun, defcend neareft to it, by placing the aphelion of thefe at the greateft height from the fun [b].

[a] Newt. Princ. philof. p. 525. [b] Ibid.

10. OUR

10. Our philosopher being led by his principles to explain the motions of the comets, in the manner now related, takes occasion from thence to give us his thoughts upon their nature and use. For which end he proves in the first place, that they must necessarily be solid and compact bodies, and by no means any sort of vapour or light substance exhaled from the planets or stars: because at the near distance, to which some comets approach the sun, it could not be, but the immense heat, to which they are exposed, should instantaneously disperse and scatter any such light volatile substance [a]. In particular the forementioned comet of 1680 descended so near the sun, as to come within a sixth part of the sun's diameter from the surface of it. In which situation it must have been exposed, as appears by computation, to a degree of heat exceeding the heat of the sun upon our earth no less than 28000 times; and therefore might have contracted a degree of heat 2000 times greater, than that of red hot iron [b]. Now a substance, which could endure so intense a heat, without being dispersed in vapor, must needs be firm and solid.

11. It is shewn likewise, that the comets are opake substances, shining by a reflected light, borrowed from the sun [c]. This is proved from the observation, that comets, though they are approaching the earth, yet diminish in lustre, if at the same time they recede from

the fun; and on the contrary, are found to encreafe daily in brightnefs, when they advance towards the fun, though at the fame time they move from the earth [a].

12. THE comets therefore in thefe refpects refemble the planets; that both are durable opake bodies, and both revolve about the fun in conic fections. But farther the comets, like our earth, are furrounded by an atmofphere. The air we breath is called the earth's atmofphere; and it is moſt probable, that all the other planets are invefted with the like fluid. Indeed here a difference is found between the planets and comets. The atmofpheres of the planets are of fo fine and fubtile a fubftance, as hardly to be difcerned at any diftance, by reafon of the fmall quantity of light which they reflect, except only in the planet Mars. In him there is fome little appearance of fuch a fubftance furrounding him, as ftars which have been covered by him are faid to look fomewhat dim a fmall fpace before his body comes under them, as if their light, when he is near, were obftructed by his atmofphere. But the atmofpheres which furround the comets are fo grofs and thick, as to reflect light very copioufly. They are alfo much greater in proportion to the body they furround, than thofe of the planets, if we may judge of the reft from our air; for it has been obferved of comets, that the bright light appearing in the middle of them, which

[a] Ibid. pag. 482, 483.

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is reflected from the folid body, is fcarce a ninth or tenth part of the whole comet.

13. I fpeak only of the heads of the comets, the moft lucid part of which is furrounded by a fainter light, the moft lucid part being ufually not above a ninth or tenth part of the whole in breadth [a]. Their tails are an appearance very peculiar, nothing of the fame nature appertaining in the leaft degree to any other of the celeftial bodies. Of that appearance there are feveral opinions; our author reduces them to three [b]. The two firft, which he propofes, are rejected by him; but the third he approves. The firft is, that they arife from a beam of light tranfmitted through the head of the comet, in like manner as a ftream of light is difcerned, when the fun fhines into a darkened room through a fmall hole. This opinion, as Sir Isaac Newton obferves, implies the authors of it wholly unfkilled in the principles of optics; for that ftream of light, feen in a darken'd room, arifes from the reflection of the fun beams by the duft and motes floating in the air: for the rays of light themfelves are not feen, but by their being reflected to the eye from fome fubftance, upon which they fall [c]. The next opinion examined by our author is that of the celebrated Des Cartes, who imagins thefe tails to be the light of the comet refracted in its paffage to us, and thence affording an oblong reprefentation; as the light of the fun does, when refracted

[a] Ibid. pag. 481.   [b] Ibib. pag. 509.   [c] See the fore-cited place.

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by the prifm in that noted experiment, which will have a great fhare in the third book of this difcourfe [a]. But this opinion is at once overturned from this confideration on-ly, that the planets could be no more free from this re-fraction than the comets ; nay ought to have larger or brighter tails, than they, becaufe the light of the planets is ftrongeft. However our author has thought proper to add fome farther objections againft this opinion : for inftance, that thefe tails are not variegated with colours, as is the image produced by the prifm, and which is infeparable from that unequal refraction, which produces that difpro-portioned length of the image. And befides, when the light in its paffage from different comets to the earth de-fcribes the fame path through the heavens, the refraction of it fhould of neceffity be in all refpects the fame. But this is contrary to obfervation ; for the comet in 1680, the 28th day of December, and a former comet in the year 1577, the 29th day of December, appear'd in the fame place of the heavens, that is, were feen adjacent to the fame fixed ftars, the earth likewife being in the fame place at both times; yet the tail of the latter comet de-viated from the oppofition to the fun a little to the north-ward, and the tail of the former comet declined from the oppofition of the fun five times as much fouthward [b].

14. THERE are fome other falfe opinions, though lefs regarded than thefe, which have been advanced upon this

[a] Ibid and Cartef. Princ. Phil. part. 3. § 134, &c.   [b] Vid. Phil. Nat. princ. Math. p. 511.

argument.

argument. These our excellent author passes over, hastening to explain, what he takes to be the true cause of this appearance. He thinks it is certainly owing to steams and vapours exhaled from the body, and gross atmosphere of the comets, by the heat of the sun; because all the appearances agree perfectly to this sentiment. The tails are but small, while the comet is descending to the sun, but enlarge themselves to an immense degree, as soon as ever the comet has passed its perihelion; which shews the tail to depend upon the degree of heat, which the comet receives from the sun. And that the intense heat to which comets, when nearest the sun, are exposed, should exhale from them a very copious vapour, is a most reasonable supposition; especially if we consider, that in those free and empty regions steams will more easily ascend, than here upon the surface of the earth, where they are suppressed and hindered from rising by the weight of the incumbent air: as we find by experiments made in vessels exhausted of the air, where upon removal of the air several substances will fume and discharge steams plentifully, which emit none in the open air. The tails of comets, like such a vapour, are always in the plane of the comet's orbit, and opposite to the sun, except that the upper part thereof inclines towards the parts, which the comet has left by its motion; resembling perfectly the smoak of a burning coal, which, if the coal remain fixed, ascends from it perpendicularly; but, if the coal be in motion, ascends obliquely, inclining from the motion of the coal. And besides, the tails of comets may be compared to this smoak in another

3

respect,

respect, that both of them are denser and more compact on the convex side, than on the concave. The different appearance of the head of the comet, after it has paſt its perihelion, from what it had before, confirms greatly this opinion of their tails: for ſmoke raiſed by a ſtrong heat is blacker and groſſer, than when raiſed by a leſs; and accordingly the heads of comets, at the ſame diſtance from the ſun, are obſerved leſs bright and ſhining after the perihelion, than before, as if obſcured by ſuch a groſs ſmoke.

15. The obſervations of Hevelius upon the atmoſpheres of comets ſtill farther illuſtrate the ſame; who relates, that the atmoſpheres, eſpecially that part of them next the ſun, are remarkably contracted when near the ſun, and dilated again afterwards.

16. To give a more full idea of theſe tails, a rule is laid down by our author, whereby to determine at any time, when the vapour in the extremity of the tail firſt roſe from the head of the comet. By this rule it is found, that the tail does not conſiſt of a fleeting vapour, diſſipated ſoon after it is raiſed, but is of long continuance; that almoſt all the vapour, which roſe about the time of the perihelion from the comet of 1680, continued to accompany it, aſcending by degrees, being ſucceeded conſtantly by freſh matter, which rendered the tail contiguous to the comet. From this computation the tails are found to participate of another property of aſcending vapours, that when they aſcend with the greateſt velocity, they are leaſt incurvated.   I i   17. The

17. The only objection that can be made againſt this opinion is the difficulty of explaining, how a ſufficient quantity of vapour can be raiſed from the atmoſphere of a comet to fill thoſe vaſt ſpaces, through which their tails are ſometimes extended. This our author removes by the following computation: our air being an elaſtic fluid, as has been ſaid before [a], is more denſe here near the ſurface of the earth, where it is preſſed upon by the whole air above; than it is at a diſtance from the earth, where it has a leſs weight incumbent. I have obſerved, that the denſity of the air is reciprocally proportional to the compreſſing weight. From hence our author computes to what degree of rarity the air muſt be expanded, according to this rule, at an height equal to a ſemidiameter of the earth: and he finds, that a globe of ſuch air, as we breath here on the ſurface of the earth, which ſhall be one inch only in diameter, if it were expanded to the degree of rarity, which the air muſt have at the height now mentioned, would fill all the planetary regions even to the very ſphere of Saturn, and far beyond. Now ſince the air at a greater height will be ſtill immenſly more rarified, and the ſurface of the atmoſpheres of comets is uſually about ten times the diſtance from the center of the comet, as the ſurface of the comet it ſelf, and the tails are yet vaſtly farther removed from the center of the comet; the vapour, which compoſes thoſe tails, may very well be allowed to be ſo expanded, as that a moderate quantity of matter may fill all that ſpace, they are ſeen to take up. Though indeed the atmoſpheres of comets being

[a] Book I. Ch. 4. § 11.

very

very grofs, they will hardly be rarified in their tails to fo great a degree, as our air under the fame circumftances; efpecially fince they may be fomething condenfed, as well by their gravitation to the fun, as that the parts will gravitate to one another; which will hereafter be fhewn to be the univerfal property of all matter [a]. The only fcruple left is, how fo much light can be reflected from a vapour fo rare, as this computation implies. For the removal of which our author obferves, that the moft refulgent of thefe tails hardly appear brighter, than a beam of the fun's light tranfmitted into a darkened room through a hole of a fingle inch diameter; and that the fmalleft fixed ftars are vifible through them without any fenfible diminution of their luftre.

18. All thefe confiderations put it beyond doubt, what is the true nature of the tails of comets. There has indeed nothing been faid, which will account for the irregular figures, in which thofe tails are fometimes reported to have appeared; but fince none of thofe appearances have ever been recorded by aftronomers, who on the contrary afcribe the fame likenefs to the tails of all comets, our author with great judgment refers all thofe to accidental refractions by intervening clouds, or to parts of the milky way contiguous to the comets [b].

19. The difcuffion of this appearance in comets has led Sir Isaac Newton into fome fpeculations relating to their ufe, which I cannot but extreamly admire, as

[a] Ch. 5.   [b] All thefe arguments are laid down in Philof. Nat. Princ. Lib. III. from p. 509, to 517.

reprefenting

reprefenting in the ftrongeft light imaginable the exten-
five providence of the great author of nature, who,
befides the furnifhing this globe of earth, and without
doubt the reft of the planets, fo abundantly with eve-
ry thing neceffary for the fupport and continuance of the
numerous races of plants and animals, they are ftocked
with, has over and above provided a numerous train of
comets, far exceeding the number of the planets, to re-
ctify continually, and reftore their gradual decay, which
is our author's opinion concerning them [a]. For fince the
comets are fubject to fuch unequal degrees of heat, being
fometimes burnt with the moft intenfe degree of it, at
other times fcarce receiving any fenfible influence from the
fun; it can hardly be fuppofed, they are defigned for any
fuch conftant ufe, as the planets. Now the tails, which they
emit, like all other kinds of vapour, dilate themfelves as
they afcend, and by confequence are gradually difperfed and
fcattered through all the planetary regions, and thence can-
not but be gathered up by the planets, as they pafs through
their orbs: for the planets having a power to caufe all bodies
to gravitate towards them, as will in the fequel of this
difcourfe be fhewn [b]; thefe vapours will be drawn in procefs
of time into this or the other planet, which happens to
act ftrongeft upon them. And by entering the atmofpheres
of the earth and other planets, they may well be fuppofed to
contribute to the renovation of the face of things, in par-
ticular to fupply the diminution caufed in the humid parts

a Philof. Nat. Princ. Lib. III. p. 515 .   b Ch.5.

by

by vegetation and putrefaction. For vegetables are nourished by moisture, and by putrefaction are turned in great part into dry earth; and an earthy substance always subsides in fermenting liquors; by which means the dry parts of the planets must continually increase, and the fluids diminish, nay in a sufficient length of time be exhausted, if not supplied by some such means. It is farther our great author's opinion, that the most subtile and active parts of our air, upon which the life of things chiefly depends, is derived to us, and supplied by the comets. So far are they from portending any hurt or mischief to us, which the natural fears of men are so apt to suggest from the appearance of any thing uncommon and astonishing.

20. THAT the tails of comets have some such important use seems reasonable, if we consider, that those bodies do not send out those fumes merely by their near approach to the sun; but are framed of a texture, which disposes them in a particular manner to fume in that sort: for the earth, without emitting any such steam, is more than half the year at a less distance from the sun, than the comet of 1664 and 1665 approached it, when nearest; likewise the comets of 1682 and 1683 never approached the sun much above a seventh part nearer than Venus, and were more than half as far again from the sun as Mercury; yet all these emitted tails.

21. FROM the very near approach of the comet of 1680 our author draws another speculation; for if the

sun

fun have an atmofphere about it, the comet mentioned feems to have defcended near enough to the fun to enter within it. If fo, it muft have been fomething retarded by the refiftance it would meet with, and confequently in its next defcent to the fun will fall nearer than now; by which means it will meet with a greater refiftance, and be again more retarded. The event of which muft be, that at length it will impinge upon the fun's furface, and thereby fupply any decreafe, which may have happened by fo long an emiffion of light, or otherwife. And fomething like this our author conjectures may be the cafe of thofe fixed ftars, which by an additional increafe of their luftre have for a certain time become vifible to us, though ufually they are out of fight. There is indeed a kind of fixed ftars, which appear and difappear at regular and equal intervals : here fome more fteady caufe muft be fought for ; perhaps thefe ftars turn round their own axis's, as our fun does [a], and have fome part of their body more luminous than the other, whereby they are feen, when the moft lucid part is next to us, and when the darker part is turned toward us, they vanifh out of fight.

22. WHETHER the fun does really diminifh, as has been here fuggefted, is difficult to prove; yet that it either does fo, or that the earth increafes, if not both, is rendered probable from Dr. HALLEY's obfervation [b], that by comparing

[a] See Ch. 1. § 11.

[b] Newt. Princ. Philof. pag. 525, 526. An account of all the ftars of both thefe kinds, which have appeared within the laft 150 years may be feen in the Philofophical tranfactions, vol. 29. numb. 346.

the

the proportion, which the periodical time of the moon bore to that of the fun in former times, with the proportion between them at prefent, the moon is found to be fomething accelerated in refpect of the fun.   But if the fun diminifh, the periods of the primary planets will be lengthened ; and if the earth be encreafed, the period of the moon will be fhortened : as will appear by the next chapter, wherein it fhall be fhewn, that the power of the fun and earth is the refult of the fame power being lodg'd in all their parts, and that this principle of producing gravitation in other bodies is proportional to the folid matter in each body.

C H A P. V.

Of the B O D I E S of the S U N and P L A N E T S.

OUR author, after having difcovered that the celeftial motions are performed by a force extended from the fun and primary planets, follows this power into the deepeft receffes of thofe bodies themfelves, and proves the fame to accompany the fmalleft particle, of which they are compofed.

2. PREPARATIVE hereto he fhews firft, that each of the heavenly bodies attracts the reft, and all bodies, with fuch different degrees of force, as that the force of the fame attracting

tracting body is exerted on others exactly in proportion to the quantity of matter in the body attracted [a].

3. Of this the first proof he brings is from experiments made here upon the earth. The power by which the moon is influenced was above shewn to be the same, with that power here on the surface of the earth, which we call gravity [b]. Now one of the effects of the principle of gravity is, that all bodies descend by this force from the same height in equal times. Which has been long taken notice of; particular methods having been invented to shew that the only cause, why some bodies were observed to fall from the same height sooner than others, was the resistance of the air. This we have above related [c]; and proved from hence, that since bodies resist to any change of their state from rest to motion, or from motion to rest, in proportion to the quantity of matter contained in them; the power that can move different quantities of matter equally, must be proportional to the quantity. The only objection here is, that it can hardly be made certain, whether this proportion in the effect of gravity on different bodies holds perfectly exact or not from these experiments; by reason that the great swiftness, with which bodies fall, prevents our being able to determine the times of their descent with all the exactness requisite. Therefore to remedy this inconvenience, our author substitutes another more certain experiment in the room of these made upon falling bodies. Pen-

[a] Newt. Princ. Philos. Nat. Lib. III. prop. 6.   [b] Ch. 3. § 6.   [c] Book I. Ch. 2. § 24.

dulums

dulums are caufed to vibrate by the fame principle, as makes bodies defcend ; the power of gravity putting them in motion, as well as the other. But if the ball of any pendulum, of the fame length with another, were more or lefs attracted in proportion to the quantity of folid matter in the ball, that pendulum muft accordingly move fafter or flower than the other. Now the vibrations of pendulums continue for a great length of time, and the number of vibrations they make may eafily be determined without fufpicion of error ; fo that this experiment may be extended to what exactnefs one pleafes : and our author affures us , that he examined in this way feveral fubftances, as gold, filver, lead, glafs, fand, common falt, wood, water, and wheat ; in all which he found not the leaft deviation from the proportion mentioned, though he made the experiment in fuch a manner, that in bodies of the fame weight a difference in the quantity of their matter lefs than a thoufandth part of the whole would have difcovered it felf [a]. It appears therefore, that all bodies are made to defcend by the power of gravity here, near the furface of the earth, with the fame degree of fwiftnefs. We have above obferved this defcent to be after the rate of $16\frac{1}{3}$ feet in the firft fecond of time from the beginning of their fall. Moreover it was alfo obferved, that if any body, which fell here at the furface of the earth after this rate, were to be conveyed up to the height of the moon, it would

[a] Newt. Princ. Lib. III. prop. 6.

K k   defcend

descend from thence just with the same degree of velocity, as that with which the moon is attracted toward the earth; and therefore the power of the earth upon the moon bears the same proportion to the power it would have upon those bodies at the same distance, as the quantity of matter in the moon bears to the quantity in those bodies.

4. THUS the assertion laid down is proved in the earth, that the power of the earth on every body it attracts is, at the same distance from the earth, proportional to the quantity of solid matter in the body acted on. As to the sun, it has been shewn, that the power of the sun's action upon the same primary planet is reciprocally in the duplicate proportion of the distance; and that the power of the sun decreases throughout in the same proportion, the motion of comets traversing the whole planetary region testifies. This proves, that if any planet were removed from the sun to any other distance whatever, the degree of its acceleration toward the sun would yet remain reciprocally in the duplicate proportion of its distance. But it has likewise been shewn, that the degree of acceleration, which the sun gives to every one of the planets, is reciprocally in the duplicate proportion of their respective distances. All which compared together puts it out of doubt, that the power of the sun upon any planet, removed into the place of any other, would give it the same velocity of descent, as it gives that other; and consequently, that the sun's action upon different planets at the same distance would be proportional to the quantity of matter in each. It has farther been

been fhewn, that the fun attracts the primary planets, and their refpective fecondary, when at the fame diftance, fo as to communicate to both the fame degree of velocity ; and therefore the force, wherewith the fun acts on the fecondary planet, bears the fame proportion to the force, wherewith at the fame diftance it attracts the primary, as the quantity of folid matter in the fecondary planet bears to the quantity of matter in the primary.

5. This property therefore is proved of both kinds of planets, in refpect of the fun. Therefore the fun poffeffes the quality found in the earth, of acting on bodies with a degree of force proportional to the quantity of matter in the body, which receives the influence.

6. That the power of attraction, with which the other planets are endued, fhould differ from that of the earth, can hardly be fuppofed, if we confider the fimilitude between thofe bodies ; and that it does not in this refpect, is farther proved from the fatellites of Saturn and Jupiter, which are attracted by their refpective primary according to the fame law, that is, in the fame proportion to their diftances, as the primary are attracted by the fun: fo that what has been concluded of the fun in relation to the primary planets, may be juftly concluded of thefe primary in refpect of their fecondary, and in confequence of that, in regard likewife to all other bodies, viz. that they will attract every body in proportion to the quantity of folid matter it contains.

3   K k 2   7. Hence

7. Hence it follows, that this attraction extends itself to every particle of matter in the attracted body : and that no portion of matter whatever is exempted from the influence of thofe bodies, to which we have proved this attractive power to belong.

8. Before we proceed farther, we may here remark, that this attractive power both of the fun and planets now appears to be quite of the fame nature in all; for it acts in each in the fame proportion to the diftance, and in the fame manner acts alike upon every particle of matter. This power therefore in the fun and other planets is not of a different nature from this power in the earth; which has been already fhewn to be the fame with that, which we call gravity [a].

9. And this lays open the way to prove, that the attracting power lodged in the fun and planets, belongs likewife to every part of them : and that their refpective powers upon the fame body are proportional to the quantity of matter, of which they are compofed; for inftance, that the force with which the earth attracts the moon, is to the force, with which the fun would attract it at the fame diftance, as the quantity of folid matter contained in the earth, to the quantity contained in the fun [b].

10. The firft of thefe affertions is a very evident confequence from the latter. And before we proceed to the proof,

[a] Ch. 3. § 6.   [b] Newt. Princ. philof. Lib. III. prop. 7. cor. 1.

it

it muſt firſt be ſhewn, that the third law of motion, which makes action and reaction equal, holds in theſe attractive powers.   The moſt remarkable attractive force,   next to the power of gravity, is that, by which the loadſtone attracts iron. Now if a loadſtone were laid upon water, and ſupported by ſome proper ſubſtance, as wood or cork, ſo that it might ſwim ; and if a piece of iron were cauſed to ſwim upon the water in like manner: as ſoon as the loadſtone begins to attract the iron, the iron ſhall move toward the ſtone, and the ſtone ſhall alſo move toward the iron ; when they meet, they ſhall ſtop each other, and remain fixed together without any motion.   This ſhews, that the velocities,   wherewith they meet, are reciprocally proportional to the quantities of ſolid matter in each ;   and that by the ſtone's attracting the iron, the ſtone itſelf receives as much motion, in the ſtrict philoſophic ſenſe of that word [a], as it communicates to the iron: for it has been declared above to be an effect of the percuſſion of two bodies,   that if they meet with velocities reciprocally proportional to the reſpective bodies,   they ſhall be ſtopped by the concourſe, unleſs their elaſticity put them into freſh motion;   but if they meet with any other velocities,   they ſhall retain ſome motion after meeting [b].   Amber, glaſs, ſealing-wax, and many other ſubſtances acquire by rubbing a power, which from its having been remarkable,   particularly in amber,   is called electrical.   By this power they will for ſome time after

[a] See Book I. Ch. 1. § 25.   [b] Ibid. § 5, 6.

rubbing

rubbing attract light bodies, that shall be brought within the sphere of their activity. On the other hand Mr. Boyle found, that if a piece of amber be hung in a perpendicular position by a string, it shall be drawn itself toward the body whereon it was rubbed, if that body be brought near it. Both in the loadstone and in electrical bodies we usually ascribe the power to the particular body, whose presence we find necessary for producing the effect. The loadstone and any piece of iron will draw each other, but in two pieces of iron no such effect is ordinarily observed; therefore we call this attractive power the power of the loadstone: though near a loadstone two pieces of iron will also draw each other. In like manner the rubbing of amber, glass, or any such body, till it is grown warm, being necessary to cause any action between those bodies and other substances, we ascribe the electrical power to those bodies. But in all these cases if we would speak more correctly, and not extend the sense of our expressions beyond what we see; we can only say that the neighbourhood of a loadstone and a piece of iron is attended with a power, whereby the loadstone and the iron are drawn toward each other; and the rubbing of electrical bodies gives rise to a power, whereby those bodies and other substances are mutually attracted. Thus we must also understand in the power of gravity, that the two bodies are mutually made to approach by the action of that power. When the sun draws any planet, that planet also draws the sun; and the motion, which the planet receives from the sun, bears the same proportion to the motion, which the sun it self receives, as

the

the quantity of folid matter in the fun bears to the quantity of folid matter in the planet.   Hitherto, for brevity fake in fpeaking of thefe forces, we have generally afcribed them to the body, which is leaft moved ; as when we called the power, which exerts itfelf between the fun and any planet, the attractive power of the fun ; but to fpeak more correctly, we fhould rather call this power in any cafe the force, which acts between the fun and earth, between the fun and Jupiter, between the earth and moon, &c. for both the bodies are moved by the power acting between them, in the fame manner, as when two bodies are tied together by a rope, if that rope fhrink by being wet, or otherwife, and thereby caufe the bodies to approach, by drawing both, it will communicate to both the fame degree of motion, and caufe them to approach with velocities reciprocally proportional to the refpective bodies.   From this mutual action between the fun and planet it follows, as has been obferved above [a], that the fun and planet do each move about their common center of gravity. Let A (in fig. 108.) reprefent the fun, B a planet, C their common center of gravity.   If thefe bodies were once at reft, by their mutual attraction they would directly approach each other with fuch velocities, that their common center of gravity would remain at reft, and the two bodies would at length meet in that point.   If the planet B were to receive an impulfe, as in the direction of the line DF, this would prevent the two bodies from falling together ;

[a] Chap. 2. § 8.

but

but their common center of gravity would be put into motion in the direction of the line C F equidiſtant from B E. In this caſe Sir Isaac Newton proves[a], that the ſun and planet would deſcribe round their common center of gravity ſimilar orbits, while that center would proceed with an uniform motion in the line C F; and ſo the ſyſtem of the two bodies would move on with the center of gravity without end. In order to keep the ſyſtem in the ſame place, it is neceſſary, that when the planet received its impulſe in the direction B E, the ſun ſhould alſo receive ſuch an impulſe the contrary way, as might keep the center of gravity C without motion; for if theſe began once to move without giving any motion to their common center of gravity, that center would always remain fixed.

II. By this may be underſtood in what manner the action between the ſun and planets is mutual. But farther, we have ſhewn above[b], that the power, which acts between the ſun and primary planets, is altogether of the ſame nature with that, which acts between the earth and the bodies at its ſurface, or between the earth and its parts, and with that which acts between the primary planets and their ſecondary; therefore all theſe actions muſt be aſcribed to the ſame cauſe[c]. Again, it has been already proved, that in different planets the force of the ſun's action upon each at the ſame diſtance would be proportional to the quantity of ſolid matter in the planet[d]; therefore the reaction of each planet

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[a] Newt. Princ. Lib. I. prop. 63.   [b] § 8.   [c] See Introd. § 23.   [d] § 4, 5.

Fig. 104.

Fig. 105.
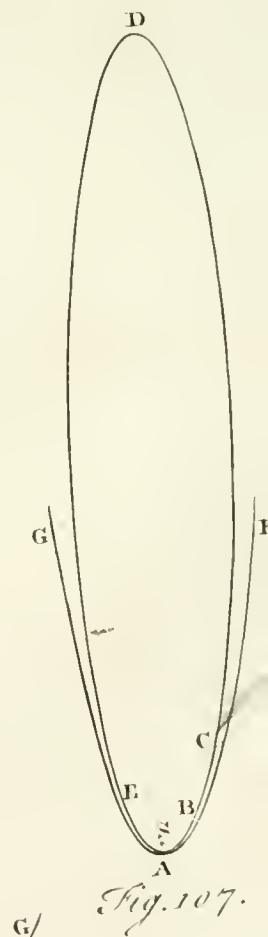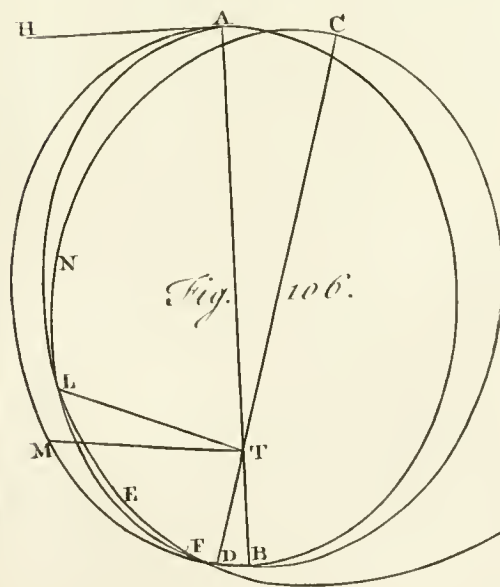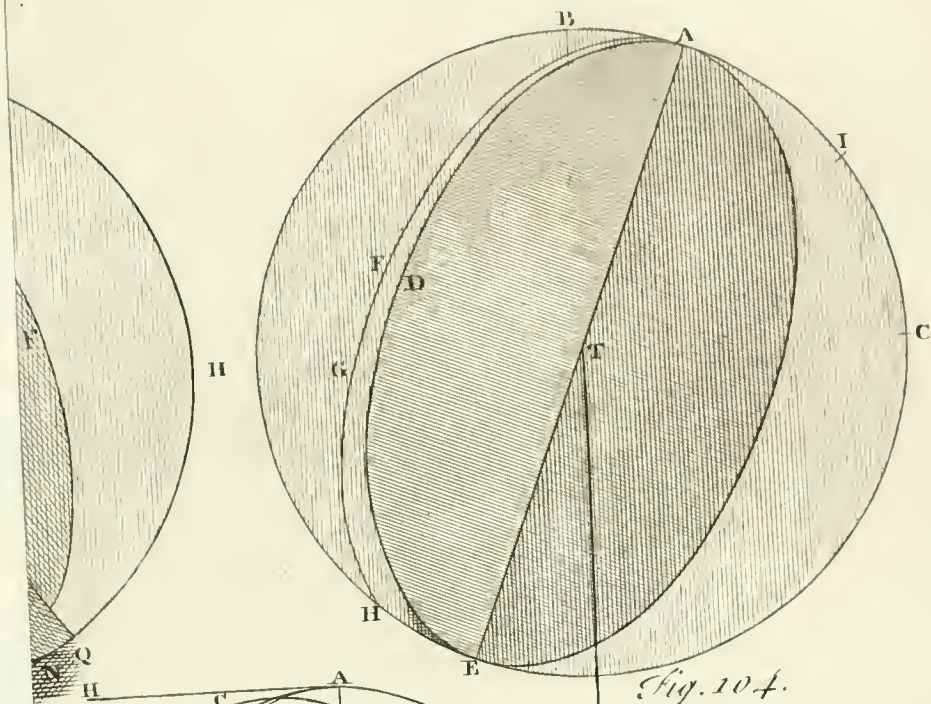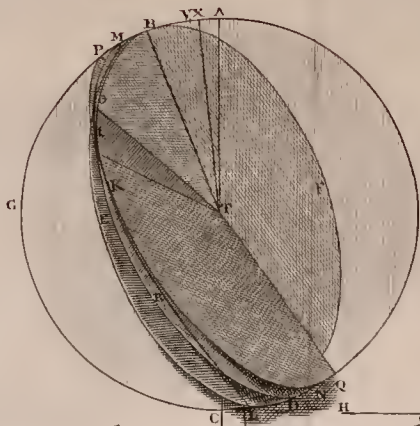
Fig. 106.

Fig. 107.
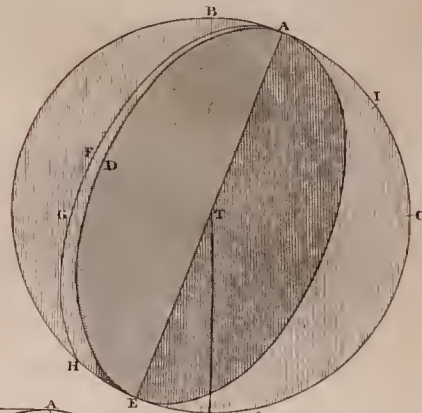
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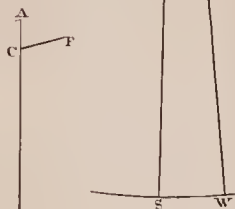
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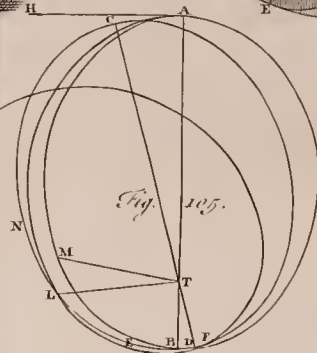
Fig. 105.

Fig. 106.
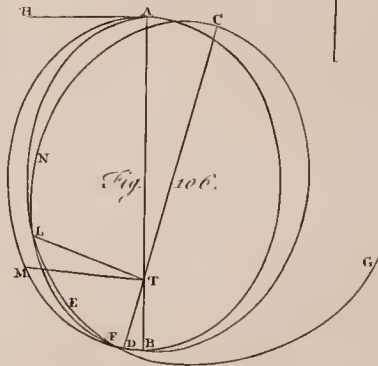
Fig. 107.

Fig. 108.

on the fun at the fame diftance, or the motion, which the fun would receive from each planet, would alfo be proportional to the quantity of matter in the planet; that is, thefe planets at the fame diftance would act on the fame body with' degrees of ftrength proportional to the quantity of folid matter in each.

12. In the next place, from what has been now proved, our great author has deduced this farther confequence, no lefs furprizing than elegant; that each of the particles, out of which the bodies of the fun and planets are framed, exert their power of gravitation by the fame law, and in the fame proportion to the diftance, as the great bodies which they compofe. For this purpofe he firft demonftrates, that if a globe were compounded of particles, which will attract the particles of any other body reciprocally in the duplicate proportion of their diftances, the whole globe will attract the fame in the reciprocal duplicate proportion of their diftances from the center of the globe; provided the globe be of uniform denfity throughout [a]. And from this our author deduces the reverfe, that if a globe acts upon diftant bodies by the law juft now fpecified, and the power of the globe is derived from its being compofed of attractive particles; each of thofe particles will attract after the fame proportion [b]. The manner of deducing this is not fet down at large by our author, but is as follows. The globe is

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[a] Newt. Princ. philof. Lib. 1. prop. 74.   [b] Ibid. coroll. 3.

fuppofed

fuppofed to act upon the particles of a body without it con-
ftantly in the reciprocal duplicate proportion of their diftan-
ces from its center; and therefore at the fame diftance from
the globe, on which fide foever the body be placed, the
globe will act equally upon it.   Now becaufe, if the parti-
cles, of which the globe is compofed, acted upon thofe with-
out in the reciprocal duplicate proportion of their diftances,
the whole globe would act upon them in the fame manner as
it does ; therefore, if the particles of the globe have not all
of them that property, fome muft act ftronger than in that
proportion, while others act weaker: and if this be the con-
dition of the globe, it is plain, that when the body attracted
is in fuch a fituation in refpect of the globe, that the greater
number of the ftrongeft particles are neareft to it, the body
will be more forcibly attracted; than when by turning the
globe about, the greater quantity of weak particles fhould
be neareft, though the diftance of the body fhould remain
the fame from the center of the globe.   Which is contrary
to what was at firft remarked, that the globe on all fides of
it acts with the fame ftrength at the fame diftance. Whence it
appears, that no other conftitution of the globe can agree to it.

13. From thefe propofitions it is farther collected, that
if all the particles of one globe attract all the particles of an-
other in the proportion fo often mentioned, the attracting
globe will act upon the other in the fame proportion to the
diftance between the center of the globe which attracts, and
the center of that which is attracted [a] : and farther, that this

[a] Lib. I. Prop. 75. and Lib. III. prop. 8.

proportion

proportion holds true, though either or both the globes be compofed of diffimilar parts, fome rarer and fome more denfe; provided only, that all the parts in the fame globe equally diftant from the center be homogeneous [a]. And alfo, if both the globes attract each other [b]. All which place it beyond contradiction, that this proportion obtains with as much exactnefs near and contiguous to the furface of attracting globes, as at greater diftances from them.

14. Thus our author, without the pompous pretence of explaining the caufe of gravity, has made one very important ftep toward it, by fhewing that this power in the great bodies of the univerfe, is derived from the fame power being lodged in every particle of the matter which compofes them: and confequently, that this property is no lefs than univerfal to all matter whatever, though the power be too minute to produce any vifible effects on the fmall bodies, wherewith we converfe, by their action on each other [c]. In the fixed ftars indeed we have no particular proof that they have this power; for we find no apperance to demonftrate that they either act, or are acted upon by it. But fince this power is found to belong to all bodies, whereon we can make obfervation; and we fee that it is not to be altered by any change in the form of bodies, but always accompanies them in every fhape without diminution, remaining ever proportional to the quantity of folid matter in each; fuch a power muft without doubt belong univerfally to all matter.

[a] Lib. I. Prop.76.   [b] Ibid. cor.5.   [c] Vid Lib.III. Prop. 7. coroll. 1.

15. This

15. This therefore is the univerfal law of matter; which recommends it felf no lefs for its great plainnefs and fimplicity, than for the furprizing difcoveries it leads us to. By this principle we learn the different weight, which the fame body will have upon the furfaces of the fun and of diverfe planets; and by the fame we can judge of the compofition of thofe celeftial bodies, and know the denfity of each ; which is formed of the moft compact, and which of the moft rare fubftance. Let the adverfaries of this philofophy reflect here, whether loading this principle with the appellation of an occult quality, or perpetual miracle, or any other reproachful name, be fufficient to diffuade us from cultivating it; fince this quality, which they call occult, leads to the knowledge of fuch things, that it would have been reputed no lefs than madnefs for any one, before they had been difcovered, even to have conjectured that our faculties fhould ever have reached fo far.

16. See how all this naturally follows from the foregoing principles in thofe planets, which have fatellites moving about them. By the times, in which thefe fatellites perform their revolutions, compared with their diftances from their refpective primary, the proportion between the power, with which one primary attracts his fatellites, and the force with which any other attracts his will be known; and the proportion of the power with which any planet attracts its fecondary, to the power with which it attracts a body at its furface is found, by comparing the diftance of the fecondary planet from the center of the primary, to the

the diftance of the primary planet's furface from the fame :
and from hence is deduced the proportion between the power
of gravity upon the furface of one planet, to the gravity upon
the furface of another.   By the like method of comparing
the periodical time of a primary planet about the fun, with
the revolution of a fatellite about its primary, may be found
the proportion of gravity, or of the weight of any body up-
on the furface of the fun, to the gravity, or to the weight of
the fame body upon the furface of the planet, which carries
about the fatellite.

17. Bʏ thefe kinds of computation it is found, that the
weight of the fame body upon the furface of the fun will
be about 23 times as great, as here upon the furface of the
earth ; about 10⅔ times as great, as upon the furface of Jupi-
ter ; and near 19 times as great, as upon the furface of Saturn [a].

18. Tʜᴇ quantity of matter, which compofes each of
thefe bodies, is proportional to the power it has upon a
body at a given diftance.   By this means it is found, that the
fun contains 1067 times as much matter as Jupiter ;   Jupi-
ter 158⅔ times as much as the earth, and 2⅚ times as much
as Saturn [b].   The diameter of the fun is about 92 times,
that of Jupiter about 9 times, and that of Saturn about
7 times the diameter of the earth.

[a] Newt. Princ. Lib. III. prop. 8. coroll. 1.   [b] Ibid. coroll. 2.

19. By making a comparison between the quantity of matter in these bodies and their magnitudes, to be found from their diameters, their respective densities are readily deduced ; the density of every body being measured by the quantity of matter contained under the same bulk, as has been above remarked [a].   Thus the earth is found $4\frac{1}{4}$ times more dense than Jupiter ; Saturn has between $\frac{2}{3}$ and $\frac{1}{4}$ of the density of Jupiter ; but the sun has one fourth part only of the density of the earth [b]. From which this observation is drawn by our author; that the sun is rarified by its great heat, and that of the three planets named, the more dense is nearer the sun than the more rare ; as was highly reasonable to expect, the densest bodies requiring the greatest heat to agitate and put their parts in motion ; as on the contrary, the planets which are more rare, would be rendered unfit for their office, by the intense heat to which the denser are exposed.   Thus the waters of our seas, if removed to the distance of Saturn from the sun, would remain perpetually frozen; and if as near the sun as Mercury, would constantly boil [c].

20. The densities of the three planets Mercury, Venus, and Mars, which have no satellites, cannot be expresly assigned ; but from what is found in the others, it is very probable, that they also are of such different degrees of density, that universally the planet which is nearest to the sun, is formed of the most compact substance.

[a] Book I. Ch. 4. § 2.   [b] Newt. Princ. Lib. III. prop. 8. coroll. 3.   [c] Ibid. coroll. 4.

CHAP.

Chap. VI.

Of the FLUID PARTS of the PLANETS.

THIS globe, that we inhabit, is compofed of two parts; the folid earth, which affords us a foundation to dwell upon; and the feas and other waters, that furnifh rains and vapours neceffary to render the earth fruitful, and productive of what is requifite for the fupport of life. And that the moon, though but a fecondary planet, is compofed in like manner, is generally thought, from the different degrees of light which appear on its furface; the parts of that planet, which reflect a dim light, being fuppofed to be fluid, and to imbibe the fun's rays, while the folid parts reflect them more copioufly. Some indeed do not allow this to be a conclufive argument: but whether we can diftinguifh the fluid part of the moon's furface from the reft or not; yet it is moft probable that there are two fuch different parts, and with ftill greater reafon we may afcribe the like to the other primary planets, which yet more nearly refemble our earth. The earth is alfo encompaffed by another fluid the air, and we have before remarked, that probably the reft of the planets are furrounded by the like. Thefe fluid parts in particular engage our author's attention, both by reafon of fome remarkable appearances peculiar to them, and likewife of fome effects they have upon the whole bodies to which they belong.

4　　　　2. Fluids

2. Fluids have been already treated of in general, with respect to the effect they have upon solid bodies moving in them [a]; now we muft confider them in reference to the operation of the power of gravity upon them. By this power they are rendered weighty, like all other bodies, in proportion to the quantity of matter, which is contained in them. And in any quantity of a fluid the upper parts prefs upon the lower as much, as any folid body would prefs on another, whereon it fhould lie. But there is an effect of the preffure of fluids on the bottom of the veffel, wherein they are contained, which I fhall particularly explain. The force fupported by the bottom of fuch a veffel is not fimply the weight of the quantity of the fluid in the veffel, but is equal to the weight of that quantity of the fluid, which would be contained in a veffel of the fame bottom and of equal width throughout, when this veffel is filled up to the fame height, as that to which the veffel propofed is filled. Suppofe water were contained in the veffel A B C D (in fig. 109.) filled up to E F. Here it is evident, that if a part of the bottom, as G H, which is directly under any part of the fpace E F, be confidered feparately; it will appear at once, that this part fuftains the weight of as much of the fluid, as ftands perpendicularly over it up to the height of E F; that is, the two perpendiculars G I and H K being drawn, the part G H of the bottom will fuftain the whole weight of the fluid included between thefe two perpendiculars. Again, I fay, every other part of the bottom equally broad with this, will fuftain as great a preffure. Let the part L M be of the

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[a] Book I Ch. 4.

fame

same breadth with G H.   Here the perpendiculars L O and M N being drawn, the quantity of water contained between these perpendiculars is not so great, as that contained between the perpendiculars G I and H K; yet, I say, the pressure on L M will be equal to that on G H.   This will appear by the following considerations.   It is evident, that if the part of the vessel between O and N were removed, the water would immediately flow out, and the surface E F would subside ; for all parts of the water being equally heavy, it must soon form itself to a level surface, if the form of the vessel, which contains it, does not prevent.   Therefore since the water is prevented from rising by the side N O of the vessel, it is manifest, that it must press against N O with some degree of force. In other words, the water between the perpendiculars L O and M N endeavours to extend itself with a certain degree of force; or more correctly, the ambient water presses upon this, and endeavours to force this pillar or column of water into a greater length.   But since this column of water is sustained between N O and L M, each of these parts of the vessel will be equally pressed against by the power, wherewith this column endeavours to extend.   Consequently L M bears this force over and above the weight of the column of water between L O and M N.   To know what this expansive force is, let the part O N of the vessel be removed, and the perpendiculars L O and M N be prolonged ;   then by means of some pipe fixed over N O let water be filled between these perpendiculars up to P Q an equal height with E F. Here the water between the perpendiculars L P and M Q is of an equal height with the highest part of the water in the vessel ; therefore the water in the

<center>M m</center>   vessel

veffel cannot by its preffure force it up higher, nor can the water in this column fubfide; becaufe, if it fhould, it would raife the water in the veffel to a greater height than itfelf. But it follows from hence, that the weight of water contained between P O and Q N is a juft balance to the force, wherewith the column between L O and M N endeavours to extend. So the part L M of the bottom, which fuftains both this force and the weight of the water between L O and M N, is pref-fed upon by a force equal to the united weight of the water between L O and M N, and the weight of the water between P O and Q N; that is, it is preffed on by a force equal to the weight of all the water contained between L P and M Q. And this weight is equal to that of the water contained between G I and H K, which is the weight fuftained by the part G H of the bottom. Now this being true of every part of the bottom B C, it is evident, that if another veffel R S T V be formed with a bottom R V equal to the bottom B C, and be throughout its whole height of one and the fame breadth; when this veffel is filled with water to the fame height, as the veffel A B C D is filled, the bottoms of thefe two veffels fhall be preffed upon with equal force. If the veffel be broader at the top than at the bottom, it is evident, that the bottom will bear the preffure of fo much of the fluid, as is perpen-dicularly over it, and the fides of the veffel will fupport the reft. This property of fluids is a corollary from a propofi-tion of our author [a]; from whence alfo he deduces the ef-fects of the preffure of fluids on bodies refting in them.

[a] Lib. II. prop. 20. cor. 2.

Thefe

These are, that any body heavier than a fluid will sink to the bottom of the vessel, wherein the fluid is contained, and in the fluid will weigh as much as its own weight exceeds the weight of an equal quantity of the fluid; any body uncompressible of the same density with the fluid, will rest any where in the fluid without suffering the least change either in its place or figure from the pressure of such a fluid, but will remain as undisturbed as the parts of the fluid themselves; but every body of less density than the fluid will swim on its surface, a part only being received within the fluid. Which part will be equal in bulk to a quantity of the fluid, whose weight is equal to the weight of the whole body; for by this means the parts of the fluid under the body will suffer as great a pressure as any other parts of the fluid as much below the surface as these.

3. In the next place, in relation to the air, we have above made mention, that the air surrounding the earth being an elastic fluid, the power of gravity will have this effect on it, to make the lower parts near the surface of the earth more compact and compressed together by the weight of the air incumbent, than the higher parts, which are pressed upon by a less quantity of the air, and therefore sustain a less weight [a]. It has been also observed, that our author has laid down a rule for computing the exact degree of density in the air at all heights from the earth [b]. But there is a farther effect from the air's being compressed by

[a] Chap. 4. § 17.   [b] Ibid.

the power of gravity, which he has diſtinctly conſidered. The air being elaſtic and in a ſtate of compreſſion, any tremulous body will propagate its motion to the air, and excite therein vibrations, which will ſpread from the body that occaſions them to a great diſtance. This is the efficient cauſe of ſound : for that ſenſation is produced by the air, which, as it vibrates, ſtrikes againſt the organ of hearing. As this ſubject was extremely difficult, ſo our great author's ſucceſs is ſurprizing.

4. Our author's doctrine upon this head I ſhall endeavour to explain ſomewhat at large. But preliminary thereto muſt be ſhewn, what he has delivered in general of preſſure propagated through fluids ; and alſo what he has ſet down relating to that wave-like motion, which appears upon the ſurface of water, when agitated by throwing any thing into it, or by the reciprocal motion of the finger, &c.

5. Concerning the firſt, it is proved, that preſſure is ſpread through fluids, not only right forward in a ſtreight line, but alſo laterally, with almoſt the ſame eaſe and force. Of which a very obvious exemplification by experiment is propoſed : that is, to agitate the ſurface of water by the reciprocal motion of the finger forwards and backwards only ; for though the finger have no circular motion given it, yet the waves excited in the water will diffuſe themſelves on each hand of the direction of the motion, and ſoon ſurround the finger. Nor is what we obſerve in ſounds unlike to this, which do not proceed in ſtraight lines only, but are heard though a

mountain

mountain intervene, and when they enter a room in any part of it, they fpread themfelves into every corner ; not by reflection from the walls, as fome have imagined, but as far as the fenfe can judge, directly from the place where they enter.

6. How the waves are excited in the furface of ftagnant water, may be thus conceived. Suppofe in any place, the water raifed above the reft in form of a fmall hillock ; that water will immediately fubfide, and raife the circumambient water above the level of the parts more remote, to which the motion cannot be communicated under longer time. And again, the water in fubfiding will acquire, like all falling bodies, a force, which will carry it below the level furface, till at length the preffure of the ambient water prevailing, it will rife again, and even with a force like to that wherewith it defcended, which will carry it again above the level. But in the mean time the ambient water before raifed will fubfide, as this did, finking below the level ; and in fo doing, will not only raife the water, which firft fubfided, but alfo the water next without itfelf. So that now befide the firft hillock, we fhall have a ring invefting it, at fome diftance raifed above the plain furface likewife ; and between them the water will be funk below the reft of the furface. After this, the firft hillock, and the new made annular rifing, will defcend ; raifing the water between them, which was before depreffed, and likewife the adjacent part of the furface without. Thus will thefe annular waves be fucceffively fpread more and more. For, as the hillock fubfiding produces one ring, and that ring fub-

<div align="right">fiding</div>

fiding raifes again the hillock, and a fecond ring ; fo the hillock and fecond ring fubfiding together raife the firft ring, and a third ; then this firft and third ring fubfiding together raife the firft hillock, the fecond ring, and a fourth ; and fo on continually, till the motion by degrees ceafes. Now it is demonftrated, that thefe rings afcend and defcend in the manner of a pendulum ; defcending with a motion continually accelerated, till they become even with the plain furface of the fluid, which is half the fpace they defcend ; and then being retarded again by the fame degrees as thofe, whereby they were accelerated, till they are depreffed below the plain furface, as much as they were before raifed above it: and that this augmentation and diminution of their velocity proceeds by the fame degrees, as that of a pendulum vibrating in a cycloid, and whofe length fhould be a fourth part of the diftance between any two adjacent waves: and farther, that a new ring is produced every time a pendulum, whofe length is four times the former, that is, equal to the interval between the fummits of two waves, makes one ofcillation or fwing [a].

7. This now opens the way for underftanding the motion confequent upon the tremors of the air, excited by the vibrations of fonorous bodies: which we muft conceive to be performed in the following manner.

8. Let A, B, C, D, E, F, G, H (in fig. 110.) reprefent a feries of the particles of the air, at equal diftances from each other. I K L a mufical chord, which I fhall ufe for the tre-

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[a] Vid. Newt. Princ. Lib. II. prop. 46.

mulous

mulcus and fonorous body, to make the conception as fim-
ple as may be.   Suppofe this chord ftretched upon the points
I and L, and forcibly drawn into the fituation I K L,   fo that
it become contiguous to the particle A in its middle point K:
and let the chord from this fituation begin to recoil, preffing
againft the particle A, which will thereby be put into motion
towards B : but the particles A, B, C being equidiftant, the
elaftic power, by which B avoids A, is equal to, and balan-
ced by the power, by which it avoids C ; therefore the elaftic
force, by which B is repelled from A, will not put B into any
degree of motion, till A is by the motion of the chord brought
nearer to B, than B is to C: but as foon as that is done, the
particle B will be moved towards C; and being made to ap-
proach C, will in the next place move that; which will up-
on that advance, put D likewife into motion, and fo on :
therefore the particle A being moved by the chord, the fol-
lowing particles of the air B, C, D, &c. will fucceffively be
moved.   Farther, if the point K of the chord moves for-
ward with an accelerated velocity, fo that the particle A fhall
move againft B with an advancing pace, and gain ground of
it, approaching nearer and nearer continually ; A by approach-
ing will prefs more upon B, and give it a greater velocity
likewife, by reafon that as the diftance between the particles
diminifhes, the elaftic power, by which they fly each other,
increafes.   Hence the particle B, as well as A, will have its
motion gradually accelerated, and by that means will more
and more approach to C.   And from the fame caufe C will
more and more approach D ; and fo of the reft.   Suppofe
now, fince the agitation of thefe particles has been fhewn to
<div align="right">be</div>

be fucceflive, and to follow one another, that E be the re-
moteft particle moved, while the chord is moving from its
curve fituation I K L into that of a ftreight line, as I k L; and
F the firft which remains unaffected, though juft upon the
point of being put into motion.   Then fhall the particles
A, B, C, D, E, F, G, when the point K is moved into k, have
acquired the rangement reprefented by the adjacent points
a, b, c, d, e, f, g: in which a is nearer to b than b to c, and
b nearer to c than c to d, and c nearer to d than d to e, and
d nearer to e than e to f, and laftly e nearer to f than f to g.

9. But now the chord having recovered its rectilinear fi-
tuation I k L, the following motion will be changed, for the
point K, which before advanced with a motion more and
more accelerated, though by the force it has acquired it will
go on to move the fame way as before, till it has advanced
near as far forwards, as it was at firft drawn backwards; yet
the motion of it will henceforth be gradually leffened.   The
effect of which upon the particles a, b, c, d, e, f, g will be,
that by the time the chord has made its utmoft advance, and
is upon the return, thefe particles will be put into a contrary
rangement; fo that f fhall be nearer to g, than e to f, and
e nearer to f than d to e; and the like of the reft, till you
come to the firft particles a, b, whofe diftance will then be
nearly or quite what it was at firft.   All which will appear
as follows.   The prefent diftance between a and b is fuch,
that the elaftic power, by which a repels b, is ftrong enough to
maintain that diftance, though a advance with the velocity,
with which the ftring refumes its rectilinear figure; and the

motion

motion of the particle *a* being afterwards flower, the present elasticity between *a* and *b* will be more than sufficient to preserve the distance between them. Therefore while it accelerates *b* it will retard *a*. The distance *bc* will still diminish, till *b* come about as near to *c*, as it is from *a* at present; for after the distances *ab* and *bc* are become equal, the particle *b* will continue its velocity superior to that of *c* by its own power of inactivity, till such time as the increase of elasticity between *b* and *c* more than shall be between *a* and *b* shall suppress its motion: for as the power of inactivity in *b* made a greater elasticity necessary on the side of *a* than on the side of *c* to push *b* forward, so what motion *b* has acquired it will retain by the same power of inactivity, till it be suppressed by a greater elasticity on the side of *c*, than on the side of *a*. But as soon as *b* begins to slacken its pace the distance of *b* from *c* will widen as the distance *ab* has already done. Now as *a* acts on *b*, so will *b* on *c*, *c* on *d*, &c. so that the distances between all the particles *a, b, c, d, e, f, g* will be successively contracted into the distance of *a* from *b*, and then dilated again. Now because the time, in which the chord describes this present half of its vibration, is about equal to that it took up in describing the former; the particles *a, b* will be as long in dilating their distance, as before in contracting it, and will return nearly to their original distance. And farther, the particles *b, c*, which did not begin to approach so soon as *a, b*, are now about as much longer, before they begin to recede; and likewise the particles *c, d*, which began to approach after *b, c*, begin to separate later. Whence it appears that the particles, whose distance began to be lessened, when

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that of a, b was firſt enlarged, viz. the particles f, g, ſhould be about their neareſt diſtance, when a and b have recovered their prime interval. Thus will the particles a, b, c, d, e, f, g have changed their ſituation in the manner aſſerted. But farther, as the particles f, g or F, G gradually approach each other, they will move by degrees the ſucceeding particles to as great a length, as the particles A, B did by a like approach. So that, when the chord has made its greateſt advance, being arrived into the ſituation I x L, the particles moved by it will have the rangement noted by the points a, β, γ, δ, ϵ, ζ, η, θ, λ, μ, ν, ξ. Where a, β are at the original diſtance of the particles in the line A H; ζ, η are the neareſt of all, and the diſtance $\nu \xi$ is equal to that between a and β.

10. By this time the chord I x L begins to return, and the diſtance between the particles a and β being enlarged to its original magnitude, a has loſt all that force it had acquired by its motion, being now at reſt; and therefore will return with the chord, making the diſtance between a and β greater than the natural; for β will not return ſo ſoon, becauſe its motion forward is not yet quite ſuppreſſed, the diſtance $\beta \gamma$ not being already enlarged to its prime dimenſion: but the receſs of a, by diminiſhing the preſſure upon β by its elaſticity, will occaſion the motion of β to be ſtopt in a little time by the action of γ, and then ſhall β begin to return: at which time the diſtance between γ and δ ſhall by the ſuperior action of δ above β be enlarged to the dimenſion of the diſtance $\beta \gamma$, and therefore ſoon after to that of $a \beta$. Thus it appears, that each of theſe particles goes on to move forward, till its diſtance from

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the preceding one be equal to its original diſtance ;   the whole chain a, β, γ, δ, ϵ, ζ, η, having an undulating motion forward, which is ſtopt gradually by the exceſs of the expanſive power of the preceding parts above that of the hinder.   Thus are theſe parts ſucceſſively ſtopt, as before they were moved ; ſo that when the chord has regained its rectilinear ſituation, the expanſion of the parts of the air will have advanced ſo far, that the interval between ζ η, which at preſent is moſt contracted, will then be reſtored to its natural ſize: the diſtances between η and θ, θ and λ, λ and μ, μ and ν, ν and ξ, being ſucceſſively contracted into the preſent diſtance of ζ from η, and again enlarged ; ſo that the ſame effect ſhall be produced upon the parts beyond ζ η, by the enlargement of the diſtance between thoſe two particles, as was occaſioned upon the particles a, β, γ, δ, ϵ, ζ, η, θ, λ, μ, ν, ξ, by the enlargement of the diſtance a β to its natural extent.   And therefore the motion in the air will be extended half as much farther as at preſent, and the diſtance between ν and ξ contracted into that, which is at preſent between ζ and η,   all the particles of the air in motion taking the rangement expreſſed in figure III. by the points a, β, γ, δ, ϵ, ζ, η, θ, λ, μ, ν, ξ, π, ρ, σ, τ, φ ; wherein the particles from a to ξ have their diſtances from each other gradually diminiſhed, the diſtances between the particles ν, ξ being contracted the moſt from the natural diſtance between thoſe particles, and the diſtance between a, β as much augmented, and the diſtance between the middle particles ζ, η becoming equal to the natural. The particles π, ρ, σ, τ, φ, which follow ξ, have their diſtances gradually greater

and

and greater, the particles $v, \xi, \pi, \rho, \sigma, \tau, \varphi$ being ranged like the particles a, b, c, d, e, f, g, or like the particles $\zeta, \eta, \theta, \lambda, \mu, v, \xi$ in the former figure. Here it will be underſtood, by what has been before explained, that the particles ζ, η being at their natural diſtance from each other, the particle ζ is at reſt, the particles $\epsilon, \delta, \gamma, \beta, \alpha$ between them and the ſtring being in motion backward, and the reſt of the particles $\eta, \theta, \lambda, \mu, v, \xi, \pi, \rho, \sigma, \tau$ in motion forward : each of the particles between η and ξ moving faſter than that, which immediately follows it ; but of the particles from ξ to φ, on the contrary, thoſe behind moving on faſter than thoſe, which precede.

11. But now the ſtring having recovered its rectilinear figure, though it ſhall go on recoiling, till it return near to its firſt ſituation I K L, yet there will be a change in its motion; ſo that whereas it returned from the ſituation I $\varkappa$ L with an accelerated motion, its motion ſhall from hence be retarded again by the ſame degrees, as accelerated before. The effect of which change upon the particles of the air will be this. As by the accelerated motion of the chord α contiguous to it moved faſter than β, ſo as to make the interval $\alpha \beta$ greater than the interval $\beta \gamma$, and from thence β was made likewiſe to move faſter than γ, and the diſtance between β and γ rendered greater than the diſtance between γ and δ, and ſo of the reſt ; now the motion of α being diminiſhed, β ſhall overtake it, and the diſtance between α and β be reduced into that, which is at preſent between β and γ, the interval between β and γ being inlarged into the preſent

fent diftance between α and β; but when the interval $\beta\gamma$ is increafed to that, which is at prefent between α and β, the diftance between γ and δ fhall be enlarged to the prefent diftance between γ and β, and the diftance between δ and ϵ inlarged into the prefent diftance between γ and δ; and the fame of the reft.   But the chord more and more. flackening its pace, the diftance between α and β fhall be more and more diminifhed; and in confequence of that the. diftance between β and γ fhall be again contracted, firft into its prefent dimenfion, and afterwards into a narrower fpace; while the interval $\gamma\delta$ fhall dilate into that at prefent. between α and β, and as foon as it is fo much enlarged, it fhall. contract again.   Thus by the reciprocal expanfion and contraction of the air between α and ζ, by that time the chord is got into. the fituation I K L, the interval $\zeta\eta$ fhall be expanded into the prefent diftance between α and β; and by that time likewife the prefent diftance of α from β will be contracted into their natural interval: for this diftance will be about the fame time in contracting it felf, as has been. taken up in its dilatation; feeing the ftring will be as long in returning from its rectilinear figure, as it has been in recovering it from its fituation I κ L.   This is the change which will be made in the particles between α and ζ.   As for thofe between ζ and ξ, becaufe each preceding particle advances fafter than that, which immediately follows it, their diftances will fucceffively be dilated into that, which is at prefent between ζ and η.   And as foon as any two particles are arrived at their natural diftance, the hindermoft of them fhall be ftopt, and immediately after return;,

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the diſtances between the returning particles being greater than the natural. And this dilatation of theſe diſtances ſhall extend ſo far, by that time the chord is returned into its firſt ſituation I K L, that the particles $\nu \xi$ ſhall be removed to their natural diſtance. But the dilatation of $\nu \xi$ ſhall contract the interval $\tau \varphi$ into that at preſent between ν and ξ, and the contraction of the diſtance between thoſe two particles τ and φ will agitate a part of the air beyond; ſo that when the chord is returned into the ſituation I K L, having made an intire vibration, the moved particles of the air will take the rangement expreſſed by the points, $l, m, n, o, p, q, r, s,$ $t, u, w, x, y, z, 1, 2, 3, 4, 5, 6, 7, 8$: in which $l m$, are at the natural diſtance of the particles, the diſtance $m n$ greater than $l m$, and $n o$ greater than $m n$, and ſo on, till you come to $q r$, the wideſt of all: and then the diſtances gradually diminiſh not only to the natural diſtance, as $w x$, but till they are contracted as much as $\xi \tau$ was before; which falls out in the points $2, 3$, from whence the diſtances augment again, till you come to the part of the air untouched.

12. This is the motion, into which the air is put, while the chord makes one vibration, and the whole length of air thus agitated in the time of one vibration of the chord our author calls the length of one pulſe. When the chord goes on to make another vibration, it will not only continue to agitate the air at preſent in motion, but ſpread the pulſation of the air as much farther, and by the ſame degrees, as before. For when the chord returns into its rectilinear ſituation I k L, $l m$ ſhall be brought into its moſt contracted

<div align="right">ſtate</div>

state, qr now in the state of greatest dilatation shall be reduced to its natural distance, the points w, x now at their natural distance shall be at their greatest distance, the points 2, 3 now most contracted enlarged to their natural distance, and the points 7, 8 reduced to their most contracted state: and the contraction of them will carry the agitation of the air as far beyond them, as that motion was carried from the chord, when it first moved out of the situation I K L into its rectilinear figure.   When the chord is got into the situation I x L, lm shall recover its natural dimensions, qr be reduced to its state of greatest contraction, wx brought to its natural dimension, the distance 2 3 enlarged to the utmost, and the points 7, 8 shall have recovered their natural distance; and by thus recovering themselves they shall agitate the air to as great a length beyond them, as it was moved beyond the chord, when it first came into the situation I x L.   When the chord is returned back again into its rectilinear situation, lm shall be in its utmost dilatation, qr restored again to its natural distance, wx reduced into its state of greatest contraction,   2 3 shall recover its natural dimension, and 7 8 be in its state of greatest dilatation. By which means the air shall be moved as far beyond the points 7, 8, as it was moved beyond the chord, when it before made its return back to its rectilinear situation ; for the particles 7, 8 have been changed from their state of rest and their natural distance into a state of contraction, and then have proceeded to the recovery of their natural distance, and after that to a dilatation of it, in the same manner as the particles contiguous to the chord were agitated before.   In

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the laſt place, when the chord is returned into the ſituation
I K L, the particles of air from *l* to *a* ſhall acquire their pre-
ſent rangement, and the motion of the air be extended as
much farther.   And the like will happen after every com-
pleat vibration of the ſtring.

13. Concerning this motion of ſound, our author
ſhews how to compute the velocity thereof, or in what time
it will reach to any propoſed diſtance from the ſonorous
body.   For this he requires to know the height of air, hav-
ing the ſame denſity with the parts here at the ſurface of
the earth, which we breath, that would be equivalent in
weight to the whole incumbent atmoſphere.   This is to
be found by the barometer, or common weatherglaſs.   In
that inſtrument quickſilver is included in a hollow glaſs
cane firmly cloſed at the top.   The bottom is open, but
immerged into quickſilver contained in a veſſel open to the
air. Care is taken when the lower end of the cane is immer-
ged, that the whole cane be full of quickſilver, and that no air
inſinuate itſelf. When the inſtrument is thus fixed, the quick-
ſilver in the cane being higher than that in the veſſel, if
the top of the cane were open, the fluid would ſoon ſink
out of the glaſs cane, till it came to a level with that in
the veſſel.   But the top of the cane being cloſed up, ſo
that the air, which has free liberty to preſs on the quick-
ſilver in the veſſel, cannot bear at all on that, which is with-
in the cane, the quickſilver in the cane will be ſuſpended
to ſuch a height, as to balance the preſſure of the air on
the quickſilver in the veſſel.   Here it is evident, that the

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weight of the quickfilver in the glafs cane is equivalent to the preffure of fo much of the air, as is perpendicularly over the hollow of the cane; for if the cane be opened that the air may enter, there will be no farther ufe of the quickfilver to fuftain the preffure of the air without; for the quickfilver in the cane, as has already been obferved, will then fubfide to a level with that without. Hence therefore if the proportion between the denfity of quickfilver and of the air we breath be known, we may know what height of fuch air would form a column equal in weight to the column of quickfilver within the glafs cane. When the quickfilver is fuftained in the barometer at the height of 30 inches, the height of fuch a column of air will be about 29725 feet; for in this cafe the air has about $\frac{1}{870}$ of the denfity of water, and the denfity of quickfilver exceeds that of water about $13\frac{2}{3}$ times, fo that the denfity of quickfilver exceeds that of the air about 11890 times; and fo many times 30 inches make 29725 feet. Now Sir Isaac Newton determines, that while a pendulum of the length of this column fhould make one vibration or fwing, the fpace, which any found will have moved, fhall bear to this length the fame proportion, as the circumference of a circle bears to the diameter thereof ; that is, about the proportion of 355 to 113 [a]. Only our author here confiders fingly the gradual progrefs of found in the air from particle to particle in the manner we have explained, without taking into confideration the magnitude of thofe particles. And though there requires time for the motion to be propagated from one par-

[a] Princ. philof. Lib. II. prop. 49.

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ticle

ticle to another, yet it is communicated to the whole of the fame particle in an inftant: therefore whatever proportion the thicknefs of thefe particles bears to their diftance from each other, in the fame proportion will the motion of found be fwifter. Again the air we breath is not fimply compofed of the elaftic part, by which found is conveyed, but partly of vapours, which are of a different nature; and in the computation of the motion of found we ought to find the height of a column of this pure air only, whofe weight fhould be equal to the weight of the quickfilver in the cane of the barometer, and this pure air being a part only of that we breath, the column of this pure air will be higher than 29725 feet. On both thefe accounts the motion of found is found to be about 1142 feet in one fecond of time, or near 13 miles in a minute, whereas by the computation propofed above, it fhould move but 979 feet in one fecond.

14. We may obferve here, that from thefe demonftrations of our author it follows, that all founds whether acute or grave move equally fwift, and that found is fwifteft, when the quickfilver ftands higheft in the barometer.

15. Thus much of the appearances, which are caufed in thefe fluids from their gravitation toward the earth.. They alfo gravitate toward the moon; for in the laft chapter it has been proved, that the gravitation between the earth and moon is mutual, and that this gravitation of the whole bodies arifes from that power acting in all their parts; fo that

every

every particle of the moon gravitates toward the earth, and every particle of the earth toward the moon. But this gravitation of thefe fluids toward the moon produces no fenfible effect, except only in the fea, where it caufes the tides.

16. That the tides depend upon the influence of the moon has been the receiv'd opinion of all antiquity; nor is there indeed the leaft fhadow of reafon to fuppofe otherwife, confidering how fteadily they accompany the moon's courfe. Though how the moon caufed them, and by what principle it was enabled to produce fo diftinguifh'd an appearance, was a fecret left for this philofophy to unfold: which teaches, that the moon is not here alone concerned, but that the fun likewife has a confiderable fhare in their production; though they have been generally afcribed to the other luminary, becaufe its effect is greateft, and by that means the tides more immediately fuit themfelves to its motion; the fun difcovering its influence more by enlarging or reftraining the moon's power, than by any diftinct effects. Our author finds the power of the moon to bear to the power of the fun about the proportion of $4\frac{1}{2}$ to 1. This he deduces from the obfervations made at the mouth of the river Avon, three miles from Briftol, by Captain Sturmey, and at Plymouth by Mr. Colepresse, of the height to which the water is raifed in the conjunction and oppofition of the luminaries, compared with the elevation of it, when the moon is in either quarter; the firft being caufed

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by the united actions of the fun and moon, and the o-
ther by the difference of them, as fhall hereafter be fhewn.

17. That the fun fhould have a like effect on the fea,
as the moon, is very manifeft; fince the fun likewife attracts
every fingle particle, of which this earth is compofed. And
in both luminaries fince the power of gravity is reciprocally
in the duplicate proportion of the diftance, they will not
draw all the parts of the waters in the fame manner; but
muft act upon the neareft parts ftronger, than upon the re-
moteft, producing by this inequality an irregular motion.
We fhall now attempt to fhew how the actions of the fun
and moon on the waters, by being combined together, pro-
duce all the appearances obferved in the tides.

18. To begin therefore, the reader will remember what
has been faid above, that if the moon without the fun would
have defcribed an orbit concentrical to the earth, the action
of the fun would make the orbit oval, and bring the moon
nearer to the earth at the new and full, than at the quarters[a].
Now our excellent author obferves, that if inftead of one moon,
we fuppofe a ring of moons, contiguous and occupying the
whole orbit of the moon, his demonftration would ftill take
place, and prove that the parts of this ring in paffing from the
quarter to the conjunction or oppofition would be accelerated,
and be retarded again in paffing from the conjunction or op-
pofition to the next quarter. And as this effect does not de-

[a] Chap. 3. § 18.

pend

pend on the magnitude of the bodies, whereof the ring is compofed, the fame would hold, though the magnitude of thefe moons were fo far to be diminifhed, and their number increafed, till they fhould form a fluid [a]. Now the earth turns round continually upon its own center, caufing thereby the alternate change of day and night, while by this revolution each part of the earth is fucceffively brought toward the fun, and carried off again in the fpace of 24 hours. And as the fea revolves round along with the earth itfelf in this diurnal motion, it will reprefent in fome fort fuch a fluid ring.

19. But as the water of the fea does not move round with fo much fwiftnefs, as would carry it about the center of the earth in the circle it now defcribes, without being fupported by the body of the earth; it will be neceffary to confider the water under three diftinct cafes. The firft cafe fhall fuppofe the water to move with the degree of fwiftnefs, required to carry a body round the center of the earth difingaged from it in a circle at the diftance of the earth's femidiameter, like another moon. The fecond cafe is, that the waters make but one turn about the axis of the earth in the fpace of a month, keeping pace with the moon; fo that all parts of the water fhould preferve continually the fame fituation in refpect of the moon. The third cafe fhall be the real one of the waters moving with a velocity between thefe two, neither fo fwift as the firft cafe requires, nor fo flow as the fecond.

[a] Newt. Princ. philof. Lib. I. prop. 66. coroll. 18.

20. In

20. In the firſt caſe the waters, like the body which they equalled in velocity, by the action of the moon would be brought nearer the center under and oppoſite to the moon, than in the parts in the middle between theſe eaſtward or weſtward. That ſuch a body would ſo alter its diſtance by the moon's action upon it, is clear from what has been mentioned of the like changes in the moon's motion cauſed by the ſun [a]. And computation ſhews, that the difference between the greateſt and leaſt diſtance of ſuch a body would not be much above $4\frac{1}{2}$ feet. But in the ſecond caſe, where all the parts of the water preſerve the ſame ſituation continually in reſpect of the moon, the weight of thoſe parts under and oppoſite to the moon will be diminiſhed by the moon's action, and the parts in the middle between theſe will have their weight increaſed: this being effected juſt in the ſame manner, as the ſun diminiſhes the attraction of the moon towards the earth in the conjunction and oppoſition, but increaſes that attraction in the quarters. For as the firſt of theſe conſequences from the ſun's action on the moon is occaſioned by the moon's being attracted by the ſun in the conjunction more than the earth, and in the oppoſition leſs than it, and therefore in the common motion of the earth and moon, the moon is made to advance toward the ſun in one caſe too faſt, and in the other is left as it were behind; ſo the earth will not have its middle parts drawn towards the moon ſo ſtrongly as the nearer parts, and yet more forcibly than the remoteſt: and therefore ſince the earth and moon move each

[a] § 8.

month

month round their common center of gravity [a], while the earth moves round this center, the fame effect will be produced, on the parts of the water neareft to that center or to the moon, as the moon feels from the fun when in conjunction, and the water on the contrary fide of the earth will be affected by the moon, as the moon is by the fun, when in oppofition [b]; that is, in both cafes the weight of the water, or its propenfity towards the center of the earth, will be diminifhed. The parts in the middle between thefe will have their weight increafed, by being preffed towards the center of the earth through the obliquity of the moon's action upon them to its action upon the earth's center, juft as the fun increafes the gravitation of the moon in the quarters from the fame caufe [c]. But now it is manifeft, that where the weight of the fame quantity of water is leaft, there it will be accumulated; while the parts, which have the greateft weight, will fubfide. Therefore in this cafe there would be no tide or alternate rifing and falling of the water, but the water would form it felf into an oblong figure, whofe axis prolonged would pafs through the moon. By Sir Isaac Newton's computation the excefs of this axis above the diameters perpendicular to it, that is, the height of the waters under and oppofite to the moon above their height in the middle between thefe places eaftward or weftward caufed by the moon, is about $8\frac{2}{3}$ feet.

[a] Ch. 3. § 5.   [b] Ch. 3 § 17.   [c] Ibid

21. Thus

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21. THUS the difference of height in this latter supposition is little short of twice that difference in the preceding. But the case of the sea is a middle between these two: for a body, which should revolve round the center of the earth at the distance of a semidiameter without pressing on the earth's surface, must perform its period in less than an hour and half, whereas the earth turns round but once in a day; and in the case of the waters keeping pace with the moon it should turn round but once in a month: so that the real motion of the water is between the motions required in these two cases. Again, if the waters moved round as swiftly as the first case required, their weight would be wholly taken off by their motion; for this case supposes the body to move so, as to be kept revolving in a circle round the earth by the power of gravity without pressing on the earth at all, so that its motion just supports its weight. But if the power of gravity had been only $\frac{1}{289}$ part of what it is, the body could have moved thus without pressing on the earth, and have been as long in moving round, as the earth it self is. Consequently the motion of the earth takes off from the weight of the water in the middle between the poles, where its motion is swiftest, $\frac{1}{289}$ part of its weight and no more. Since therefore in the first case the weight of the waters must be intirely taken off by their motion, and by the real motion of the earth they lose only $\frac{1}{289}$ part thereof, the motion of the water will so little diminish their weight, that their figure will much nearer resemble the case of their keeping pace with the moon than the other. Upon the whole, if the waters moved with the

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velocity neceſſary to carry a body round the center of the earth at the diſtance of the earth's ſemidiameter without bearing on its ſurface, the water would be loweſt under the moon, and riſe gradually as it moved on with the earth eaſtward, till it came half way toward the place oppoſite to the moon; from thence it would ſubſide again, till it came to the oppoſition, where it would become as low as at firſt; afterwards it would riſe again, till it came half way to the place under the moon; and from hence it would ſubſide, till it came a ſecond time under the moon. But in caſe the water kept pace with the moon, it would be higheſt where in the other caſe it is loweſt, and loweſt where in the other it is higheſt; therefore the diurnal motion of the earth being between the motions of theſe two caſes, it will cauſe the higheſt place of the water to fall between the places of the greateſt height in theſe two caſes. The water as it paſſes from under the moon ſhall for ſome time riſe, but deſcend again before it arrives half way to the oppoſite place, and ſhall come to its leaſt height before it becomes oppoſite to the moon; then it ſhall riſe again, continuing ſo to do till it has paſſed the place oppoſite to the moon, but ſubſide before it comes to the middle between the places oppoſite to and under the moon; and laſtly it ſhall come to its loweſt, before it comes a ſecond time under the moon. If A (in fig. 112, 113, 114.) repreſent the moon, B the center of the earth, the oval C D E F in fig. 112. will repreſent the ſituation of the water in the firſt caſe; but if the water kept pace with the moon, the line C D E F in fig. 113. would repreſent the ſitua-

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tion of the water; but the line CDEF in fig. 114. will reprefent the fame in the real motion of the water, as it accompanies the earth in its diurnal rotation: in all thefe figures C and E being the places where the water is loweft, and D and F the places where it is higheft. Purfuant to this determination it is found, that on the fhores, which lie expofed to the open fea, the high water ufually falls out about three hours after the moon has paffed the meridian of each place.

22. Let this fuffice in general for explaining the manner, in which the moon acts upon the feas. It is farther to be noted, that thefe effects are greateft, when the moon is over the earth's equator [a], that is, when it fhines perpendicularly upon the parts of the earth in the middle between the poles. For if the moon were placed over either of the poles, it could have no effect upon the water to make it afcend and defcend. So that when the moon declines from the equator toward either pole, it's action muft be fomething diminifhed, and that the more, the farther it declines. The tides likewife will be greateft, when the moon is neareft to the earth, it's action being then the ftrongeft.

23. Thus much of the action of the moon. That the fun fhould produce the very fame effects, though in a lefs degree, is too obvious to require a particular explanation: but as was remarked before, this action of the

[a] See below § 44.

fun

sun being weaker than that of the moon, will cause the
tides to follow more nearly the moon's course, and princi-
pally shew it self by heightening or diminishing the ef-
fects of the other luminary.   Which is the occasion, that
the highest tides are found about the conjunction and oppo-
sition of the luminaries, being then produced by their uni-
ted action, and the weakest tides about the quarters of
the moon ; because the moon in this case raising the water
where the sun depresses it, and depressing it where the
sun raises it, the stronger action of the moon is in part
retunded and weakened by that of the sun.   Our author
computes that the sun will add near two feet to the height
of the water in the first case, and in the other take from
it as much.   However the tides in both comply with the
same hour of the moon.   But at other times, between
the conjunction or opposition and quarters, the time de-
viates from that forementioned,   towards the hour in which
the sun would make high water,   though still it keeps much
nearer to the moon's hour than to the sun's.

24. Again the tides have some farther varieties from
the situation of the places where they happen northward
or southward.   Let p P (in fig. 115.) represent the axis, on
which the earth daily revolves,   let h p H P represent the
figure of the water, and let n B N D be a globe inscri-
bed within this figure.   Suppose the moon to be advanced
from the equator toward the north pole, so that h H the
axis of the figure of the water p A H P E h shall decline
towards the north pole N ; take any place G nearer to
the

the north pole than to the fouth, and from the center of the earth C draw C G F; then will G F denote the altitude to which the water is raifed by the tide, when the moon is above the horizon: in the fpace of twelve hours, the earth having turned half round its axis, the place G will be removed to g; but the axis h H will have kept its place preferving its fituation in refpect of the moon, at leaft will have moved no more than the moon has done in that time, which it is not neceffary here to take into confideration. Now in this cafe the height of the water will be equal to $g\,f$, which is not fo great as G F. But whereas G F is the altitude at high water, when the moon is above the horizon, $g\,f$ will be the altitude of the fame, when the moon is under the horizon. The contrary happens toward the fouth pole, for K L is lefs than $k\,l$. Hence is proved, that when the moon declines from the equator, in thofe places, which are on the fame fide of the equator as the moon, the tides are greater, when the moon is above the horizon, than when under it; and the contrary happens on the other fide of the equator.

25. Now from thefe principles may be explained all the known appearances in the tides; only by the affiftance of this additional remark, that the fluctuating motion, which the water has in flowing and ebbing, is of a durable nature, and would continue for fome time, though the action of the luminaries fhould ceafe; for this prevents the difference between the tide when the moon is above

the

the horizon, and the tide when the moon is below it from being fo great, as the rule laid down requires. This likewife makes the greateft tides not exactly upon the new and full moon, but to be a tide or two after ; as at Briftol and Plymouth they are found the third after.

26. THIS doctrine farther fhews us, why not only the fpring tides fall out about the new and full moon, and the neap tides about the quarters ; but likewife how it comes to pafs, that the greateft fpring tides happen about the equinoxes ; becaufe the luminaries are then one of them over the equator, and the other not far from it. It appears too, why the neap tides, which accompany thefe, are the leaft of all ; for the fun ftill continuing over the equator continues to have the greateft power of leffening the moon's action, and the moon in the quarters being far removed toward one of the poles, has its power thereby weakned.

27. MOREOVER the action of the moon being ftronger, when near the earth, than when more remote ; if the moon, when new fuppofe, be at its neareft diftance from the earth, it fhall when at the full be fartheft off ; whence it is, that two of the very largeft fpring tides do never immediately fucceed each other.

28. BECAUSE the fun in its paffage from the winter folftice to the fummer recedes from the earth, and paffing from the fummer folftice to the winter approaches it, and is therefore nearer the earth before the vernal equinox than after,

after, but nearer after the autumnal equinox than before; the greateſt tides oftner precede the vernal equinox than follow it, and in the autumnal equinox on the contrary they oftner follow it than come before it.

29. The altitude, to which the water is raiſed in the open ocean, correſponds very well to the forementioned calculations; for as it was ſhewn, that the water in ſpring tides ſhould riſe to the height of 10 or 11 feet, and the neap tides to 6 or 7; accordingly in the Pacific, Atlantic and Ethiopic oceans in the parts without the tropics, the water is obſerved to riſe about 6, 9, 12 or 15 feet. In the Pacific ocean this elevation is ſaid to be greater than in the other, as it ought to be by reaſon of the wide extent of that ſea. For the ſame reaſon in the Ethiopic ocean between the tropics the aſcent of the water is leſs than without, by reaſon of the narrowneſs of the ſea between the coaſts of Africa and the ſouthern parts of America. And iſlands in ſuch narrow ſeas, if far from ſhore, have leſs tides than the coaſts. But now in thoſe ports where the water flows in with great violence upon fords and ſhoals, the force it acquires by that means will carry it to a much greater height, ſo as to make it aſcend and deſcend to 30, 40 or even 50 feet and more; inſtances of which we have at Plymouth, and in the Severn near Chepſtow; at St. Michael's and Auranches in Normandy; at Cambay and Pegu in the Eaſt Indies.

30. Again the tides take a conſiderable time in paſſing through long ſtraits, and ſhallow places. Thus the tide,

3   which

which is made on the weſt coaſt of Ireland and on the coaſt of Spain at the third hour after the moon's coming to the meridian, in the ports eaſtward toward the Britiſh channel falls out later, and as the flood paſſes up that channel ſtill later and later, ſo that the tide takes up full twelve hours in coming up to London bridge.

31. In the laſt place tides may come to the ſame port from different ſeas, and as they may interfere with each other, they will produce particular effects.   Suppoſe the tide from one ſea come to a port at the third hour after the moon's paſſing the meridian of the place, but from another ſea to take up ſix hours more in its paſſage.   Here one tide will make high water, when by the other it ſhould be loweſt; ſo that when the moon is over the equator, and the two tides are equal, there will be no riſing and falling of the water at all; for as much as the water is carried off by one tide, it will be ſupplied by the other.   But when the moon declines from the equator, the ſame way as the port is ſituated, we have ſhewn that of the two tides of the ocean, which are made each day, that tide, which is made when the moon is above the horizon, is greater than the other.   Therefore in this caſe, as four tides come to this port each day the two greateſt will come on the third, and on the ninth hour after the moon's paſſing the meridian, and the two leaſt at the fifteenth and at the twenty firſt hour.   Thus from the third to the ninth hour more water will be in this port by the two greateſt tides than from the ninth to the fifteenth, or from the twenty firſt to the

following

following third hour, where the water is brought by one great and one small tide ; but yet there will be more water brought by thefe tides, than what will be found between the two leaft tides, that is, between the fifteenth and twenty firft hour. Therefore in the middle between the third and ninth hour, or about the moon's fetting, the water will be at its greateft height; in the middle between the ninth and fifteenth, as alfo between the twenty firft and following third hour it will have its mean height; and be loweft in the middle between the fifteenth and twenty firft hour, that is, at the moon's rifing. Thus here the water will have but one flood and one ebb each day. When the moon is on the other fide of the equator, the flood will be turned into ebb, and the ebb into flood ; the high water falling out at the rifing of the moon, and the low water at the fetting. Now this is the cafe of the port of Batfham in the kingdom of Tunquin in the Eaft Indies; to which port there are two inlets, one between the continent and the iflands which are called the Manillas, and the other between the continent and Borneo.

32. The next thing to be confidered is the effect, which thefe fluids of the planets have upon the folid part of the bodies to which they belong. And in the firft place I fhall fhew, that it was neceffary upon account of thefe fluid parts to form the bodies of the planets into a figure fomething different from that of a perfect globe. Becaufe the diurnal rotation, which our earth performs about its axis, and the like motion we fee in fome of the other planets, which

(which is an ample conviction that they all do the like) will diminish the force, with which bodies are attracted upon all the parts of their surfaces, except at the very poles, upon which they turn.   Thus a stone or other weighty substance resting upon the surface of the earth, by the force which it receives from the motion communicated to it by the earth, if its weight prevented not, would continue that motion in a straight line from the point where it received it, and according to the direction, in which it was given, that is, in a line which touches the surface at that point; insomuch that it would move off from the earth in the same manner, as a weight fasten'd to a string and whirled about endeavours continually to recede from the center of motion, and would forthwith remove it self to a greater distance from it, if loosed from the string which retains it.   And farther, as the centrifugal force, with which such a weight presses from the center of its motion, is greater, by how much greater the velocity is, with which it moves; so such a body, as I have been supposing to lie on the earth, would recede from it with the greater force, the greater the velocity is, with which the part of the earth's surface it rests upon is moved, that is, the farther distant it is from the poles.   But now the power of gravity is great enough to prevent bodies in any part of the earth from being carried off from it by this means;   however it is plain that bodies having an effort contrary to that of gravity, though much weaker than it, their weight, that is, the degree of force, with which they are pressed to the earth, will be diminished thereby, and be the more diminished,

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the greater this contrary effort is ; or in other words, the
fame body will weigh heavier at either of the poles, than
upon any other part of the earth ; and if any body be
removed from the pole towards the equator, it will lofe
of its weight more and more ,   and be lighteft of all at
the equator, that is, in the middle between the poles.

33. T h i s now is eafily applied to the waters of the feas,
and fhews that the water under the poles will prefs more forci-
bly to the earth, than at or near the equator : and confequent-
ly that which preffes leaft, muft give place, till by afcend-
ing it makes room for receiving a greater quantity, which by
its additional weight may place the whole upon a ballance.
To illuftrate this more particularly I fhall make ufe of fig. 116
In which let A C B D be a circle, by whofe revolution about
the diameter A B a globe fhould be formed, reprefenting a
globe of folid earth.   Suppofe this globe covered on all fides
with water to the fame height, fuppofe that of E A or B F,
at which diftance the circle E G F H furrounds the circle
A C B D ; then it is evident, if the globe of earth be at reft,
the water which furrounds it will reft in that fituation.
But if the globe be turned inceffantly about its axis A B,
and the water have likewife the fame motion, it is alfo
evident, from what has been explained, that the water be-
tween the circles E H F G and A D B C will remain no longer
in the prefent fituation, the parts of it between H and D, and
between G and C being by this rotation become lighter, than
the parts between E and A and between B and F ;   fo that the
water over the poles A and B muft of neceffity fubfide, and the

water

water be accumulated over D and C, till the greater quantity in thefe latter places fupply the defect of its weight. This would be the cafe, were the globe all covered with water. And the fame figure of the furface would alfo be preferved, if fome part of the water adjoining to the globe in any part of it were turned into folid earth, as is too evident to need any proof; becaufe the parts of the water remaining at reft, it is the fame thing, whether they continue in the ftate of being eafily feparable, which denominates them fluid, or were to be confolidated together, fo as to make a hard body: and this, though the water fhould in fome places be thus confolidated, even to the furface of it. Which fhews that the form of the folid part of the earth makes no alteration in the figure the water will take: and by confequence in order to the preventing fome parts of the earth from being entirely overflowed, and other parts quite deferted, the folid parts of the earth muft have given them much the fame figure, as if the whole earth were covered on all fides with water.

34. FARTHER, I fay, this figure of the earth is the fame, as it would receive, were it entirely a globe of water, provided that water were of the fame denfity as the fubftance of the globe. For fuppofe the globe ACBD to be liquified, and that the globe EHFG, now entirely water, by its rotation about its axis fhould receive fuch a figure as we have been defcribing, and then the globe ACBD fhould be confolidated again, the figure of the water would plainly not be altered, by fuch a confolidation.

35. BUT

35. But from this laſt obſervation our author is enabled to determine the proportion between the axis of the earth drawn from pole to pole, and the diameter of the equator, upon the ſuppoſition that all the parts of the earth are of equal denſity; which he does by computing in the firſt place the proportion of the centrifugal force of the parts under the equator to the power of gravity; and then by conſidering the earth as a ſpheroid, made by the revolution of an ellipſis about its leſſer axis, that is, ſuppoſing the line MILK to be an exact ellipſis, from which it can differ but little, by reaſon that the difference between the leſſer axis M L and the greater I K is but very ſmall. From this ſuppoſition, and what was proved before, that all the particles which compoſe the earth have the attracting power explained in the preceding chapter, he finds at what diſtance the parts under the equator ought to be removed from the center, that the force, with which they ſhall be attracted to the center, diminiſhed by their centrifugal force, ſhall be ſufficient to keep thoſe parts in a ballance with thoſe which lie under the poles. And upon the ſuppoſition of all the parts of the earth having the ſame degree of denſity, the earth's ſurface at the equator muſt be above 17 miles more diſtant from the center, than at the poles [a].

36. After this it is ſhewn, from the proportion of the equatorial diameter of the earth to its axis, how the ſame may be determined of any other planet, whoſe denſity in

[a] Newton Princ. Lib. III prop. 19.

compariſon

comparifon of the denfity of the earth, and the time of its revolution about its axis, are known.   And by the rule delivered for this, it is found, that the diameter of the equator in Jupiter fhould bear to its axis about the proportion of 10 to 9 [a], and accordingly this planet appears of an oval form to the aftronomers.   The moft confiderable effects of this fpheroidical figure our author takes likewife into confideration; one of which is that bodies are not equally heavy in all diftances from the poles; but near the equator, where the diftance from the center is greateft, they are lighter than towards the poles: and nearly in this proportion, that the actual power, by which they are drawn to the center, refulting from the difference between their abfolute gravity and centrifugal force, is reciprocally as the diftance from the center.   That this may not appear to contradict what has before been faid of the alteration of the power of gravity, in proportion to the change of the diftance from the center, it is proper carefully to remark, that our author has demonftrated three things relating hereto: the firft is, that decreafe of the power of gravity as we recede from the center, which has been fully explained in the laft chapter, upon fuppofition that the earth and planets are perfect fpheres, from which their difference is by many degrees too little to require notice for the purpofes there intended: the next is, that whether they be perfect fpheres, or exactly such fpheroids as have now been mentioned, the power of gravity, as we defcend in the fame line to the center, is at all diftances as the diftance from the center, the parts of the

[a] Lib. 1II prop. 19.

earth above the body by drawing the body towards them lessening its gravitation towards the center [a]; and both these assertions relate to gravity alone: the third is what we mentioned in this place, that the actual force on different parts of the surface, with which bodies are drawn to the center, is in the proportion here assigned [b].

38. The next effect of this figure of the earth is an obvious consequence of the former: that pendulums of the same length do not in different distances from the pole make their vibrations in the same time; but towards the poles, where the gravity is strongest, they move quicker than near the equator, where they are less impelled to the center; and accordingly pendulums, that measure the same time by their vibrations, must be shorter near the poles than at a greater distance. Both which deductions are found true in fact; of which our author has recounted particularly several experiments, in which it was found, that clocks exactly adjusted to the true measure of time at Paris, when transported nearer to the equator, became erroneous and moved too slow, but were reduced to their true motion by contracting their pendulums. Our author is particular in remarking, how much they lost of their motion, while the pendulums remained unaltered; and what length the observers are said to have shortened them, to bring them to time. And the experiments, which appear to be most carefully made, shew the earth to be raised in the middle between the poles, as much as our author found it by his computation [c].

[a] Lib. I. prop. 73.　　[b] Lib, III. prop. 20.　　[c] Ibid.

39. These

39. THESE experiments on the pendulum our author has been very exact in examining, inquiring particularly how much the extenſion of the rod of the pendulum by the great heats in the torrid zone might make it neceſſary to ſhorten it.   For by an experiment made by PICART, and another made by DE LA HIRE, heat, though not very intenſe, was found to increaſe the length of rods of iron.   The experiment of PICART was made with a rod one foot long, which in winter, at the time of froſt, was found to increaſe in length by being heated at the fire.   In the experiment of DE LA HIRE a rod of ſix foot in length was found, when heated by the ſummer ſun only, to grow to a greater length, than it had in the aforeſaid cold ſeaſon.   From which obſervations a doubt has been raiſed, whether the rod of the pendulums in the aforementioned experiments was not extended by the heat of thoſe warm climates to all that exceſs of length, the obſervers found themſelves obliged to leſſen them by.   But the experiments now mentioned ſhew the contrary.   For in the firſt of them the rod of a foot long was lengthened no more than $\frac{1}{9}$ part of what the pendulum under the equator muſt be diminiſhed; and therefore a rod of the length of the pendulum would not have been extended above $\frac{1}{3}$ of that length.   In the experiment of DE LA HIRE, where the heat was leſs, the rod of ſix foot long was extended no more than $\frac{3}{10}$ of what the pendulum muſt be ſhortened; ſo that a rod of the length of the pendulum would not have gained above $\frac{3}{10}$ or $\frac{1}{7}$ of that length. And the heat in this latter experiment, though leſs than in the former, was yet greater than the rod of a pendulum can or-

dinarily contract in the hotteſt country; for metals receive a great heat when expoſed to the open ſun, certainly much greater than that of a human body. But pendulums are not uſually ſo expoſed, and without doubt in theſe experiments were kept cool enough to appear ſo to the touch; which they would do in the hotteſt place, if lodged in the ſhade. Our author therefore thinks it enough to allow about $\frac{1}{10}$ of the difference obſerved upon account of the greater warmth of the pendulum.

40. There is a third effect, which the water has on the earth by changing its figure, that is taken notice of by our author; for the explaining of which we ſhall firſt prove, that bodies deſcend perpendicularly to the ſurface of the earth in all places. The manner of collecting this from obſervation, is as follows. The ſurfaces of all fluids reſt parallel to that part of the ſurface of the ſea, which is in the ſame place with them, to the figure of which, as has been particularly ſhewn, the figure of the whole earth is formed. For if any hollow veſſel, open at the bottom, be immerſed into the ſea; it is evident, that the ſurface of the ſea within the veſſel will retain the ſame figure it had, before the veſſel incloſed it; ſince its communication with the external water is not cut off by the veſſel. But all the parts of the water being at reſt, it is as clear, that if the bottom of the veſſel were cloſed, the figure of the water could receive no change thereby, even though the veſſel were raiſed out of the ſea; any more than from the inſenſible alteration of the power of gravity, conſequent upon the augmentation of the

the diftance from the center. But now it is clear, that bodies defcend in lines perpendicular to the furfaces of quiefcent fluids; for if the power of gravity did not act perpendicularly to the furface of fluids, bodies which fwim on them could not reft, as they are feen to do ; becaufe, if the power of gravity drew fuch bodies in a direction oblique to the furface whereon they lay, they would certainly be put in motion, and be carried to the fide of the veffel, in which the fluid was contained, that way the action of gravity inclined.

41. HENCE it follows, that as we ftand, our bodies are perpendicular to the furface of the earth.   Therefore in going from north to fouth our bodies do not keep in a parallel direction.   Now in all diftances from the pole the fame length gone on the earth will not make the fame change in the pofition of our bodies, but the nearer we are to the poles, we muft go a greater length to caufe the fame variation herein.   Let MILK (in fig. 117) reprefent the figure of the earth, M, L the poles, I, K two oppofite points in the middle between thefe poles.   Let TV and PO be two arches, T V being moft remote from the pole L; draw TW, VX, PQ, OR, each perpendicular to the furface of the earth, and let TW, VX meet in Y, and PQ, OR in S.   Here it is evident, that in paffing from V to T the pofition of a man's body would be changed by the angle under TYV, for at V he would ftand in the line YV continued upward, and at T in the line YT; but in paffing from O to P the pofition of his body would be changed by

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the angle under OSP. Now I fay, if thefe two angles are equal the arch OP is longer than TV: for the figure MILK being oblong, and IK longer than ML, the figure will be more incurvated toward I than toward L; fo that the lines TW and VX will meet in Y before they are drawn out to fo great a length as the lines PQ and OR muft be continued to, before they will meet in S. Since therefore YT and YV are fhorter than PS and SV, TV muft be lefs than OP. If thefe angles under TYV and OSP are each $\frac{1}{90}$ part of the angle made by a perpendicular line, they are faid each to contain one degree. And the unequal length of thefe arches OP and VT gives occafion to the affertion, that in paffing from north to fouth the degrees on the earth's furface are not of an equal length, but thofe near the pole longer than thofe toward the equator. For the length of the arch on the earth lying between the two perpendiculars, which make an angle of a degree with each other, is called the length of a degree on the earth's furface.

42. This figure of the earth has fome effect on eclipfes. It has been obferved above, that fometimes the nodes of the moon's orbit lie in a ftraight line drawn from the fun to the earth; in which cafe the moon will crofs the plane of the earth's motion at the new and full. But whenever the moon paffes near the plane at the full, fome part of the earth will intercept the fun's light, and the moon fhining only with light borrow'd from the fun, when that light is prevented from falling on any part of the moon, fo much of her body will be darkened. Alfo when the moon at the

new

new is near the plane of the earth's motion, the inhabitants
on fome part of the earth will fee the moon come under
the fun, and the fun thereby be covered from them either
wholly or in part.   Now the figure, which we have fhewn
to belong to the earth, will occafion the fhadow of the
earth on the moon not to be perfectly round, but caufe the
diameter from eaft to weft to be fomewhat longer than the
diameter from north to fouth.   In eclipfes of the fun this
figure of the earth will make fome little difference in the
place, where the fun fhall appear wholly or in any given
part covered.   Let A B C D (in fig. 118.) reprefent the earth,
A C the axis whereon it turns daily, E the center.   Let F A G C
reprefent a perfect globe infcribed within the earth.   Let H I
be a line drawn through the centers of the fun and moon, crof-
fing the furface of the earth in K,   and the furface of the
globe infcribed in L. Draw E L, which will be perpendicular
to the furface of the globe in L : and draw likewife K M,
fo that it fhall be perpendicular to the furface of the earth
in K.   Now whereas the eclipfe would appear central at L,
if the earth were the globe A G C F, and does really appear
fo at K; I fay, the latitude of the place K on the real earth
is different from the latitude of the place L on the globe
F A G C. What is called the latitude of any place is determined
by the angle which the line perpendicular to the furface of
the earth at that place makes with the axis; the difference
between this angle,   and that made by a perpendicular line or
fquare being called the latitude of each place. But it might
here be proved, that the angle which K M makes with M C
is lefs, than the angle made between L E and E C: confe-

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quently the latitude of the place K is greater, than the latitude, which the place L would have.

43. The next effect, which follows from this figure of the earth, is that gradual change in the distance of the fixed stars from the equinoctial points, which astronomers observe. But before this can be explained, it is necessary to say something more particular, than has yet been done, concerning the manner of the earth's motion round the sun.

44. It has already been said, that the earth turns round each day on its own axis, while its whole body is carried round the sun once in a year. How these two motions are joined together may be conceived in some degree by the motion of a bowl on the ground, where the bowl in rouling on continually turns upon its axis, and at the same time the whole body thereof is carried straight on. But to be more express let A (in fig. 119) represent the sun BCDE four different situations of the earth in its orbit moving about the sun. In all these let FG represent the axis, about which the earth daily turns. The points F, G are called the poles of the earth; and this axis is supposed to keep always parallel to it self in every situation of the earth; at least that it would do so, were it not for a minute deviation, the cause whereof will be explained in what follows. When the earth is in B, the half HIK will be illuminated by the sun, and the other half HLK will be in darkness. Now if on the globe any point be taken

in

in the middle between the poles, this point fhall defcribe
by the motion of the globe the circle M N, half of which
is in the enlightened part of the globe, and half in the
dark part. But the earth is fuppofed to move round its axis
with an equable motion ; therefore on this point of the
globe the fun will be feen juft half the day, and be invifi-
ble the other half. And the fame will happen to every
point of this circle, in all fituations of the earth during its
whole revolution round the fun. This circle M N is called
the equator, of which we have before made mention.

45. Now fuppofe any other point taken on the furface
of the globe toward the pole F, which in the diurnal re-
volution of the globe fhall defcribe the circle O P. Here
it appears that more than half this circle is enlightned by
the fun, and confequently that in any particular point of
this circle the fun will be longer feen than lie hid, that is
the day will be longer than the night. Again if we con-
fider the fame circle O P on the globe fituated in D the op-
pofite part of the orbit from B, we fhall fee, that here in
any place of this circle the night will be as much longer
than the day.

46. In thefe fituations of the globe of earth a line
drawn from the fun to the center of the earth will be
obliquely inclined toward the axis F G. Now fuppofe, that
fuch a line drawn from the fun to the center of the earth,
when in C or E, would be perpendicular to the axis F G ;

in

in which cafes the fun will fhine perpendicularly upon the
equator, and confequently the line drawn from the center
of the earth to the fun will crofs the equator, as it paffes
through the furface of the earth ; whereas in all other fitu-
ations of the globe this line will pafs through the furface
of the globe at a diftance from the equator either north-
ward or fouthward. Now in both thefe cafes half the cir-
cle O P will be in the light, and half in the dark ; and there-
fore to every place in this circle the day will be equal
to the night. Thus it appears, that in thefe two oppofite
fituations of the earth the day is equal to the night in all
parts of the globe; but in all other fituations this equality
will only be found in places fituated in the very middle
between the poles, that is, on the equator.

47. The times, wherein this univerfal equality between
the day and night happens, are called the equinoxes. Now
it has been long obferved by aftronomers, that after the
earth hath fet out from either equinox, fuppofe from E
(which will be the fpring equinox, if F be the north pole)
the fame equinox fhall again return a little before the earth
has made a compleat revolution round the fun. This re-
turn of the equinox preceding the intire revolution of the
earth is called the preceffion of the equinox, and is caufed
by the protuberant figure of the earth.

48. Since the fun fhines perpendicularly upon the e-
quator, when the line drawn from the fun to the center
of the earth is perpendicular to the earth's axis, in this cafe
the

the plane, which fhould cut through the earth at the e-
quator, may be extended to pafs through the fun; but it
will not do fo in any other pofition of the earth. Now
let us confider the prominent part of the earth about the
equator, as a folid ring moving with the earth round the
fun. At the time of the equinoxes, this ring will have
the fame kind of fituation in refpect of the fun, as the
orbit of the moon has, when the line of the nodes is di-
rected to the fun; and at all other times will refemble the
moon's orbit in other fituations. Confequently this ring,
which otherwife would keep throughout its motion paral-
lel to it felf, will receive fome change in its pofition from
the action of the fun upon it, except only at the time of
the equinox. The manner of this change may be under-
ftood as follows. Let A B C D (in fig. 120) reprefent this ring,
E the center of the earth, S the fun, A F C G a circle de-
fcribed in the plane of the earth's motion to the center E.
Here A and C are the two points, in which the earth's e-
quator croffes the plane of the earth's motion; and the time
of the equinox falls out, when the ftraight line A C con-
tinued would pafs through the fun. Now let us recollect
what was faid above concerning the moon, when her or-
bit was in the fame fituation with this ring. From thence
it will be underftood, if a body were fuppofed to be mo-
ving in any part of this circle A B C D, what effect the acti-
on of the fun on the body would have toward changing
the pofition of the line A C. In particular H I being drawn
perpendicular to S E, if the body be in any part of this
circle between A and H, or between C and I, the line A C

would.

would be fo turned, that the point A fhall move toward B, and the point C toward D; but if it were in any other part of the circle, either between H and C, or between I and A, the line A C would be turned the contrary way. Hence it follows, that as this folid ring turns round the center of the earth, the parts of this ring between A and H, and between C and I, are fo influenced by the fun, that they will endeavour, fo to change the fituation of the line A C as to caufe the point A to move toward B, and the point C to move toward D; but all the parts of the ring between H and C, and between I and A, will have the oppofite tendency, and difpofe the line A C to move the contrary way. And fince thefe laft named parts are larger than the other, they will prevail over the other, fo that by the action of the fun upon this ring, the line A C will be fo turned, that A fhall continually be more and more moving toward D, and C toward B. Thus no fooner fhall the fun in its vifible motion have departed from A, but the motion of the line A C fhall haften its meeting with C, and from thence the motion of this line fhall again haften the fun's fecond conjunction with A; for as this line fo turns, that A is continually moving toward D, fo the fun's vifible motion is the fame way as from S toward T.

49. The moon will have on this ring the like effect as the fun, and operate on it more ftrongly, in the fame proportion as its force on the fea exceeded that of the fun on the fame. But the effect of the action of both luminaries will be greatly diminifhed by reafon of this ring's being connect-

ed

ed to the reft of the earth; for by this means the fun and moon have not only this ring to move, but likewife the whole globe of the earth, upon whofe fpherical part they have no immediate influence.   Befide the effect is alfo rendred lefs, by reafon that the prominent part of the earth is not collected all under the equator, but fpreads gradually from thence toward both poles.   Upon the whole, though the fun alone carries the nodes of the moon through an intire revolution in about 19 years, the united force of both luminaries on the prominent parts of the earth will hardly carry round the equinox in a lefs fpace of time than 26000 years.

50. To this motion of the equinox we muft add another confequence of this action of the fun and moon upon the elevated parts of the earth, that this annular part of the earth about the equator, and confequently the earth's axis, will twice a year and twice a month change its inclination to the plane of the earth's motion, and be again reftored, juft as the inclination of the moon's orbit by the action of the fun is annually twice diminifhed, and as often recovers its original magnitude.   But this change is very infenfible.

51. I fhall now finifh the prefent chapter with our great author's inquiry into the figure of the fecondary planets, particularly cf our moon, upon the figure of which its fluid parts will have an influence.   The moon turns always the fame fide towards the earth,   and confequently revolves but once round its axis in the fpace of an entire month;

S f   for

for a fpectator placed without the circle, in which the moon moves, would in that time obferve all the parts of the moon fucceffively to pafs once before his view and no more, that is, that the whole globe of the moon has turned once round. Now the great flownefs of this motion will render the centrifugal force of the parts of the waters very weak, fo that the figure of the moon cannot, as in the earth, be much affected by this revolution upon its axis: but the figure of thofe waters are made different from fpherical by another caufe, viz. the action of the earth upon them; by which they will be reduced to an oblong oval form, whofe axis prolonged would pafs through the earth; for the fame reafon, as we have above obferved, that the waters of the earth would take the like figure, if they had moved fo flowly, as to keep pace with the moon. And the folid part of the moon muft correfpond with this figure of the fluid part: but this elevation of the parts of the moon is nothing near fo great as is the protuberance of the earth at the equator, for it will not exceed 93 englifh feet.

52. The waters of the moon will have no tide, except what will arife from the motion of the moon round the earth. For the converfion of the moon about her axis is equable, whereby the inequality in the motion round the earth difcovers to us at fome times fmall parts of the moon's furface towards the eaft or weft, which at other times lie hid; and as the axis, whereon the moon turns, is oblique to her motion round the earth, fometimes fmall parts of her

furface

furface toward the north, and fometimes the like toward the fouth are vifible, which at other times are out of fight. Thefe appearances make what is called the libration of the moon, difcovered by HEVELIUS.   But now as the axis of the oval figure of the waters will be pointed towards the earth, there muft arife from hence fome fluctuation in them; and befide, by the change of the moon's diftance from the earth, they will not always have the very fame height.

J. Creson del.   J. Pine Sculp.

Sf 2   BOOK III.

BOOK III.

Chap I.

Concerning the caufe of COLOURS inherent in the LIGHT.

AFTER this view which has been taken of Sir Isaac Newton's mathematical principles of philofophy, and the ufe he has made of them, in explaining the fyftem of the world, &c. the courfe of my defign directs us to turn our eyes to that other philofophical work, his treatife of Optics, in which we fhall find our great author's inimitable genius difcovering it felf no lefs, than in the

the former; nay perhaps even more, fince this work gives as many inftances of his fingular force of reafoning, and of his unbounded invention, though unaffifted in great meafure by thofe rules and general precepts, which facilitate the invention of mathematical theorems.   Nor yet is this work inferior to the other in ufefulnefs; for as that has made known to us one great principle in nature, by which the celeftial motions are continued, and by which the frame of each globe is preferved; fo does this point out to us another principle no lefs univerfal, upon which depends all thofe operations in the fmaller parts of matter, for whofe fake the greater frame of the univerfe is erected; all thofe immenfe globes, with which the whole heavens are filled, being without doubt only defign'd as fo many convenient apartments for carrying on the more noble operations of nature in vegetation and animal life.   Which fingle confideration gives abundant proof of the excellency of our author's choice, in applying himfelf carefully to examine the action between light and bodies, fo neceffary in all the varieties of thefe productions, that none of them can be fuccefsfully promoted without the concurrence of heat in a greater or lefs degree.

2. 'T is true, our author has not made fo full a difcovery of the principle, by which this mutual action between light and bodies is caufed; as he has in relation to the power, by which the planets are kept in their courfes: yet he has led us to the very entrance upon it, and pointed out the path fo plainly which muft be followed to reach it; that one may

be bold to fay, whenever mankind fhall be bleffed with this improvement of their knowledge, it will be derived fo directly from the principles laid down by our author in this book, that the greateft fhare of the praife due to the difcovery will belong to him.

3. IN fpeaking of the progrefs our author has made, I fhall diftinctly purfue three things, the two firft relating to the colours of natural bodies : for in the firft head fhall be fhewn, how thofe colours are derived from the properties of the light itfelf ; and in the fecond upon what properties of the bodies they depend : but the third head of my difcourfe fhall treat of the action of bodies upon light in refracting, reflecting, and inflecting it.

4. THE firft of thefe, which fhall be the bufinefs of the prefent chapter, is contained in this one propofition : that the fun's direct light is not uniform in refpect of colour, not being difpofed in every part of it to excite the idea of whitenefs, which the whole raifes; but on the contrary is a compofition of different kinds of rays, one fort of which if alone would give the fenfe of red, another of orange, a third of yellow, a fourth of green, a fifth of light blue, a fixth of indigo, and a feventh of a violet purple ; that all thefe rays together by the mixture of their fenfations imprefs upon the organ of fight the fenfe of whitenefs, though each ray always imprints there its own colour; and all the difference between the colours of bodies when viewed in open day light arifes from this, that coloured bodies

do

do not reflect all the forts of rays falling upon them in e-
qual plenty, but some forts much more copiously than o-
thers; the body appearing of that colour, of which the
light coming from it is moft compofed.

5. THAT the light of the fun is compounded, as has been
faid, is proved by refracting it with a prifm. By a prifm I here
mean a glafs or other body of a triangular form, fuch as is re-
prefented in fig. 121. But before we proceed to the illuftration
of the propofition we have juft now laid down, it will be ne-
ceffary to fpend a few words in explaining what is meant by
the refraction of light; as the defign of our prefent labour is
to give fome notion of the fubject, we are engaged in, to
fuch as are not verfed in the mathematics.

6. IT is well known, that when a ray of light paffing
through the air falls obliquely upon the furface of any tranf-
parent body, fuppofe water or glafs, and enters it, the ray
will not pafs on in that body in the fame line it defcribed
through the air, but be turned off from the furface, fo
as to be lefs inclined to it after paffing it, than before. Let
ABCD (in fig. 122.) reprefent a portion of water, or glafs,
AB the furface of it, upon which the ray of light EF falls
obliquely; this ray fhall not go right on in the courfe de-
lineated by the line FG, but be turned off from the fur-
face AB into the line FH, lefs inclined to the furface AB
than the line EF is, in which the ray is incident upon that
furface.

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7. ON

7. On the other hand, when the light paffes out of any fuch body into the air, it is inflected the contrary way, being after its emergence rendred more oblique to the furface it paffes through, than before. Thus the ray F H, when it goes out of the furface C D, will be turned up towards that furface, going out into the air in the line H I.

8. This turning of the light out of its way, as it paffes from one tranfparent body into another is called its refraction. Both thefe cafes may be tried by an eafy experiment with a bafon and water. For the firft cafe fet an empty bafon in the funfhine or near a candle, making a mark upon the bottom at the extremity of the fhadow caft by the brim of the bafon, then by pouring water into the bafon you will obferve the fhadow to fhrink, and leave the bottom of the bafon enlightned to a good diftance from the mark. Let A B C (in fig. 123.) denote the empty bafon, E A D the light fhining over the brim of it, fo that all the part A B D be fhaded. Then a mark being made at D, if water be poured into the bafon (as in fig. 124.) to F G, you fhall obferve the light, which before went on to D, now to come much fhort of the mark D, falling on the bottom in the point H, and leaving the mark D a good way within the enlightened part; which fhews that the ray E A, when it enters the water at I, goes no longer ftraight forwards, but is at that place incurvated, and made to go nearer the perpendicular. The other cafe may be tryed by putting any fmall body into an empty bafon, placed lower than your eye, and then receding from the bafon, till you can but juft

fee

fee the body over the brim. After which, if the bafon be filled with water, you fhall prefently obferve the body to be vifible, though you go farther off from the bafon. Let A B C (in fig. 125.) denote the bafon as before, D the body in it, E the place of your eye, when the body is feen juft over the edge A, while the bafon is empty. If it be then filled with water, you will obferve the body ftill to be vifible, though you take your eye farther off. Suppofe you fee the body in this cafe juft over the brim A, when your eye is at F, it is plain that the rays of light, which come from the body to your eye have not come ftraight on, but are bent at A, being turned downwards, and more inclined to the furface of the water, between A and your eye at F, than they are between A and the body D.

9. THIS we hope is fufficient to make all our readers apprehend, what the writers of optics mean, when they mention the refraction of the light, or fpeak of the rays of light being refracted. We fhall therefore now go on to prove the affertion advanced in the forementioned propofition, in relation to the different kinds of colours, that the direct light of the fun exhibits to our fenfe: which may be done in the following manner.

10. IF a room be darkened, and the fun permitted to fhine into it through a fmall hole in the window fhutter, and be made immediately to fall upon a glafs prifm, the beam of light fhall in paffing through fuch a prifm be parted into rays, which exhibit all the forementioned colours. In this man-

T t

ner if A B (in fig. 126) reprefent the window fhutter; C the hole in it; D E F the prifm; Z Y a beam of light coming from the fun, which paffes through the hole, and falls upon the prifm at Y, and if the prifm were removed would go on to X, but in entring the furface E F of the glafs it fhall be turned off, as has been explained, into the courfe Y W falling upon the fecond furface of the prifm D F in W, going out of which into the air it fhall be again farther inflected. Let the light now, after it has paffed the prifm, be received upon a fheet of paper held at a proper diftance, and it fhall paint upon the paper the picture, image, or fpectrum L M of an oblong figure, whofe length fhall much exceed its breadth; though the figure fhall not be oval, the ends L and M being femicircular and the fides ftraight. But now this figure will be variegated with colours in this manner. From the extremity M to fome length, fuppofe to the line *n o*, it fhall be of an intenfe red; from *n o* to *p q* it fhall be an orange; from *p q* to *r s* it fhall be yellow; from thence to *t u* it fhall be green; from thence to *w x* blue; from thence to *y z* indigo; and from thence to the end violet.

11. Thus it appears that the fun's white light by its paffage through the prifm, is fo changed as now to be divided into rays, which exhibit all thefe feveral colours. The queftion is, whether the rays while in the fun's beam before this refraction poffeffed thefe properties diftinctly; fo that fome part of that beam would without the reft have given a red colour, and another part alone have given an
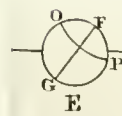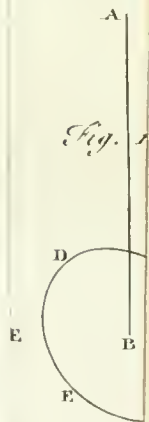
orange,

$L.9$

A B I K D F

G H

Fig. 109.

A

Fig. 1

D

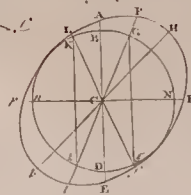E B

E
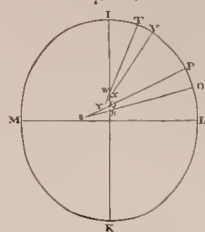
O F

G P

E

9. O A

O C F

G P

F

I

Fig. 110. Fig. 111. Fig. 115. Fig. 116. Fig. 109. Fig. 117. Fig. 118. Fig. 112. Fig. 113. Fig. 114. Fig. 120. Fig. 119. Fig. 121. Fig. 122. Fig. 123. Fig. 124. Fig. 125.
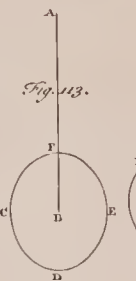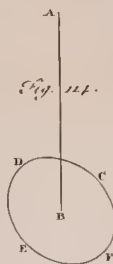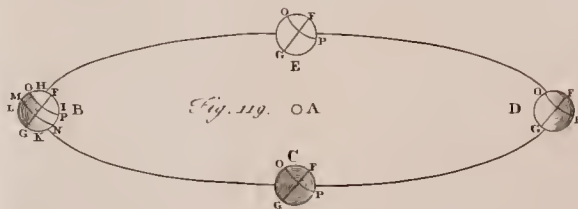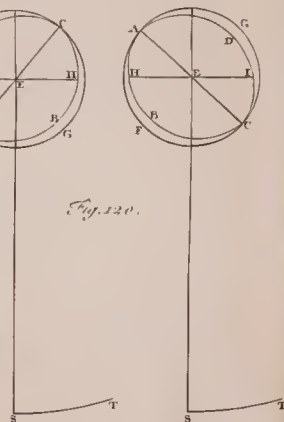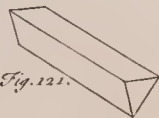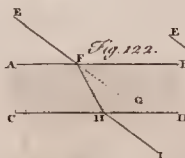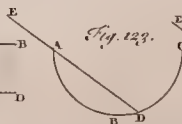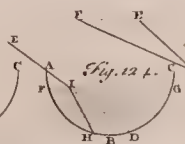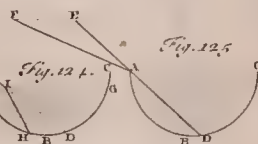
orange, &c. That this is poffible to be the cafe, appears from
hence; that if a convex glafs be placed between the paper
and the prifm, which may collect all the rays proceeding
out of the prifm into its focus, as a burning glafs does the
fun's direct rays; and if that focus fall upon the paper, the
fpot formed by fuch a glafs upon the paper fhall appear
white, juft like the fun's direct light. The reft remaining
as before, let P Q (in fig. 127.) be the convex glafs, cau-
fing the rays to meet upon the paper H G I K in the point
N, I fay that point or rather fpot of light fhall appear white,
without the leaft tincture of any colour. But it is evident
that into this fpot are now gathered all thofe rays, which be-
fore when feparate gave all thofe different colours; which
fhews that whitenefs may be made by mixing thofe colours:
efpecially if we confider, it can be proved that the glafs
P Q does not alter the colour of the rays which pafs
through it. Which is done thus: if the paper be made
to approach the glafs P Q, the colours will manifeft them-
felves as far as the magnitude of the fpectrum, which the
paper receives, will permit. Suppofe it in the fituation *h g i k*,
and that it then receive the fpectrum *l m*, this fpectrum
fhall be much fmaller, than if the glafs P Q were removed,
and therefore the colours cannot be fo much feparated; but
yet the extremity *m* fhall manifeftly appear red, and
the other extremity *l* fhall be blue; and thefe colours as
well as the intermediate ones fhall difcover themfelves more
perfectly, the farther the paper is removed from N, that
is, the larger the fpectrum is: the fame thing happens, if
the paper be removed farther off from P Q than N. Sup-

pose into the position $\theta \gamma \eta \varkappa$, the spectrum $\lambda \mu$ painted upon it shall again discover its colours, and that more distinctly, the farther the paper is removed, but only in an inverted order: for as before, when the paper was nearer the convex glass, than at N, the upper part of the image was blue, and the under red; now the upper part shall be red, and the under blue: because the rays cross at N.

12. N a y farther that the whiteness at the focus N, is made by the union of the colours may be proved without removing the paper out of the focus, by intercepting with any opake body part of the light near the glass; for if the under part, that is the red, or more properly the red-making rays, as they are styled by our author, are intercepted, the spot shall take a bluish hue; and if more of the inferior rays are cut off, so that neither the red-making nor orange-making rays, and if you please the yellow-making rays likewise, shall fall upon the spot; then shall the spot incline more and more to the remaining colours. In like manner if you cut off the upper part of the rays, that is the violet coloured or indigo-making rays, the spot shall turn reddish, and become. more so, the more of those opposite colours are intercepted

13. This I think abundantly proves that whiteness may be produced by a mixture of all the colours of the spectrum. At least there is but one way of evading the present arguments, which is, by asserting that the rays of light after passing the prism have no different properties to exhibit this or the other colour, but are in that respect perfectly

fectly homogeneal, so that the rays which pass to the under and red part of the image do not differ in any properties whatever from those, which go to the upper and violet part of it; but that the colours of the spectrum are produced only by some new modifications of the rays, made at their incidence upon the paper by the different terminations of light and shadow: if indeed this assertion can be allowed any place, after what has been said; for it seems to be sufficiently obviated by the latter part of the preceding experiment, that by intercepting the inferior part of the light, which comes from the prism, the white spot shall receive a bluish cast, and by stopping the upper part the spot shall turn red, and in both cases recover its colour, when the intercepted light is permitted to pass again; though in all these trials there is the like termination of light and shadow. However our author has contrived some experiments expresly to shew the absurdity of this supposition; all which he has explained and enlarged upon in so distinct and expressive a manner, that it would be wholly unnecessary to repeat them in this place[a]. I shall only mention that of them, which may be tried in the experiment before us. If you draw upon the paper H G I K, and through the spot N, the straight line $w x$ parallel to the horizon, and then if the paper be much inclined into the situation $r s v t$ the line $w x$ still remaining parallel to the horizon, the spot N shall lose its whiteness and receive a blue tincture; but if it be inclined as much the contrary way, the same spot shall exchange its white colour for a reddish dye.

[a] Opt. B. I. part 2. prop 1.

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All which can never be accounted for by any difference in the termination of the light and shadow, which here is none at all ; but are easily explained by supposing the upper part of the rays, whenever they enter the eye, disposed to give the sensation of the dark colours blue, indigo and violet; and that the under part is fitted to produce the bright colours yellow, orange and red : for when the paper is in the situation *r s t u*, it is plain that the upper part of the light falls more directly upon it, than the under part, and therefore those rays will be most plentifully reflected from it; and by their abounding in the reflected light will cause it to incline to their colour. Just so when the paper is inclined the contrary way, it will receive the inferior rays most directly, and therefore ting the light it reflects with their colour.

14. It is now to be proved that these dispositions of the rays of light to produce some one colour and some another, which manifest themselves after their being refracted, are not wrought by any action of the prism upon them, but are originally inherent in those rays ; and that the prism only affords each species an occasion of shewing its distinct quality by separating them one from another, which before, while they were blended together in the direct beam of the sun's light, lay conceal'd. But that this is so, will be proved, if it can be shewn that no prism has any power upon the rays, which after their passage through one prism are rendered uncompounded and contain in them but one colour, either to divide that colour into several, as the sun's

light

light is divided, or so much as to change it into any other colour. This will be proved by the following experiment [a]. The same thing remaining, as in the first experiment, let another prism NO (in fig. 128.) be placed either immediately, or at some distance after the first, in a perpendicular posture, so that it shall refract the rays issuing from the first sideways. Now if this prism could divide the light falling upon it into coloured rays, as the first has done, it would divide the spectrum breadthwise into colours, as before it was divided lengthwise; but no such thing is observed. If LM were the spectrum, which the first prism DEF would paint upon the paper HGIK; PQ lying in an oblique posture shall be the spectrum projected by the second, and shall be divided lengthwise into colours corresponding to the colours of the spectrum LM, and occasioned like them by the refraction of the first prism, but its breadth shall receive no such division; on the contrary each colour shall be uniform from side to side, as much as in the spectrum LM, which proves the whole assertion.

15. THE same is yet much farther confirmed by another experiment. Our author teaches that the colours of the spectrum LM in the first experiment are yet compounded, though not so much as in the sun's direct light. He shews therefore how, by placing the prism at a distance from the hole, and by the use of a convex glass, to separate the colours of the spectrum, and make them uncompounded to any degree of exactness [b]. And he shews when this

[a] Newt. Opt. B. 1. part 1. experim. 5.　　[b] Ibid prop. 4.

is

is done fufficiently, if you make a fmall hole in the paper whereon the fpectrum is received, through which any one fort of rays may pafs, and then let that coloured ray fall fo upon a prifm, as to be refracted by it, it fhall in no cafe whatever change its colour; but fhall always retain it perfectly as at firft, however it be refracted [a].

16. Nor yet will thefe colours after this full feparation of them fuffer any change by reflection from bodies of different colours; on the other hand they make all bodies placed in thefe colours appear of the colour which falls upon them [b]: for minium in red light will appear as in open day light; but in yellow light will appear yellow; and which is more extraordinary, in green light will appear green, in blue, blue; and in the violet-purple coloured light will appear of a purple colour; in like manner verdigreafe, or blue bife, will put on the appearance of that colour, in which it is placed: fo that neither bife placed in the red light fhall be able to give that light the leaft blue tincture, or any other different from red; nor fhall minium in the indigo or violet light exhibit the leaft appearance of red, or any other colour diftinct from that it is placed in. The only difference is, that each of thefe bodies appears moft luminous and bright in the colour, which correfponds with that it exhibits in the day light, and dimmeft in the colours moft remote from that; that is, though minium and bife placed in blue light fhall both appear blue, yet the bife fhall appear of a bright blue, and the minium of a dusky and obfcure blue: but

[a] Newt. Opt. B. 1. part 2. exper. 5.   [b] Ibid exper. 6.

if minium and bife be compared together in red light, the minium fhall afford a brisk red, the bife a duller colour, though of the fame fpecies.

17. AND this not only proves the immutability of all thefe fimple and uncompounded colours; but likewife un-folds the whole myftery, why bodies appear in open day-light of fuch different colours, it confifting in nothing more than this, that whereas the white light of the day is com-pofed of all forts of colours, fome bodies reflect the rays of one fort in greater abundance than the rays of any other [a]. Though it appears by the forecited experiment, that almoft all thefe bodies reflect fome portion of the rays of every colour, and give the fenfe of particular colours only by the predominancy of fome forts of rays above the reft. And what has before been explained of compofing white by mingling all the colours of the fpectrum together fhews clearly, that nothing more is required to make bodies look white, than a power to reflect indifferently rays of every colour. But this will more fully appear by the following method: if near the coloured fpectrum in our firft experiment a piece of white paper be fo held, as to be illuminated equally by all the parts of that fpectrum, it fhall appear white; where-as if it be held nearer to the red end of the image, than to the other, it fhall turn reddifh; if nearer the blue end, it fhall feem bluifh [b].

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[a] Newton Opt. B. I. prop. 10.   [b] Ibid exp. 9.

18. OUR

18. Our indefatigable and circumfpect author farther examined his theory by mixing the powders which painters ufe of feveral colours, in order if poffible to produce a white powder by fuch a compofition [a]. But in this he found fome difficulties for the following reafons. Each of thefe coloured powders reflects but part of the light, which is caft upon them; the red powders reflecting little green or blue, and the blue powders reflecting very little red or yellow, nor the green powders reflecting near fo much of the red or indigo and purple, as of the other colours: and befides, when any of thefe are examined in homogeneal light, as our author calls the colours of the prifm, when well feparated, though each appears more bright and luminous in its own day-light colour, than in any other; yet white bodies, fuppofe white paper for inftance, in thofe very colours exceed thefe coloured bodies themfelves in brightnefs; fo that white bodies reflect not only more of the whole light than coloured bodies do in the day-light, but even more of that very colour which they reflect moft copioufly. All which confiderations make it manifeft that a mixture of thefe will not reflect fo great a quantity of light, as a white body of the fame fize; and therefore will compofe fuch a colour as would refult from a mixture of white and black, fuch as are all grey and dun colours, rather than a ftrong white. Now fuch a colour he compounded of certain ingredients, which he particularly fets down, in fo much that when the compofition was ftrongly illuminated by the fun's direct beams, it would appear much whiter than even white pa-

[a] Newt. Opt. B. I. part 1. exp 15.

per, if confiderably fhaded. Nay he found by trials how to proportion the degree of illumination of the mixture and paper, fo that to a fpectator at a proper diftance it could not well be determined which was the more perfect colour; as he experienced not only by himfelf, but by the concurrent opinion of a friend, who chanced to vifit him while he was trying this experiment. I muft not here o-mit another method of trying the whitenefs of fuch a mix-ture, propofed in one of our author's letters on this fub-ject [a]: which is to enlighten the compofition by a beam of the fun let into a darkened room, and then to receive the light reflected from it upon a piece of white paper, obfer-ving whether the paper appears white by that reflection; for if it does, it gives proof of the compofition's being white; becaufe when the paper receives the reflection from any coloured body, it looks of that colour. Agreeable to this is the trial he made upon water impregnated with foap, and agitated into a froth [b]: for when this froth after fome fhort time exhibited upon the little bubbles, which compo-fed it, a great variety of colours, though thefe colours to a fpectator at a fmall diftance difcover'd themfelves diftinctly; yet when the eye was fo far removed, that each little bub-ble could no longer be diftinguifhed, the whole froth by the mixture of all thefe colours appeared intenfly white.

19. Our author having fully fatisfied himfelf by thefe and many other experiments, what the refult is of mixing

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[a] Philof. Tranfact. N. 88, p. 5099.   [b] Opt. B. 1. par. 2. exp. 14.

together

together all the prifmatic colours; he proceeds in the next place to examine, whether this appearance of whitenefs be raifed by the rays of thefe different kinds acting fo, when they meet, upon one another, as to caufe each of them to imprefs the fenfe of whitenefs upon the optic nerve; or whether each ray does not make upon the organ of fight the fame impreffion, as when feparate and alone; fo that the idea of whitenefs is not excited by the impreffion from any one part of the rays, but refults from the mixture of all thofe different fenfations.   And that the latter fentiment is the true one, he evinces by undeniable experiments.

20. In particular the foregoing experiment [a], wherein the convex glafs was ufed, furnifhes proofs of this: in that when the paper is brought into the fituation $\theta\gamma\eta\varkappa$, beyond N the colours, that at N difappeared, begin to emerge again; which fhews that by mingling at N they did not lofe their colorific qualities, though for fome reafon they lay conceal-ed.   This farther appears by that part of the experiment, when the paper, while in the focus, was directed to be en-clined different ways; for when the paper was in fuch a fituation, that it muft of neceffity reflect the rays, which before their arrival at the point N would have given a blue colour, thofe rays in this very point itfelf by abounding in the reflected light tinged it with the fame colour; fo when the paper reflects moft copioufly the rays, which before they come to the point N exhibit rednefs, thofe fame rays tin-

[a] Ibid. exp. 10.

&ture

ͣture the light reflected by the paper from that very point with their own proper colour.

21. There is a certain condition relating to fight, which affords an opportunity of examining this ftill more fully: it is this, that the impreffions of light remain fome fhort fpace upon the eye; as when a burning coal is whirl'd about in a circle, if the motion be very quick, the eye fhall not be able to diftinguifh the coal, but fhall fee an entire circle of fire.   The reafon of which appearance is, that the impreffion made by the coal upon the eye in any one fituation is not worn out, before the coal returns again to the fame place, and renews the fenfation.   This gives our author the hint to try, whether thefe colours might not be tranfmitted fucceffively to the eye fo quick, that no one of the colours fhould be diftinctly perceived, but the mixture of the fenfations fhould produce a uniform whitenefs; when the rays could not act upon each other, becaufe they never fhould meet, but come to the eye one after another. And this thought he executed by the following expedient [a].   He made an inftrument in fhape like a comb, which he applied near the convex glafs, fo that by moving it up and down flowly the teeth of it might intercept fometimes one and fometimes another colour; and accordingly the light reflected from the paper, placed at N, fhould change colour continually. But now when the comb-like inftrument was moved very quick, the eye loft all preception of the diftinct colours, which came to it from time to time, a perfect whitenefs refulting from the

[a] Opt. pag. 122.

mixture

mixture of all thofe diftinct impreffions in the fenforium. Now in this cafe there can be no fufpicion of the feveral coloured rays acting upon one another, and making any change in each other's manner of affecting the eye, feeing they do not fo much as meet together there.

22. Our author farther teaches us how to view the fpectrum of colours produced in the firft experiment with a-nother prifm, fo that it fhall appear to the eye under the fhape of a round fpot and perfectly white [a]. And in this cafe if the comb be ufed to intercept alternately fome of the colours, which compofe the fpectrum, the round fpot fhall change its colour according to the colours intercepted; but if the comb be moved too fwiftly for thofe changes to be diftinctly perceived, the fpot fhall feem always white, as before [b].

23. Besides this whitenefs, which refults from an uni-verfal compofition of all forts of colours, our author par-ticularly explains the effects of other lefs compounded mix-tures; fome of which compound other colours like fome of the fimple ones, but others produce colours different from any of them. For inftance, a mixture of red and yellow compound a colour like in appearance to the orange, which in the fpectrum lies between them; as a compofition of yel-low and blue is made ufe of in all dyes to make a green. But red and violet purple compounded make purples un-like to any of the prifmatic colours, and thefe joined with

[a] Opt, B. I. part 2. exp. 11.   [b] Ibid prop. 4, 6.

yellow

yellow or blue make yet new colours. Besides one rule is here to be observed, that when many different colours are mixed, the colour which arises from the mixture grows languid and degenerates into whiteness. So when yellow green and blue are mixed together, the compound will be green; but if to this you add red and purple, the colour shall first grow dull and less vivid, and at length by adding more of these colours it shall turn to whiteness, or some other colour [a].

24. ONLY here is one thing remarkable of those compounded colours, which are like in appearance to the simple ones; that the simple ones when viewed through a prism shall still retain their colour, but the compounded colours seen through such a glass shall be parted into the simple ones of which they are the aggregate. And for this reason any body illuminated by the simple light shall appear through a prism distinctly, and have its minutest parts observable, as may easily be tried with flies, or other such little bodies, which have very small parts; but the same viewed in this manner when enlighten'd with compounded colours shall appear confused, their smallest parts not being distinguishable. How the prism separates these compounded colours, as likewise how it divides the light of the sun into its colours, has not yet been explained; but is reserved for our third chapter.

25. IN the mean time what has been said, I hope, will suffice to give a taste of our author's way of arguing, and

[a] Opt. pag. 51.

im

in fome meafure to illuftrate the propofition laid down in this chapter.

26. There are methods of feparating the heterogeneous rays of the fun's light by reflection, which perfectly confpire with and confirm this reafoning.   One of which ways may be this.   Let A B (in fig. 129) reprefent the window fhutter of a darkened room; C a hole to let in the fun's rays; D E F, G H I two prifms fo applied together, that the fides E F and G I be contiguous, and the fides D F, G H parallel; by this means the light will pafs through them without any feparation into colours: but if it be afterwards received by a third prifm I K L, it fhall be divided fo as to form upon any white body P Q the ufual colours, violet at *m*, blue at *n*, green at *o*, yellow at *r*, and red at *s*.   But becaufe it never happens that the two adjacent furfaces E F and G I perfectly touch, part only of the light incident upon the furface E F fhall be tranfmitted, and part fhall be reflected.   Let now the reflected part be received by a fourth prifm △ Ə Λ, and paffing through it paint upon a white body Z Γ the colours of the prifm, red at *t*, yellow at *u*, green at *w*, blue at *x*, violet at *y*.   If the prifms D E F, G H I be flowly turned about while they remain contiguous, the colours upon the body P Q fhall not fenfibly change their fituation, till fuch time as the rays become pretty oblique to the furface E F; but then the light incident upon the furface E F fhall begin to be wholly reflected.   And firft of all the violet light fhall be wholly reflected, and thereupon will difappear at *m* , appearing inftead thereof

at

at *y*, and increasing the violet light falling there , the other colours remaining as before.   If the prisms D E F, G H I be turned a little farther about, that the incident rays become yet more inclined to the surface E F, the blue shall be totally reflected, and shall disappear in *n*, but appear at *x* by making the colour there more intense.   And the same may be continued, till all the colours are successively removed from the surface P Q to z Γ.   But in any case, suppose when the violet and the blue have forsaken the surface P Q, and appear upon the surface z Γ, the green, yellow, and red only remaining upon the surface P Q; if the light be received upon a paper held any where in its whole passage between the light's coming out of the prisms D E F, G I H and its incidence upon the prism I K L, it shall appear of the colour compounded of all the colours seen upon P Q; and the reflected ray, received upon a piece of white paper held any where between the prisms D E F and Δ ⊙ Σ, shall exhibit the colour compounded of those the surface P Q is deprived of mixed with the sun's light : whereas before any of the light was reflected from the surface E F, the rays between the prisms G H I and I K L would appear white ; as will likewise the reflected ray both before and after the total reflection, provided the difference of refraction by the surfaces D F and D E be inconsiderable. I call here the sun's light white, as I have all along done; but it is more exact to ascribe to it something of a yellowish tincture, occasioned by the brighter colours abounding in it ; which caution is necessary in examining the colours of the reflected beam, when all the violet and blue are in it : for this

X x   yellowish

yellowifh turn of the fun's light caufes the blue not to be quite fo vifible in it, as it fhould be, were the light perfectly white ; but makes the beam of light incline rather towards a pale white.

Chap. II.

Of the properties of BODIES, upon which their COLOURS depend.

AFTER having fhewn in the laft chapter, that the difference between the colours of bodies viewed in open day-light is only this, that fome bodies are difpofed to reflect rays of one colour in the greateft plenty, and other bodies rays of fome other colour; order now requires us to examine more particularly into the property of bodies, which gives them this difference. But this our author fhews to be nothing more, than the different magnitude of the particles, which compofe each body : this I queftion not will appear no fmall paradox. And indeed this whole chaper will contain fcarce any affertions, but what will be almoft incredible, though the arguments for them are fo ftrong and convincing, that they force our affent. In the former chapter have been explained properties of light, not in the leaft thought of before our author's difcovery of them ; yet are they not difficult to admit, as foon as experiments are known to give proof of their reality ; but fome of the propofitions to be ftated here will, I fear, be accounted almoft paft belief; notwithftanding that the arguments, by which they

they are eſtabliſhed are unanſwerable. For it is proved by our author, that bodies are rendered tranſparent by the minuteneſs of their pores, and become opake by having them large; and more, that the moſt tranſparent body by being reduced to a great thinneſs will become leſs pervious to the light.

2. BUT whereas it had been the received opinion, and yet remains ſo among all who have not ſtudied this philoſophy, that light is reflected from bodies by its impinging againſt their ſolid parts, rebounding from them, as a tennis ball or other elaſtic ſubſtance would do, when ſtruck againſt any hard and reſiſting ſurface; it will be proper to begin with declaring our author's ſentiment concerning this, who ſhews by many arguments that reflection cannot be cauſed by any ſuch means [a]: ſome few of his proofs I ſhall ſet down, referring the reader to our author himſelf for the reſt.

3. IT is well known, that when light fails upon any tranſparent body, glaſs for inſtance, part of it is reflected and part tranſmitted; for which it is ready to account, by ſaying that part of the light enters the pores of the glaſs, and part impinges upon its ſolid parts. But when the tranſmitted light arrives at the farther ſurface of the glaſs, in paſſing out of glaſs into air there is as ſtrong a reflection cauſed, or rather ſomething ſtronger. Now it is not to be conceived, how the light ſhould find as many ſolid parts in the air to ſtrike againſt as in the glaſs, or even a greater num-

[a] Opt. Book II. prop. 8.

ber

ber of them.   And to augment the difficulty, if water
be placed behind the glafs, the reflection becomes much
weaker. Can we therefore fay, that water has fewer folid
parts for the light to ftrike againft, than the air? And if
we fhould, what reafon can be given for the reflection's be-
ing ftronger, when the air by the air-pump is removed
from behind the glafs, than when the air receives the rays
of light.   Befides the light may be fo inclined to the hin-
der furface of the glafs, that it fhall wholly be reflected,
which happens when the angle which the ray makes with
the furface does not exceed about $49\frac{1}{3}$ degrees; but if the
inclination be a very little increafed, great part of the light
will be tranfmitted ; and how the light in one cafe fhould
meet with nothing but the folid parts of the air, and by
fo fmall a change of its inclination find pores in great plenty,
is wholly inconceivable.   It cannot be faid, that the light
is reflected by ftriking againft the folid parts of the furface
of the glafs; becaufe without making any change in that
furface, only by placing water contiguous to it inftead of
air, great part of that light fhall be tranfmitted, which could
find no paffage through the air.   Moreover in the laft ex-
periment recited in the preceding chapter, when by turn-
ing the prifms DEF, GHI, the blue light became wholly
reflected, while the reft was moftly tranfmitted, no poffible
reafon can be affigned, why the blue-making rays fhould
meet with nothing but the folid parts of the air between
the prifms, and the reft of the light in the very fame obli-
quity find pores in abundance. Nay farther, when two glaff-
es touch each other, no reflection at all is made ; though

it

it does not in the leaft appear, how the rays fhould avoid the folid parts of glafs, when contiguous to other glafs, any more than when contiguous to air. But in the laft place upon this fuppofition it is not to be comprehended, how the moft polifhed fubftances could reflect the light in that regular manner we find they do; for when a polifhed looking glafs is covered over with quick filver, we cannot fuppofe the particles of light fo much larger than thofe of the quickfilver, that they fhould not be fcattered as much in reflection, as a parcel of marbles thrown down upon a rugged pavement. The only caufe of fo uniform and regular a reflection muft be fome more fecret caufe, uniformly fpread over the whole furface of the glafs.

4. But now, fince the reflection of light from bodies does not depend upon its impinging againft their folid parts, fome other reafon muft be fought for. And firft it is paft doubt that the leaft parts of almoft all bodies are tranfparent, even the microfcope fhewing as much [a]; befides that it may be experienced by this method. Take any thin plate of the opakeft body, and apply it to a fmall hole defigned for the admiffion of light into a darkened room; however opake that body may feem in open day-light, it fhall under thefe circumftances fufficiently difcover its tranfparency, provided only the body be very thin. White metals indeed do not eafily fhew themfelves tranfparent in thefe trials, they reflecting almoft all the light incident upon them at their firft fuperficies; the caufe of which will appear in what

[a] Opt. Book II. par. 3. prop. 1.

follows

follows[a]. But yet thefe fubftances, when reduced into parts of extraordinary minutenefs by being diffolved in aqua fortis or the like corroding liquors do alfo become tranfparent.

5. Since therefore the light finds free paffage through the leaft parts of bodies, let us confider the largenefs of their pores, and we fhall find, that whenever a ray of light has paffed through any particle of a body, and is come to its farther furface, if it finds there another particle contiguous, it will without interruption pafs into that particle; juft as light will pafs through one piece of glafs into another piece in contact with it without any impediment, or any part being reflected : but as the light in paffing out of glafs, or any other tranfparent body, fhall part of it be reflected back, if it enter into air or other tranfparent body of a different denfity from that it paffes out of ; the fame thing will happen in the light's paffage through any particle of a body, whenever at its exit out of that particle it meets no other particle contiguous, but muft enter into a pore, for in this cafe it fhall not all pafs through, but part of it be reflected back. Thus will the light, every time it enters a pore, be in part reflected; fo that nothing more feems neceffary to opacity, than that the particles, which compofe any body, touch but in very few places, and that the pores of it are numerous and large, fo that the light may in part be reflected from it, and the other part, which enters too deep to be returned out of the body, by numerous reflections may be ftifled and loft [b]; which in all probabi-

[a] § 17.   [b] Opt. Book II. par. 3. prop. 4.

lity

lity happens, as often as it impinges againſt the ſolid part of the body, all the light which does ſo not being reflected back, but ſtopt, and deprived of any farther motion [a].

6. This notion of opacity is greatly confirmed by the obſervation, that opake bodies become tranſparent by filling up the pores with any ſubſtance of near the ſame denſity with their parts. As when paper is wet with water or oyl; when linnen cloth is either dipt in water, oyled, or varniſhed; or the oculus mundi ſtone ſteeped in water [b]. All which experiments confirm both the firſt aſſertion, that light is not reflected by ſtriking upon the ſolid parts of bodies; and alſo the ſecond, that its paſſage is obſtructed by the reflections it undergoes in the pores; ſince we find it in theſe trials to paſs in greater abundance through bodies, when the number of their ſolid parts is increaſed, only by taking away in great meaſure thoſe reflections; which filling the pores with a ſubſtance of near the ſame denſity with the parts of the body will do. Beſides as filling the pores of a dark body makes it tranſparent; ſo on the other hand evacuating the pores of a body tranſparent, or ſeparating the parts of ſuch a body, renders it opake. As ſalts or wet paper by being dried, glaſs by being reduced to powder or the ſurface made rough; and it is well known that glaſs veſſels diſcover cracks in them by their opacity. Juſt ſo water itſelf becomes impervious to the light by being formed into many ſmall bubbles, whether in froth, or by being mixed and agitated with any quantity of a liquor

[a] Opt. Book II. pag. 241.   [b] Ibid. pag. 224.

with

with which it will not incorporate, fuch as oyl of turpentine, or oyl olive.

7. A certain electrical experiment made by Mr. Hauksbee may not perhaps be ufelefs to clear up the prefent fpeculation, by fhewing that fomething more is neceffary befides mere porofity for tranfmitting freely other fine fubftances. The experiment is this; that a glafs cane rubbed till it put forth its electric quality would agitate leaf brafs inclofed under a glafs veffel, though not at fo great a diftance, as if no body had intervened; yet the fame cane would lofe all its influence on the leaf brafs by the interpofition of a piece of the fineft muflin, whofe pores are immenfely larger and more patent than thofe of glafs.

8. Thus I have endeavoured to fmooth my way, as much as I could, to the unfolding yet greater fecrets in nature; for I fhall now proceed to fhew the reafon why bodies appear of different colours. My reader no doubt will be fufficiently furprized, when I inform him that the knowledge of this is deduced from that ludicrous experiment, with which children divert themfelves in blowing bubbles of water made tenacious by the folution of foap. And that thefe bubbles, as they gradually grow thinner and thinner till they break, change fucceffively their colours from the fame principle, as all natural bodies preferve theirs.

9. Our author after preparing water with foap, fo as to render it very tenacious, blew it up into a bubble, and placing

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ing it under a glafs, that it might not be irregularly agitated by the air, obferved as the water by fubfiding changed the thicknefs of the bubble, making it gradually lefs and lefs till the bubble broke; there fucceffively appeared colours at the top of the bubble, which fpread themfelves into rings furrounding the top and defcending more and more, till they vanifhed at the bottom in the fame order in which they appeared[a]. The colours emerged in this order: firft red, then blue; to which fucceeded red a fecond time, and blue immediately followed; after that red a third time, fucceeded by blue; to which followed a fourth red, but fucceeded by green; after this a more numerous order of colours, firft red, then yellow, next green, and after that blue, and at laft purple; then again red, yellow, green, blue, violet followed each other in order; and in the laft place red, yellow, white, blue; to which fucceeded a dark fpot, which reflected fcarce any light, though our author found it did make fome very obfcure reflection, for the image of the fun or a candle might be faintly difcerned upon it; and this laft fpot fpread itfelf more and more, till the bubble at laft broke. Thefe colours were not fimple and uncompounded colours, like thofe which are exhibited by the prifm, when due care is taken to feparate them; but were made by a various mixture of thofe fimple colours, as will be fhewn in the next chapter: whence thefe colours, to which I have given the name of blue, green, or red, were not all alike, but differed as follows. The blue, which appeared next the dark fpot, was a pure colour, but very faint, refembling the sky-colour; the

[a] Ibid. Obf. 17. &c.

Y y

white

white next to it a very ſtrong and intenſe white, bright-
er much than the white, which the bubble reflected, before
any of the colours appeared.   The yellow which preced-
ed this was at firſt pretty good, but ſoon grew dilute; and
the red which went before the yellow at firſt gave a tin-
ćture of ſcarlet inclining to violet, but ſoon changed into
a brighter colour; the violet of the next ſeries was deep
with little or no redneſs in it; the blue a briſk colour, but
came much ſhort of the blue in the next order; the green
was but dilute and pale; the yellow and red were very
bright and full, the beſt of all the yellows which appeared
among any of the colours: in the preceding orders the pur-
ple was reddiſh, but the blue, as was juſt now ſaid, the bright-
eſt of all; the green pretty lively better than in the order
which appeared before it, though that was a good willow
green; the yellow but ſmall in quantity, though bright; the
red of this order not very pure: thoſe which appeared be-
fore yet more obſcure, being very dilute and dirty; as were
likewiſe the three firſt blues.

10. Now it is evident, that theſe colours aroſe at the
top of the bubble, as it grew by degrees thinner and thin-
ner: but what the expreſs thickneſs of the bubble was, where
each of theſe colours appeared upon it, could not be de-
termined by theſe experiments; but was found by another
means, viz. by taking the object glaſs of a long teleſcope,
which is in a ſmall degree convex, and placing it upon a
flat glaſs, ſo as to touch it in one point, and then water be-
ing put between them, the ſame colours appeared as in the
bubble,

bubble, in the form of circles or rings furrounding the point where the glaffes touched, which appeared black for want of any reflection from it, like the top of the bubble when thinneft [a]: next to this fpot lay a blue circle, and next without that a white one; and fo on in the fame order as before, reckoning from the dark fpot. And henceforward I fhall fpeak of each colour, as being of the firft, fecond, or any following order, as it is the firft, fecond, or any following one, counting from the black fpot in the center of thefe rings; which is contrary to the order in which I muft have mentioned them, if I fhould have reputed them the firft, fecond, or third, &c. in order, as they arife after one another upon the top of the bubble.

11. But now by meafuring the diameters of each of thefe rings, and knowing the convexity of the telefcope glafs, the thicknefs of the water at each of thofe rings may be determined with great exactnefs: for inftance the thicknefs of it, where the white light of the firft order is reflected, is about $3\frac{7}{8}$ fuch parts, of which an inch contains 1000000 [b]. And this meafure gives the thicknefs of the bubble, where it appeared of this white colour, as well as of the water between the glaffes; though the tranfparent body which furrounds the water in thefe two cafes be very different: for our author found, that the condition of the ambient body would not alter the fpecies of the colour at all, though it might its ftrength and brightnefs; for pieces of Mufcovy glafs, which were fo thin as to appear coloured by being

[a] Ibid. Obf. 10.   [b] Ibid. pag. 206.

Y y 2

wet

wet with water, would have their colours faded and made
lefs bright thereby; but he could not obferve their fpecies
at all to be changed. So that the thicknefs of any tranf-
parent body determines its colour, whatever body the light
paffes through in coming to it [a].

12. But it was found that different tranfparent bodies
would not under the fame thickneffes exhibit the fame co-
lours: for if the forementioned glaffes were laid upon each
other without any water between their furfaces, the air it-
felf would afford the fame colours as the water, but more
expanded, infomuch that each ring had a larger diameter,
and all in the fame proportion. So that the thicknefs of the
air proper to each colour was in the fame proportion larger,
than the thicknefs of the water appropriated to the fame [b].

13. If we examine with care all the circumftances of thefe
colours, which will be enumerated in the next chap-
ter, we fhall not be furprized, that our author takes them to
bear a great analogy to the colours of natural bodies [c]. For
the regularity of thofe various and ftrange appearances relating
to them, which makes the moft myfterious part of the acti-
on between light and bodies, as the next chapter will fhew,
is fufficient to convince us that the principle, from which
they flow, is of the greateft importance in the frame of
nature; and therefore without queftion is defigned for no
lefs a purpofe than to give bodies their various colours, to
which end it feems very fitly fuited. For if any fuch tranf-

[a] Obfer. 21.   [b] Obferv. 5. compared with Obferv. 10.   [c] Ibid. prop. 5.

parent

parent fubftance of the thicknefs proper to produce any one colour fhould be cut into flender threads, or broken into fragments, it does not appear but thefe fhould retain the fame colour; and a heap of fuch fragments fhould frame a body of that colour. So that this is without difpute the caufe why bodies are of this or the other colour, that the particles of which they are compofed are of different fizes.   Which is farther confirmed by the analogy between the colours of thin plates, and the colours of many bodies. For example, thefe plates do not look of the fame colour when viewed obliquely, as when feen direct; for if the rings and colours between a convex and plane glafs are viewed firft in a direct manner, and then at different degrees of obliquity, the rings will be obferved to dilate themfelves more and more as the obliquity is increafed [a]; which fhews that the tranfparent fubftance between the glaffes does not exhibit the fame colour at the fame thicknefs in all fituations of the eye: juft fo the colours in the very fame part of a peacock's tail change, as the tail changes pofture in refpect of the fight.   Alfo the colours of filks, cloths, and other fubftances, which water or oyl can intimately penetrate, become faint and dull by the bodies being wet with fuch fluids, and recover their brightnefs again when dry; juft as it was before faid that plates of Mufcovy glafs grew faint and dim by wetting. To this may be added, that the colours which painters ufe will be a little changed by being ground very elaborately, without queftion by the diminution of their parts. All which particulars, and many more that

[a] Obferv. 7.

might

might be extracted from our author, give abundant proof of the
present point.   I shall only subjoin one more: these transpa-
rent plates transmit through them all the light they do not re-
flect; so that when looked through they exhibit those colours,
which result from the depriving white light of the colour re-
flected.   This may commodiously be tryed by the glasses so
often mentioned ; which if looked through exhibit coloured
rings as by reflected light, but in a contrary order ; for the mid-
dle spot, which in the other view appears black for want
of reflected light, now looks perfectly white, opposite to
the blue circle; next without this spot the light appears
tinged with a yellowish red; where the white circle ap-
peared before, it now seems dark ; and so of the rest [a].
Now in the same manner, the light transmitted through fo-
liated gold into a darkened room appears greenish by the
loss of the yellow light, which gold reflects.

14. HENCE it follows, that the colours of bodies
give a very probable ground for making conjecture concerning
the magnitude of their constituent particles [b].   My reason for
calling it a conjecture is, its being difficult to fix certainly the
order of any colour.   The green of vegetables our au-
thor judges to be of the third order, partly because of the in-
tenseness of their colour; and partly from the changes they
suffer when they wither, turning at first into a greenish or
more perfect yellow, and afterwards some of them to an o-
range or red; which changes seem to be effected from their
ranging particles growing denser by the exhalation of their

moiſture, and perhaps augmented likewiſe by the accretion of the earthy and oily parts of that moiſture.   How the mentioned colours ſhould ariſe from increaſing the bulk of thoſe particles, is evident; ſeeing thoſe colours lie without the ring of green between the glaſſes, and are therefore formed where the tranſparent ſubſtance which reflects them is thicker. And that the augmentation of the denſity of the colorific particles will conſpire to the production of the ſame effect, will be evident; if we remember what was ſaid of the different ſize of the rings, when air was included between the glaſſes, from their ſize when water was between them; which ſhewed that a ſubſtance of a greater denſity than another gives the ſame colour at a leſs thickneſs.   Now the changes likely to be wrought in the denſity or magnitude of the parts of vegetables by withering ſeem not greater, than are ſufficient to change their colour into thoſe of the ſame order; but the yellow and red of the fourth order are not full enough to agree with thoſe, into which theſe ſubſtances change, nor is the green of the ſecond ſufficiently good to be the colour of vegetables ; ſo that their colour muſt of neceſſity be of the third order.

15. The blue colour of ſyrup of violets our author ſuppoſes to be of the third order; for acids, as vinegar, with this ſyrup change it red, and ſalt of tartar or other alcalies mixed therewith turn it green.   But if the blue colour of the ſyrup were of the ſecond order, the red colour, which acids by attenuating its parts give it, muſt be of the firſt order, and the green given it by alcalies by incraſſating

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its particles fhould be of the fecond; whereas neither of thofe colours is perfect enough, efpecially the green, to anfwer thofe produced by thefe changes; but the red may well enough be allowed to be of the fecond order, and the green of the third; in which cafe the blue muft be likewife of the third order.

16. The azure colour of the skies our author takes to be of the firft order, which requires the fmalleft particles of any colour, and therefore moft like to be exhibited by vapours, before they have fufficiently coalefced to produce clouds of other colours.

17. The moft intenfe and luminous white is of the firft order, if lefs ftrong it is a mixture of the colours of all the orders. Of the latter fort he takes the colour of linnen, paper, and fuch like fubftances to be; but white metals to be of the former fort. The arguments for it are thefe. The opacity of all bodies has been fhewn to arife from the number and ftrength of the reflections made within them; but all experiments fhew, that the ftrongeft reflection is made at thofe furfaces, which intercede tranfparent bodies differing moft in denfity. Among other inftances of this, the experiments before us afford one; for when air only is included between the glaffes, the coloured rings are not only more dilated, as has before been faid, than when water is between them; but are likewife much more luminous and bright. It follows therefore, that whatever medium pervades the pores of bodies, if fo be there

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is any, thofe fubftances muft be moft opake, the denfity of whofe parts differs moft from the denfity of the medium, which fills their pores. But it has been fufficiently proved in the former part of this tract, that there is no very denfe medium lodging in, at leaft pervading at liberty the pores of bodies. And it is farther proved by the prefent experiments. For when air is inclofed by the denfer fubftance of glafs, the rings dilate themfelves, as has been faid, by being viewed obliquely; this they do fo very much, that at different obliquities the fame thicknefs of air will exhibit all forts of colours. The bubble of water, though furrounded with the thinner fubftance of air, does likewife change its colour by being viewed obliquely; but not any thing near fo much, as in the other cafe; for in that the fame colour might be feen, when the rings were viewed moft obliquely, at more than twelve times the thicknefs it appeared at under a direct view; whereas in this other cafe the thicknefs was never found confiderably above half as much again. Now the colours of bodies not depending only on the light, that is incident upon them perpendicularly, but likewife upon that, which falls on them in all degrees of obliquity; if the medium furrounding their particles were denfer than thofe particles, all forts of colours muft of neceffity be reflected from them fo copioufly, as would make the colours of all bodies white, or grey, or at beft very dilute and imperfect. But on the other hand, if the medium in the pores of bodies be much rarer than their particles, the colour reflected will be fo little changed by the obliquity of the rays, that the colour produced by the rays, which fall near the perpendicular, may

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so much abound in the reflected light, as to give the body their colour with little allay. To this may be added, that when the difference of the contiguous transparent substances is the same, a colour reflected from the denser substance reduced into a thin plate and surrounded by the rarer will be more brisk, than the same colour will be, when reflected from a thin plate formed of the rarer substance, and surrounded by the denser ; as our author experienced by blowing glass very thin at a lamp furnace, which exhibited in the open air more vivid colours, than the air does between two glasses. From these considerations it is manifest, that if all other circumstances are alike, the densest bodies will be most opake. But it was observed before, that these white metals can hardly be made so thin, except by being dissolved in corroding liquors, as to be rendred transparent; though none of them are so dense as gold, which proves their great opacity to have some other cause besides their density; and none is more fit to produce this, than such a size of their particles, as qualifies them to reflect the white of the first order.

18. For producing black the particles ought to be smaller than for exhibiting any of the colours, viz. of a size answering to the thickness of the bubble, whereby reflecting little or no light it appears colourless ; but yet they must not be too small, for that will make them transparent through deficiency of reflections in the inward parts of the body, sufficient to stop the light from going through it; but they must be of a size bordering upon that

<div align="right">disposed</div>

difpofed to reflect the faint blue of the firft order, which affords an evident reafon why blacks ufually partake a little of that colour. We fee too, why bodies diffolved by fire or putrefaction turn black : and why in grinding glaffes upon copper plates the duft of the glafs, copper, and fand it is ground with, become very black : and in the laft place why thefe black fubftances communicate fo eafily to others their hue; which is, that their particles by reafon of the great minutenefs of them eafily overfpread the groffer particles of others.

19. I shall now finifh this chapter with one remark of the exceeding great porofity in bodies neceffarily required in all that has here been faid; which, when duly confidered, muft appear very furprizing; but perhaps it will be matter of greater furprize, when I affirm that the fagacity of our author has difcovered a method, by which bodies may eafily become fo ; nay how any the leaft portion of matter may be wrought into a body of any affigned dimenfions how great fo ever , and yet the pores of that body none of them greater, than any the fmalleft magnitude propofed at pleafure; notwithftanding which the parts of the body fhall fo touch, that the body itfelf fhall be hard and folid [a]. The manner is this: fuppofe the body be compounded of particles of fuch figures, that when laid together the pores found between them may be equal in bignefs to the particles; how this may be effected, and yet the body be hard and folid, is not difficult to underftand; and the pores of fuch a bo-

[a] Opt. pag. 243.

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dy may be made of any propofed degree of fmallnefs. But the folid matter of a body fo framed will take up only half the fpace occupied by the body; and if each conftituent particle be compofed of other lefs particles according to the fame rule, the folid parts of fuch a body will be but a fourth part of its bulk; if every one of thefe leffer particles again be compounded in the fome manner, the folid parts of the whole body fhall be but one eighth of its bulk; and thus by continuing the compofition the folid parts of the body may be made to bear as fmall a proportion to the whole magnitude of the body, as fhall be defired, notwithftanding the body will be by the contiguity of its parts capable of being in any degree hard. Which fhews that this whole globe of earth, nay all the known bodies in the univerfe together, as far as we know, may be compounded of no greater a portion of folid matter, than might be reduced into a globe of one inch only in diameter, or even lefs. We fee therefore how by this means bodies may eafily be made rare enough to tranfmit light, with all that freedom pellucid bodies are found to do. Though what is the real ftructure of bodies we yet know not.

Chap. III.

Of the Refraction, Reflection, and Inflection of Light.

THUS much of the colours of natural bodies; our method now leads us to fpeculations yet greater, no lefs

lefs than to lay open the caufes of all that has hitherto been related. For it muft in this chapter be explained, how the prifm feparates the colours of the fun's light, as we found in the firft chapter; and why the thin tranfparent plates difcourfed of in the laft chapter, and confequently the particles of coloured bodies, reflect that diverfity of colours only by being of different thickneffes.

2. F o r the firft it is proved by our author, that the colours of the fun's light are manifefted by the prifm, from the rays undergoing different degrees of refraction; that the violet-making rays, which go to the upper part of the coloured image in the firft experiment of the firft chapter, are the moft refracted; that the indigo-making rays are refracted, or turned out of their courfe by paffing through the prifm, fomething lefs than the violet-making rays, but more than the blue-making rays; and the blue-making rays more than the green; the green-making rays more than the yellow; the yellow more than the orange; and the orange-making rays more than the red-making, which are leaft of all refracted. The firft proof of this, that rays of different colours are refracted unequally is this. If you take any body, and paint one half of it red and the other half blue, then upon viewing it through a prifm thofe two parts fhall appear feparated from each other; which can be caufed no otherwife than by the prifm's refracting the light of one half more than the light of the other half. But the blue half will be moft refracted; for if the body be feen through the prifm in fuch a fituation, that the body fhall appear

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lifted upwards by the refraction, as a body within a bafon of water, in the experiment mentioned in the firft chapter, appeared to be lifted up by the refraction of the water, fo as to be feen at a greater diftance than when the bafon is empty, then fhall the blue part appear higher than the red; but if the refraction of the prifm be the contrary way, the blue part fhall be depreffed more than the other. Again, after laying fine threads of black filk acrofs each of the colours, and the body well inlightened, if the rays coming from it be received upon a convex glafs, fo that it may by refracting the rays caft the image of the body upon a piece of white paper held beyond the glafs; then it will be feen that the black threads upon the red part of the image, and thofe upon the blue part, do not at the fame time appear diftinctly in the image of the body projected by the glafs; but if the paper be held fo, that the threads on the blue part may diftinctly appear, the threads cannot be feen diftinct upon the red part; but the paper muft be drawn farther off from the convex glafs to make the threads on this part vifible; and when the diftance is great enough for the threads to be feen in this red part, they become indiftinct in the other. Whence it appears that the rays proceeding from each point of the blue part of the body are fooner united again by the convex glafs than the rays which come from each point of the red parts[a]. But both thefe experiments prove that the blue-making rays, as well in the fmall refraction of the convex glafs, as in the greater refraction of the prifm, are more bent, than the red-making rays.

[a] Newt. Opt. B. I. part. 1. prop. I.

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3. THIS feems already to explain the reafon of the co-loured fpectrum made by refracting the fun's light with a prifm; though our author proceeds to examine that in particular, and proves that the different coloured rays in that fpectrum are in different degrees refracted; by fhewing how to place the prifm in fuch a pofture, that if all the rays were re-fracted in the fame manner, the fpectrum fhould of neceffi-ty be round : whereas in that cafe if the angle made by the two furfaces of the prifm, through which the light paffes, that is the angle D F E in fig. 126, be about 63 or 64 degrees, the image inftead of being round fhall be near five times as long as broad ; a difference enough to fhew a great inequality in the refractions of the rays, which go to the oppofite extremities of the image. To leave no fcruple unremoved, our author is very particular in fhewing by a great number of experiments, that this inequality of refra-ction is not cafual, and that it does not depend upon any ir-regularities of the glafs; no nor that the rays are in their paffage through the prifm each fplit and divided ; but on the contrary that every ray of the fun has its own peculiar degree of refraction proper to it, according to which it is more or lefs refracted in paffing through pellucid fubftances always in the fame manner[a]. That the rays are not fplit and multiplied by the refraction of the prifm, the third of the experiments related in our firft chapter fhews very clear-ly ; for if they were, and the length of the fpectrum in the firft refraction were thereby occafioned, the breadth fhould be no lefs dilated by the crofs refraction of the fe-

[a] Opt. B. I. part. I. prop. 2.

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cond prifm; whereas the breadth is not at all increafed, but the image is only thrown into an oblique pofture by the upper part of the rays which were at firft more refracted than the under part, being again turned fartheft out of their courfe.   But the experiment moft exprefsly adapted to prove this regular diverfity of refraction is this, which follows[a]. Two boards A B, C D (in fig. 130.) being erected in a darkened room at a proper diftance, one of them A B being near the window-fhutter E F, a fpace only being left for the prifm G H I to be placed between them; fo that the rays entring at the hole M of the window-fhutter may after pafling through the prifm be trajected through a fmaller hole K made in the board A B, and pafling on from thence go out at another hole L made in the board C D of the fame fize as the hole K, and fmall enough to tranfmit the rays of one colour only at a time; let another prifm N O P be placed after the board C D to receive the rays pafling through the holes K and L, and after refraction by that prifm let thofe rays fall upon the white furface Q R.   Suppofe firft the violet light to pafs through the holes, and to be refracted by the prifm N O P to *s*, which if the prifm N O P were removed fhould have paffed right on to W.   If the prifm G H I be turned flowly about, while the boards and prifm N O P remain fixed, in a little time another colour will fall upon the hole L, which, if the prifm N O P were taken away, would proceed like the former rays to the fame point W; but the refraction of the prifm N O P fhall not carry thefe rays to *s*, but to fome place lefs diftant from W as

[a] Opt. B. I. part 1. Expec. 6.

to *t*.   Suppofe now the rays which go to *t* to be the indigo-making rays.   It is manifeft that the boards A B, C D, and prifm N O P remaining immoveable, both the violet-making and indigo-making rays are incident alike upon the prifm N O P, for they are equally inclined to its furface O P, and enter it in the fame part of that furface; which fhews that the indigo-making rays are lefs diverted out of their courfe by the refraction of the prifm, than the violet-making rays under an exact parity of all circumftances.   Farther, if the prifm G H I be more turned about, 'till the blue-making rays pafs through the hole L, thefe fhall fall upon the furface Q R below I, as at *v*, and therefore are fubjected to a lefs refraction than the indigo-making rays.   And thus by proceeding it will be found that the green-making rays are lefs refracted than the blue-making rays, and fo of the reft, according to the order in which they lie in the coloured fpectrum.

4. This difpofition of the different coloured rays to be refracted fome more than others our author calls their refpective degrees of refrangibility.   And fince this difference of refrangibility difcovers it felf to be fo regular, the next ftep is to find the rule it obferves.

5. It is a common principle in optics, that the fine of the angle of incidence bears to the fine of the refracted angle a given proportion.   If A B (in fig. 131, 132) reprefent the furface of any refracting fubftance, fuppofe of water or glafs, and C D a ray of light incident upon that furface

face in the point D, let D E be the ray, after it has paſſed the ſurface A B; if the ray paſs out of the air into the ſubſtance whoſe ſurface is A B (as in fig. 1 3 1) it ſhall be turned from the ſurface, and if it paſs out of that ſubſtance into air it ſhall be bent towards it (as in fig. 1 3 2) But if F G be drawn through the point D perpendicular to the ſurface A B, the angle under C D F made by the incident ray and this perpendicular is called the angle of incidence; and the angle under E D G, made by this perpendicular and the ray after refraction, is called the refracted angle. And if the circle H F I G be deſcribed with any interval cutting C D in H and D E in I, then the perpendiculars H K, I L being let fall upon F G, H K is called the ſine of the angle under C D F the angle of incidence, and I L the ſine of the angle under E D G the refracted angle. The firſt of theſe ſines is called the ſine of the angle of incidence, or more briefly the ſine of incidence, the latter is the ſine of the refracted angle, or the ſine of refraction. And it has been found by numerous experiments that whatever proportion the ſine of incidence H K bears to the ſine of refraction I L in any one caſe, the ſame proportion ſhall hold in all caſes; that is, the proportion between theſe ſines will remain unalterably the ſame in the ſame refracting ſubſtance, whatever be the magnitude of the angle under C D F.

6. But now becauſe optical writers did not obſerve that every beam of white light was divided by refraction, as has been here explained, this rule collected by them can only be underſtood in the groſs of the whole beam after re-

fraction, and not fo much of any particular part of it, or at moft only of the middle part of the beam.   It therefore was incumbent upon our author to find by what law the rays were parted from each other; whether each ray apart obtained this property, and that the feparation was made by the proportion between the fines of incidence and refracti-on being in each fpecies of rays different ; or whether the light was divided by fome other rule.   But he proves by a certain experiment that each ray has its fine of incidence proportional to its fine of refraction; and farther fhews by mathematical reafoning, that it muft be fo upon condition only that bodies refract the light by acting upon it, in a direction perpendicular to the furface of the refracting body, and upon the fame fort of rays always in an equal degree at the fame diftances[a].

7. O u r great author teaches in the next place how from the refraction of the moft refrangible and leaft refrangible rays to find the refraction of all the intermediate ones [b].   The method is this: if the fine of incidence be to the fine of re-fraction in the leaft refrangible rays as A to B C, (in fig. 133) and to the fine of refraction in the moft refrangible as A to B D; if C E be taken equal to C D, and then E D be fo divided in F, G, H, I, K, L, that E D, E F, E G, E H, E I, E K, E L, E C, fhall be proportional to the eight lengths of mufical chords, which found the notes in an octave, E D being the length of the key, E F the length of the tone above

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[a] Opt. pag. 67, 68, &c.   [b] Ibid. B. I, par. 2. prop. 3.

that

that key, E G the length of the leffer third, E H of the
fourth, E I of the fifth, E K of the greater fixth, E L of
the feventh, and E C of the octave above that key ; that is if
the lines E D, E F, E G, E H, E I, E K, E L, and E C bear the fame
proportion as the numbers, 1, $\frac{9}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{9}{6}$, $\frac{1}{2}$, refpectively
then fhall B D, B F, be the two limits of the fines of refraction of
the violet-making rays, that is the violet-making rays fhall
not all of them have precifely the fame fine of refraction ,
but none of them fhall have a greater fine than B D, nor
a lefs than B F, though there are violet-making rays which
anfwer to any fine of refraction that can be taken be-
tween thefe two. In the fame manner B F and B G
are the limits of the fines of refraction of the indigo-ma-
king rays ; B G, B H are the limits belonging to the blue-
making rays; B H, B I the limits pertaining to the green-ma-
king rays, B I, B K the limits for the yellow-making rays;
B K, B L the limits for the orange-making rays; and laftly,
B L and B C the extreme limits of the fines of refraction
belonging to the red-making rays. Thefe are the propor-
tions by which the heterogeneous rays of light are feparated
from each other in refraction.

8. When light paffes out of glafs into air, our author
found A to B C as 50 to 77, and the fame A to B D as 50
to 78. And when it goes out of any other refracting fub-
ftance into air, the excefs of the fine of refraction of any
one fpecies of rays above its fine of incidence bears a con-
ftant proportion, which holds the fame in each fpecies, to
the excefs of the fine of refraction of the fame fort of rays

above

above the fine of incidence into the air out of glaſs; provided
the fines of incidence both in glaſs and the other ſubſtance
are equal.   This our author verified by tranſmitting the
light through priſms of glaſs included within a priſmatic
veſſel of water; and draws from thoſe experiments the fol-
lowing obſervations : that whenever the light in paſſing
through ſo many ſurfaces parting diverſe tranſparent ſub-
ſtances is by contrary refractions made to emerge into the
air in a direction parallel to that of its incidence, it will
appear afterwards white at any diſtance from the priſms,
where you ſhall pleaſe to examine it; but if the direction
of its emergence be oblique to its incidence, in receding
from the place of emergence its edges ſhall appear tinged
with colours: which proves that in the firſt caſe there is
no inequality in the refractions of each ſpecies of rays, but
that when any one ſpecies is ſo refracted as to emerge pa-
rallel to the incident rays, every ſort of rays after refracti-
on ſhall likewiſe be parallel to the ſame incident rays, and
to each other; whereas on the contrary, if the rays of
any one ſort are oblique to the incident light, the ſeveral
ſpecies ſhall be oblique to each other, and be gradually
ſeparated by that obliquity.   From hence he de-
duces both the forementioned theorem, and alſo this other;
that in each ſort of rays the proportion of the fine of in-
cidence to the fine of refraction, in the paſſage of the ray
out of any refracting ſubſtance into another, is compounded
of the proportion to which the fine of incidence would have to
the fine of refraction in the paſſage of that ray out of the
firſt ſubſtance into any third, and of the proportion which
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the fine of incidence would have to the fine of refraction in the paffage of the ray out of that third fubftance into the fecond.   From fo fimple and plain an experiment has our moft judicious author deduced thefe important theorems, by which we may learn how very exact and circumfpect he has been in this whole work of his optics; that notwithftanding his great particularity in explaining his doctrine, and the numerous collection of experiments he has made to clear up every doubt which could arife, yet at the fame time he has ufed the greateft caution to make out every thing by the fimpleft and eafieft means poffible.

9. Our author adds but one remark more upon refraction, which is, that if refraction be performed in the manner he has fuppofed from the light's being preffed by the refracting power perpendicularly toward the furface of the refracting body, and confequently be made to move fwifter in the body than before its incidence; whether this power act equally at all diftances or otherwife, provided only its power in the fame body at the fame diftances remain without variation the fame in one inclination of the incident rays as well as another; he obferves that the refracting powers in different bodies will be in the duplicate proportion of the tangents of the leaft angles, which the refracted light can make with the furfaces of the refracting bodies [a].   This obfervation may be explained thus.   When the light paffes into any refracting fubftance, it has been fhewn above that the fine of incidence bears a conftant proportion to the fine

[a] Opt. B. II. par. 3. prop. 10.

of refraction. Suppofe the light to pafs to the refracting body A B C D (in fig. 1 3 4) in the line E F, and to fall upon it at the point F, and then to proceed within the body in the line F G. Let H I be drawn through F perpendicular to the furface A B, and any circle K L M N be defcribed to the center F. Then from the points O and P where this circle cuts the incident and refracted ray, the perpendiculars O Q, P R being drawn, the proportion of O Q to P R will remain the fame in all the different obliquities, in which the fame ray of light can fall on the furface A B. Now O Q is lefs than F L the femidiameter of the circle K L M N, but the more the ray E F is inclined down toward the furface A B, the greater will O Q be, and will approach nearer to the magnitude of F L. But the proportion of O Q to P R remaining always the fame, when O Q is largeft, P R will alfo be greateft ; fo that the more the incident ray E F is inclined toward the furface A B, the more the ray F G after refraction will be inclined toward the fame. Now if the line F S T be fo drawn, that S V being perpendicular to F I fhall be to F L the femidiameter of the circle in the conftant proportion of P R to O Q; then the angle under N F T is that which I meant by the leaft of all that can be made by the refracted ray with this furface, for the ray after refraction would proceed in this line, if it were to come to the point F lying on the very furface A B; for if the incident ray came to the point F in any line between A F and F H, the ray after refraction would proceed forward in fome line between F T and F I. Here if N W be drawn perpendicular to F N, this line N W in the circle K L M N is called

the

the tangent of the angle under N F S. Thus much being premifed, the fenfe of the forementioned propofition is this. Let there be two refracting fubftances (in fig. 135) A B C D , and E F G H. Take a point, as I, in the furface A B, and to the center I with any femidiameter defcribe the circle K L M.   In like manner on the furface E F take fome point N, as a center, and defcribe with the fame femidiameter the circle O P Q. Let the angle under B I R be the leaft which the refracted light can make with the furface A B, and the angle under F N S the leaft which the refracted light can make with the furface E F.   Then if L T be drawn perpendicular to A B, and P V perpendicular to E F; the whole power, wherewith the fubftance A B C D acts on the light, will bear to the whole power wherewith the fubftance E F G H acts on, the light, a proportion, which is duplicate of the proportion, which L T bears to P V.

10. Upon comparing according to this rule the refractive powers of a great many bodies it is found, that unctuous bodies which abound moft with fulphureous parts refract the light two or three times more in proportion to their denfity than others : but that thofe bodies, which feem to receive in their compofition like proportions of fulphureous parts, have their refractive powers proportional to their denfities ; as appears beyond contradiction by comparing the refractive power of fo rare a fubftance as the air with that of common glafs or rock cryftal , though thefe fubftances are 2000 times denfer than air ; nay the fame proportion

portion is found to hold without fenfible difference in comparing air with pfeudo-topar and glafs of antimony, though the pfeudo-topar be 3500 times denfer than air, and glafs of antimony no lefs than 4400 times denfer. This power in other fubftances, as falts, common water, fpirit of wine, &c. feems to bear a greater proportion to their denfities than thefe laft named, according as they abound with fulphurs more than thefe; which makes our author conclude it probable, that bodies act upon the light chiefly, if not altogether, by means of the fulphurs in them; which kind of fubftances it is likely enters in fome degree the compofition of all bodies. Of all the fubftances examined by our author, none has fo great a refractive power, in refpect of its denfity, as a diamond.

11. Our author finifhes thefe remarks, and all he offers relating to refraction, with obferving, that the action between light and bodies is mutual, fince fulphureous bodies, which are moft readily fet on fire by the fun's light, when collected upon them with a burning glafs, act more upon light in refracting it, than other bodies of the fame denfity do. And farther, that the denfeft bodies, which have been now fhewn to act moft upon light, contract the greateft heat by being expofed to the fummer fun.

12. Having thus difpatched what relates to refraction, we muft addrefs ourfelves to difcourfe of the other operation of bodies upon light in reflecting it. When light paffes through a furface, which divides two tranfparent bodies

B b b

dies

dies differing in denfity, part of it only is tranfmitted, another part being reflected. And if the light pafs out of the denfer body into the rarer, by being much inclined to the forefaid furface at length no part of it fhall pafs through, but be totally reflected. Now that part of the light, which fuffers the greateft refraction, fhall be wholly reflected with a lefs obliquity of the rays, than the parts of the light which undergo a lefs degree of refraction; as is evident from the laft experiment recited in the firft chapter; where, as the prifms D E F, G H I, (in fig. 129.) were turned about, the violet light was firft totally reflected, and then the blue, next tothat the green, and fo of the reft. In confequence of which our author lays down this propofition; that the fun's light differs in reflexibility, thofe rays being moft reflexible, which are moft refrangible. And collects from this, in conjunction with other arguments, that the refraction and reflection of light are produced by the fame caufe, compaffing thofe different effects only by the difference of circumftances with which it is attended. Another proof of this being taken by our author from what he has difcovered of the paffage of light through thin tranfparent plates, viz. that any particular fpecies of light, fuppofe, for inftance, the red-making rays, will enter and pafs out of fuch a plate, if that plate be of fome certain thickneffes; but if it be of other thickneffes, it will not break through it, but be reflected back: in which is feen, that the thicknefs of the plate determines whether the power, by which that plate acts upon the light, fhall reflect it, or fuffer it to pafs through.

13. BUT

13. But this laft mentioned furprifing property of the action between light and bodies affords the reafon of all that has been faid in the preceding chapter concerning the colours of natural bodies; and muft therefore more particularly be illuftrated and explained, as being what will principally unfold the nature of the action of bodies upon light.

14. To begin: The object glafs of a long telefcope being laid upon a plane glafs, as propofed in the foregoing chapter, in open day-light there will be exhibited rings of various colours, as was there related; but if in a darkened room the coloured fpectrum be formed by the prifm, as in the firft experiment of the firft chapter, and the glaffes be illuminated by a reflection from the fpectrum, the rings fhall not in this cafe exhibit the diverfity of colours before defcribed, but appear all of the colour of the light which falls upon the glaffes, having dark rings between. Which fhews that the thin plate of air between the glaffes at fome thickneffes reflects the incident light, at other places does not reflect it, but is found in thofe places to give the light paffage; for by holding the glaffes in the light as it paffes from the prifm to the fpectrum, fuppofe at fuch a diftance from the prifm that the feveral forts of light muft be fufficiently feparated from each other, when any particular fort of light falls on the glaffes, you will find by holding a piece of white paper at a fmall diftance beyond the glaffes, that at thofe intervals, where the dark rings appeared upon the glaffes, the light is fo tranfmitted,

as

as to paint upon the paper rings of light having that co-
lour which falls upon the glaffes.   This experiment there-
fore opens to us this very ftrange property of reflection,
that in thefe thin plates it fhould bear fuch a relation to the
thicknefs of the plate, as is here fhewn.   Farther, by care-
fully meafuring the diameters of each ring it is found, that
whereas the glaffes touch where the dark fpot appears in
the center of the rings made by reflection, where the air
is of twice the thicknefs at which the light of the firft ring
is reflected, there the light by being again tranfmitted ma-
kes the firft dark ring ; where the plate has three times
that thicknefs which exhibits the firft lucid ring, it a-
gain reflects the light forming the fecond lucid ring; when
the thicknefs is four times the firft, the light is again tranf-
mitted fo as to make the fecond dark ring; where the air
is five times the firft thicknefs, the third lucid ring is made ;
where it has fix times the thicknefs, the third dark ring ap-
pears, and fo on : in fo much that the thickneffes, at which
the light is reflected, are in proportion to the numbers 1, 3,
5, 7, 9, &c. and the thickneffes, where the light is tranf-
mitted, are in the proportion of the numbers 0, 2, 4, 6, 8,
&c. And thefe proportions between the thickneffes which
reflect and tranfmit the light remain the fame in all fitua-
tions of the eye, as well when the rings are viewed obliquely,
as when looked on perpendicularly. We muft farther here
obferve, that the light, when it is reflected, as well as when it is
tranfmitted, enters the thin plate, and is reflected from its far-
ther furface; becaufe, as was before remarked, the altering
the tranfparent body behind the farther furface alters the de-
gree

gree of reflection as when a thin piece of Muſcovy glaſs has its farther ſurface wet with water, and the colour of the glaſs made dimmer by being ſo wet; which ſhews that the light reaches to the water, otherwiſe its reflection could not be influenced by it. But yet this reflection depends upon ſome power propagated from the firſt ſurface to the ſecond; for though made at the ſecond ſurface it depends alſo upon the firſt, becauſe it depends upon the diſtance between the ſurfaces; and beſides, the body through which the light paſſes to the firſt ſurface influences the reflection: for in a plate of Muſcovy glaſs, wetting the ſurface, which firſt receives the light, diminiſhes the reflection, though not quite ſo much as wetting the farther ſurface will do. Since therefore the light in paſſing through theſe thin plates at ſome thickneſſes is reflected, but at others tranſmitted without reflection, it is evident, that this reflection is cauſed by ſome power propagated from the firſt ſurface, which intermits and returns ſucceſſively. Thus is every ray apart diſpoſed to alternate reflections and tranſmiſſions at equal intervals; the ſucceſſive returns of which diſpoſition our author calls the fits of eaſy reflection, and of eaſy tranſmiſſion. But theſe fits, which obſerve the ſame law of returning at equal intervals, whether the plates are viewed perpendicularly or obliquely, in different ſituations of the eye change their magnitude. For what was obſerved before in reſpect of thoſe rings, which appear in open day-light, holds likewiſe in theſe rings exhibited by ſimple lights; namely, that theſe two alter in bigneſs according to the different angle under which they are ſeen: and our author

lay-

lays down a rule whereby to determine the thicknesses of the plate of air, which shall exhibit the same colour under different oblique views [a]. And the thickness of the aereal plate, which in different inclinations of the rays will exhibit to the eye in open day-light the same colour, is also varied by the same rule [b]. He contrived farther a method of comparing in the bubble of water the proportion between the thickness of its coat, which exhibited any colour when seen perpendicularly, to the thickness of it, where the same colour appeared by an oblique view; and he found the same rule to obtain here likewise [c]. But farther, if the glasses be enlightened successively by all the several species of light, the rings will appear of different magnitudes; in the red light they will be larger than in the orange colour, in that larger than in the yellow, in the yellow larger than in the green, less in the blue, less yet in the indigo, and least of all in the violet: which shew sthat the same thickness of the aereal plate is not fitted to reflect all colours, but that one colour is reflected where another would have been transmitted; and as the rays which are most strongly refracted form the least rings, a rule is laid down by our author for determining the relation, which the degree of refraction of each species of colour has to the thicknesses of the plate where it is reflected.

15. From these observations our author shews the reason of that great variety of colours, which appears in these thin plates in the open white light of the day. For when this white

[a] Op. B. II. par. 3. prop. 15.   [b] ibid. par. 1, observ. 7.   [c] Ibid. Observ. 19.

light

light falls on the plate, each part of the light forms rings of its own colour ; and the rings of the different colours not being of the same bignefs are varioufly intermixed, and form a great variety of tints[a].

16. I n certain experiments, which our author made with thick glaffes, he found, that thefe fits of eafy reflection and tranfmiffion returned for fome thoufands of times, and thereby farther confirmed his reafoning concerning them[b].

17. U p o n the whole, our great author concludes from fome of the experiments made by him, that the reafon why all tranfparent bodies refract part of the light incident upon them, and reflect another part, is, becaufe fome of the light, when it comes to the furface of the body, is in a fit of eafy tranfmiffion, and fome part of it in a fit of eafy reflection ; and from the durablenefs of thefe fits he thinks it probable, that the light is put into thefe fits from their firft emiffion out of the luminous body ; and that thefe fits continue to return at equal intervals without end, unlefs thofe intervals be changed by the light's entring into fome refracting fubftance[c]. He likewife has taught how to determine the change which is made of the intervals of the fits of eafy tranfmiffion and reflection, when the light paffes out of one tranfparent fpace or fubftance into another. His rule is, that when the light paffes perpendicularly to the furface, which parts any two tranfparent fubftances, thefe intervals in the fubftance, out of

[a] Opt. B. II. par. 2. pag. 199. &c.   [b] Ibid. par. 4   [c] Ibid. part. 3. prop. 13.

which

which the light paſſes, bear to the intervals in the ſubſtance, whereinto the light enters, the ſame proportion, as the ſine of incidence bears to the ſine of refraction [a]. It is farther to be obſerved, that though the fits of eaſy reflection return at conſtant intervals, yet the reflecting power never operates, but at or near a ſurface where the light would ſuffer refraction; and if the thickneſs of any tranſparent body ſhall be leſs than the intervals of the fits, thoſe intervals ſhall ſcarce be diſturbed by ſuch a body, but the light ſhall paſs through without any reflection [b].

18. What the power in nature is, whereby this action between light and bodies is cauſed, our author has not diſcovered. But the effects, which he has diſcovered, of this power are very ſurpriſing, and altogether wide from any conjectures that had ever been framed concerning it; and from theſe diſcoveries of his no doubt this power is to be deduced, if we ever can come to the knowledge of it. Sir Isaac Newton has in general hinted at his opinion concerning it; that probably it is owing to ſome very ſubtle and elaſtic ſubſtance diffuſed through the univerſe, in which ſuch vibrations may be excited by the rays of light, as they paſs through it, that ſhall occaſion it to operate ſo differently upon the light in different places as to give riſe to theſe alternate fits of reflection and tranſmiſſion, of which we have now been ſpeaking [c]. He is of opinion, that ſuch a ſubſtance may produce this, and other effects alſo in nature, though it be ſo rare as not to give any ſenſible reſiſtance to bodies in mo-

[a] Ibid. p op. 17.   [b] Ibid prop, 13.   Opt. Qu. 18, &c.

tion;

tion[a]; and therefore not inconfiftent with what has been faid above, that the planets move in fpaces free from refiftance [b].

19. In order for the more full difcovery of this action between light and bodies, our author began another fet of experiments, wherein he found the light to be acted on as it paffes near the edges of folid bodies; in particular all fmall bodies, fuch as the hairs of a man's head or the like, held in a very fmall beam of the fun's light, caft extremely broad fhadows.   And in one of thefe experiments the fhadow was 35 times the breadth of the body [c].   Thefe fhadows are alfo obferved to be bordered with colours [d].   This our author calls the inflection of light; but as he informs us, that he was interrupted from profecuting thefe experiments to any length,   I need not detain my readers with a more particular account of them.

CHAP. IV.
Of OPTIC GLASSES.

SIR Isaac Newton having deduced from his doctrine of light and colours a furprifing improvement of telefcopes, of which I intend here to give an account, I fhall firft premife fomething in general concerning thofe inftruments.

[a] See Concl. S, 2.   [c] Opt. B. III. Obf. 1.
[b] B. II. Ch. 1.   [d] Ibid. Obf. 2.

2. It will be underftood from what has been faid above, that when light falls upon the furface of glafs obliquely, after its entrance into the glafs it is more inclined to the line drawn through the point of incidence perpendicular to that furface, than before.   Suppofe a ray of light iffuing from the point A (in fig. 136) falls on a piece of glafs BCDE, whofe furface BC, whereon the ray falls, is of a fpherical or globular figure, the center whereof is F.   Let the ray proceed in the line A G falling on the furface B C in the point G, anddraw F G H.   Here the ray after its entrance into the glafs will pafs on in fome line, as G I, more inclined toward the line F G H that the line A G is inclined thereto ; for the line F G H is perpendicular to the furface B C in the point G.   By this means, if a number of rays proceeding from any one point fall on a convex fpherical furface of glafs, they fhall be inflected (as is reprefented in fig. 137,) fo as to be gathered pretty clofe together about the line drawn through the center of the glafs from the point, whence the rays proceed ; which line henceforward we fhall call the axis of the glafs : or the point from whence the rays proceed may be fo near the glafs, that the rays fhall after entring the glafs ftill go on to fpread themfelves, but not fo much as before ; fo that if the rays were to be continued backward (as in fig. 138,) they fhould gather together about the axis at a place more remote from the glafs, than the point is, whence they actually proceed.   In thefe and the following figures A denotes the point to which the rays are related before refraction, B the point to which they are directed afterwards, and C the center of the refracting furface. Here we may obferve, that it is poffible to form the glafs of fuch a figure, that all the rays which proceed from one point

<div align="right">fhall</div>

fhall after refraction be reduced again exactly into one point on the axis of the glafs. But in glaffes of a fpherical form though this does not happen ; yet the rays, which fall within a moderate diſtance from the axis, will unite extremely near together. If the light fall on a concave fpherical furface, after refraction it fhall fpread quicker than before (as in fig. 139,) unlefs the rays proceed from a point between the center and the furface of the glafs .If we fuppofe the rays of light, which fall upon the glafs, not to proceed from any point, but to move fo as to tend all to fome point in the axis of the glafs beyond the furface ; if the glafs have a convex furface, the rays fhall unite about the axis fooner, than otherwife they would do (as in fig. 140,) unlefs the point to which they tended was between the furface and the center of that furface. But if the furface be concave, they fhall not meet fo foon : nay perhaps converge. (See fig. 141 and 142.)

3. FARTHER, becaufe the light in paffing out of glafs into the air is turned by the refraction farther off from the line drawn through the point of incidence perpendicular to the refracting furface, than it was before ; the light which fpreads from a point fhall by paffing through a convex furface of glafs into the air be made either to fpread lefs than before (as in fig. 143,) or to gather about the axis beyond the glafs (as in fig. 144.) But if the rays of light were proceeding to a point in the axis of the glafs, they fhould by the refraction be made to unite fooner about that axis (as in fig. 145.) If the furface of the glafs be concave, rays which proceed from a point fhall be made to fpread fafter (as in fig. 146,) but rays which are tending to a point in the axis of

the

the glafs, fhall be made to gather about the axis farther from the glafs (as in fig. 147) or even to diverge (as in fig. 148,) unlefs the point, to which the rays are directed, lies between the furface of the glafs and its center.

4. THE rays, which fpread themfelves from a point, are called diverging; and fuch as move toward a point, are called converging rays. And the point in the axis of the glafs, about which the rays gather after refraction, is called the focus of thofe rays.

5. IF a glafs be formed of two convex fpherical furfaces (as in fig. 149,) where the glafs A B is formed of the furfaces A C B and A D B, the line drawn through the centers of the two furfaces, as the line E F, is called the axis of the glafs; and rays, which diverge from any point of this axis, by the refraction of the glafs will be caufed to converge toward fome part of the axis, or at leaft to diverge as from a point more remote from the glafs, than that from whence they proceeded; for the two furfaces both confpire to produce this effect upon the rays. But converging rays will be caufed by fuch a glafs as this to converge fooner. If a glafs be formed of two concave furfaces, as the glafs A B (in fig. 150,) the line C D drawn through the centers, to which the two furfaces are formed, is called the axis of the glafs. Such a glafs fhall caufe diverging rays, which proceed from any point in the axis of the glafs, to diverge much more, as if they came from fome place in the axis of the glafs nearer to it than the point,

<div align="right">whence</div>

Fig. 126.

Fig. 127.

Fig. 128.

Fig. 130.

Fig. 131.

Fig. 133.

Fig. 129.

Fig. 132.

Fig. 134.

Fig. 135.

Fig. 137.

Fig. 139.

Fig. 136.

Fig. 138.

Fig. 142.

Fig. 143.

Fig. 140.

Fig. 141.

Fig. 144.

Fig. 145.

Fig. 146.

Fig. 147.

Fig. 148.

whence the rays actually proceed. But converging rays will be made either to converge lefs, or even to diverge.

6. In thefe glaffes rays, which proceed from any point near the axis, will be affected as it were in the fame manner, as if they proceeded from the very axis it felf, and fuch as converge toward a point at a fmall diftance from the axis will fuffer much the fame effects from the glafs, as if they converged to fome point in the very axis. By this means any luminous body expofed to a convex glafs may have an image formed upon any white body held beyond the glafs. This may be eafily tried with a common fpectacle-glafs. For if fuch a glafs be held between a candle and a piece of white paper, if the diftances of the candle, glafs, and paper be properly adjufted, the image of the candle will appear very diftinctly upon the paper, but be feen inverted; the reafon whereof is this. Let A B (in fig. 151) be the glafs, C D an object placed crofs the axis of the glafs. Let the rays of light, which iffue from the point E, where the axis of the glafs croffes the object, be fo refracted by the glafs, as to meet again about the point F. The rays, which diverge from the point C of the object, fhall meet again almoft at the fame diftance from the glafs, but on the other fide of the axis, as at G; for the rays at the glafs crofs the axis. In like manner the rays, which proceed from the point D, will meet about H on the other fide of the axis. None of thefe rays, neither thofe which proceed from the point E in the axis, nor thofe which iffue from C or D, will meet again exactly in one point; but yet in one place, as is here fuppofed at F, G, and H, they

will

will be crouded fo clofe together, as to make a diftinct image of the object upon any body proper to reflect it, which fhall be held there.

7. If the object be too near the glafs for the rays to converge after the refraction, the rays fhall iffue out of the glafs, as if they diverged from a point more diftant from the glafs, than that from whence they really proceed (as in fig. 152,) where the rays coming from the point E of the object, which lies on the axis of the glafs A B, if-fue out of the glafs, as if they came from the point F more remote from the glafs than E; and the rays proceed-ing from the point C iffue out of the glafs, as if they pro-ceeded from the point G; likewife the rays which iffue from the point D emerge out of the glafs, as if they came from the point H. Here the point G is on the fame fide of the axis, as the point C; and the point H on the fame fide, as the point D. In this cafe to an eye placed beyond the glafs the object fhould appear, as if it were in the fitu-ation G F H.

8. If the glafs A B had been concave (as in, fig. 153,) to an eye beyond the glafs the object C D would appear in the fituation G H, nearer to the glafs than really it is. Here alfo the object will not be inverted; but the point G is on the fame fide the axe with the point C, and H on the fame fide as D.

9. Hence

9. Hence may be underftood, why fpectacles made with convex glaffes help the fight in old age : for the eye in that age becomes unfit to fee objects diftinctly, except fuch as are remov'd to a very great diftance ; whence all men, when they firft ftand in need of fpectacles, are ob-ferved to read at arm's length, and to hold the object at a greater diftance, than they ufed to do before.   But when an object is removed at too great a diftance from the fight, it cannot be feen clearly, by reafon that a lefs quantity of light from the object will enter the eye, and the whole object will alfo appear fmaller.   Now by help of a con-vex glafs an object may be held near, and yet the rays of light iffuing from it will enter the eye, as if the object were farther removed.

10. After the fame manner concave glaffes affift fuch, as are fhort fighted.   For thefe require the object to be brought inconveniently near to the eye, in order to their feeing it diftinctly; but by fuch a glafs the object may be removed to a proper diftance, and yet the rays of light enter the eye, as if they came from a place much nearer.

11. Whence thefe defects of the fight arife, that in old age objects cannot be feen diftinct within a moderate diftance, and in fhort-fightednefs not without being brought too near, will be eafily underftood, when the manner of vifion in general fhall be explain'd ; which I fhall now en-deavour to do, in order to be better underftood in what
3   follows.

follows.   The eye is form'd, as is reprefented in fig. 154.
It is of a globular figure, the fore part whereof fcarce
more protuberant than the reft is tranfparent.   Underneath
this tranfparent part is a fmall collection of an humour in
appearance like water, and it has alfo the fame refractive
power as common water ; this is called the aqueous hu-
mour, and fills the fpace A B C D in the figure.   Next beyond
lies the body D E F G; this is folid but tranfparent, it is
compofed with two convex furfaces, the hinder furface E F G
being more convex, than the anterior E D G.   Between the
outer membrane A B C, and this body E D G F is placed that
membrane, which exhibits the colours, that are feen round
the fight of the eye ; and the black fpot, which is called the
fight or pupil, is a hole in this membrane, through which the
light enters, whereby we fee.   This membrane is fixed on-
ly by its outward circuit, and has a mufcular power, where-
by it dilates the pupil in a weak light, and contracts it in
a ftrong one.   The body D E F G is called the cryftalline
humour, and has a greater refracting power than water.
Behind this the bulk of the eye is filled up with what is
called the vitreous humor, this has much the fame refra-
ctive power with water.   At the bottom of the eye toward
the inner fide next the nofe the optic glafs enters, as at
H, and fpreads it felf all over the infide of the eye, till
within a fmall diftance from A and C.   Now any object, as
I K, being placed before the eye, the rays of light iffuing
from each point of this object are fo refracted by the con-
vex furface of the aqueous humour, as to be caufed to con-
verge; after this being received by the convex furface E D G

of

of the cryftalline humour, which has a greater refractive power than the aqueous, the rays, when they are entered into this furface, ftill more converge, and at going out of the furface E F G into a humour of a lefs refractive power than the cryftalline they are made to converge yet farther. By all thefe fucceffive refractions they are brought to converge at the bottom of the eye, fo that a diftinct image of the object as L M is imprefs'd on the nerve. And by this means the object is feen.

11. It has been made a difficulty, that the image of the object impreffed on the nerve is inverted, fo that the upper part of the image is impreffed on the lower part of the eye. But this difficulty, I think, can no longer remain, if we only confider, that upper and lower are terms merely relative to the ordinary pofition of our bodies: and our bodies, when view'd by the eye, have their image as much inverted as other objects; fo that the image of our own bodies, and of other objects, are impreffed on the eye in the fame relation to one another, as they really have.

12. The eye can fee objects equally diftinct at very different diftances, but in one diftance only at the fame time. That the eye may accomodate itfelf to different diftances, fome change in its humours is requir'd. It is my opinion, that this change is made in the figure of the cryftalline humour, as I have indeavoured to prove in another place.

13. I f any of the humours of the eye are too flat, they will refract the light too little; which is the cafe in old age. If they are too convex, they refract too much; as in thofe who are fhort-fighted.

14. T h e manner of direct vifion being thus explained, I proceed to give fome account of telefcopes, by which we view more diftinctly remote objects; and alfo of microfcopes, whereby we magnify the appearance of fmall objects. In the firft place, the moft fimple fort of telefcope is com-pofed of two glaffes, either both convex, or one convex, and the other concave. (The firft fort of thefe is reprefent-ed in fig. 155, the latter in fig. 156.)

15. In fig. 155 let A B reprefent the convex glafs next the object, C D the other glafs more convex near the eye. Suppofe the object-glafs A B to form the image of the ob-ject at E F; fo that if a fheet of white paper were to be held in this place, the object would appear. Now fup-pofe the rays, which pafs the glafs A B, and are united a-bout F, to proceed to the eye glafs C D, and be there re-fracted. Three only of thefe rays are drawn in the figure, thofe which pafs by the extremities of the glafs A B, and that which paffes its middle. If the glafs C D be placed at fuch a diftance from the image E F, that the rays, which pafs by the point F, after having proceeded through the glafs diverge fo much, as the rays do that come from an object, which is at fuch a diftance from the eye as

to be feen diftinctly, thefe being received by the eye will make on the bottom of the eye a diftinct reprefentation of the point F.   In like manner the rays, which pafs through the object glafs A B to the point E after proceeding through the eye-glafs C D will on the bottom of the eye make a diftinct reprefentation of the point E.   But if the eye be placed where thefe rays, which proceed from E, crofs thofe, which proceed from F, the eye will receive the diftinct impreffion of both thefe points at the fame time; and confequently will alfo receive a diftinct impreffion from all the intermediate parts of the image E F, that is, the eye will fee the object, to which the telefcope is directed, diftinctly. The place of the eye is about the point G, where the rays H E, H F crofs, which pafs through the middle of the object-glafs A B to the points E and F; or at the place where the focus would be formed by rays coming from the point H, and refracted by the glafs C D.   To judge how much this inftrument magnifies any object, we muft firft obferve, that the angle under E H F, in which the eye at the point H would fee the image E F, is nearly the fame as the angle, under which the object appears by direct vifion; but when the eye is in G, and views the object through the telefcope, it fees the fame under a greater angle; for the rays, which coming from E and F crofs in G, make a greater angle than the rays, which proceed from the point H to thefe points E and F.   The angle at G is greater than that at H in the proportion, as the diftance between the glaffes A B and C D is greater than the diftance of the point G from the glafs C D.

16. This telescope inverts the object; for the rays, which come from the right-hand side of the object, go to the point E the left side of the image; and the rays, which come from the left side of the object, go to F the right side of the image. These rays cross again in G, so that the rays, which come from the right side of the object, go to the right side of the eye; and the rays from the left side of the object go to the left side of the eye. Therefore in this telescope the image in the eye has the same situation as the object; and seeing that in direct vision the image in the eye has an inverted situation, here, where the situation is not inverted, the object must appear so. This is no inconvenience to astronomers in celestial observations; but for objects here on the earth it is usual to add two other convex glasses, which may turn the object again (as is represented in fig. 157,) or else to use the other kind of telescope with a concave eye-glass.

17. In this other kind of telescope the effect is founded on the same principles, as in the former. The distinctness of the appearance is procured in the same manner. But here the eye-glass C D (in fig. 156) is placed between the image E F, and the object glass A B. By this means the rays, which come from the right-hand side of the object, and proceed toward E the left side of the image, being intercepted by the eye-glass are carried to the left side of the eye; and the rays, which come from the left side of the object, go to the right side of the eye; so that the impression in the eye being inverted the object appears in the same situation,

as

as when view'd by the naked eye.   The eye muſt here be placed cloſe to the glaſs.   The degree of magnifying in this inſtrument is thus to be found.   Let the rays, which paſs through the glaſs A B at H, after the refraction of the eye-glaſs C D diverge, as if they came from the point G; then the rays, which come from the extremities of the object, enter the eye under the angle at G; ſo that here alſo the object will be magnified in the proportion of the diſtance between the glaſſes, to the diſtance of G from the eye-glaſs.

18. THE ſpace, that can be taken in at one view in this teleſcope, depends on the breadth of the pupil of the eye; for as the rays, which go to the points E, F of the image, are ſomething diſtant from each other, when they come out of the glaſs C D, if they are wider aſunder than the pupil, it is evident, that they cannot both enter the eye at once.   In the other teleſcope the eye is placed in the point G, where the rays that come from the points E or F croſs each other, and therefore muſt enter the eye together.   On this account the teleſcope with convex glaſſes takes in a larger view, than thoſe with concave.   But in theſe alſo the extent of the view is limited, becauſe the eye-glaſs does not by the refraction towards its edges form ſo diſtinct a repreſentation of the object, as near the middle.

18. Microſcopes are of two ſorts.   One kind is only a very convex glaſs, by the means of which the object may be brought very near the eye, and yet be ſeen diſtinctly. This

This microfcope magnifies in proportion, as the object by being brought near the eye will form a broader impreffion on the optic nerve. The other kind made with convex glaffes produces its effects in the fame manner as the telefcope. Let the object A B (in fig. 158) be placed under the glafs C D, and by this glafs let an image be formed of this object. Above this image let the glafs G H be placed. By this glafs let the rays, which proceed from the points A and B, be refracted, as is expreffed in the figure. In particular, let the rays, which from each of thefe points pafs through the middle of the glafs C D, crofs in I, and there let the eye be placed. Here the object will appear larger, when feen through the microfcope, than if that inftrument were removed, in proportion as the angle, in which thefe rays crofs in I, is greater than the angle, which the lines would make, that fhould be drawn from I to A and B; that is, in the proportion made up of the proportion of the diftance of the object A B from I, to the diftance of I from the glafs G H; and of the proportion of the diftance between the glaffes, to the diftance of the object A B from the glafs C D.
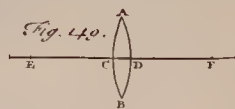
19. I shall now proceed to explain the imperfection in thefe inftruments, occafioned by the different refrangibility of the light which comes from every object. This prevents the image of the object from being formed in the focus of the object glafs with perfect diftinctnefs; fo that if the eye-glafs magnify the image overmuch, the imperfections of it muft be vifible, and make the whole appear confufed. Our author more fully to fatisfy himfelf, that the different refrangibility of the

feveral

feveral forts of rays is fufficient to produce this irregularity, underwent the labour of a very nice and difficult experiment, whofe procefs he has at large fet down, to prove, that the rays of light are refracted as differently in the fmall refraction of telefcope glaffes as in the larger of the prifm ; fo exceeding careful has he been in fearching out the true caufe of this effect.   And he ufed, I fuppofe, the greater caution, becaufe another reafon had before been generally affigned for it.   It was the opinion of all mathematicians, that this defect in telefcopes arofe from the figure, in which the glaffes were formed ;   a fpherical refracting furface not collecting into an exact point all the rays which come from any one point of an object, as has before been faid [a].   But after our author has proved, that in thefe fmall refractions, as well as in greater, the fine of incidence into air out of glafs, to the fine of refraction in the red-making rays, is as 50 to 77, and in the blue-making rays 50 to 78 ; he proceeds to compare the inequalities of refraction arifing from this different refrangibility of the rays, with the inequalities, which would follow from the figure of the glafs, were light uniformly refracted.   For this purpofe he obferves, that if rays iffuing from a point fo remote from the object glafs of a telefcope, as to be efteemed parallel, which is the cafe of the rays, which come from the heavenly bodies ; then the diftance from the glafs of the point, in which the leaft refrangible rays are united, will be to the diftance, at which the moft refrangible rays unite, as 28 to 27 ; and therefore that the leaft fpace, into which
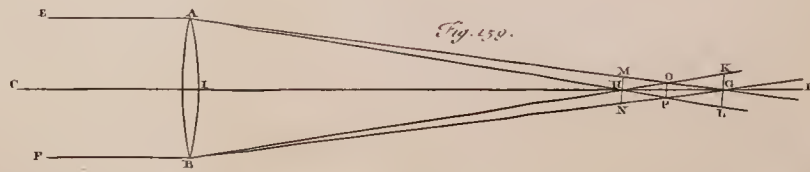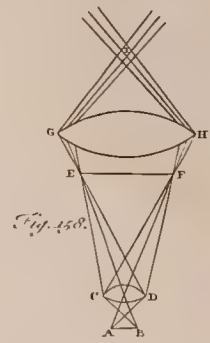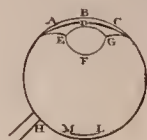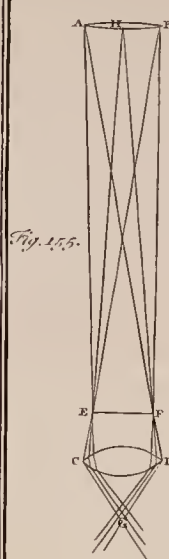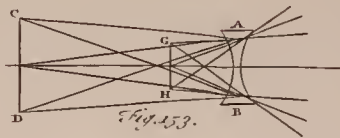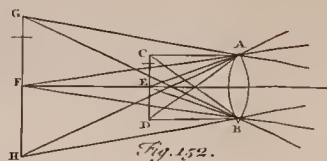
[a] § 2.

all

all the rays can be collected, will not be lefs than the 55th part of the breadth of the the glafs. For if A B (in fig .159) be the glafs, C D its axis, E A, F B two rays of the light parallel to that axis entring the glafs near its edges; after refraction let the leaft refrangible part of thefe rays meet in G, the moft refrangible in H; then, as has been faid, G I will be to I H, as 28 to 27; that is, G H will be the 28th part of G I, and the 27th part of H I; whence if K L be drawn through G, and M N through H, perpendicular to C D, M N will be the 28th part of A B, the breadth of the glafs, and K L the 27th part of the fame; fo that O P the leaft fpace, into which the rays are gathered, will be about half the mean between thefe two, that is the 55th part of A B.

20. This is the error arifing from the different refrangibility of the rays of light, which our author finds vaftly to exceed the other, confequent upon the figure of the glafs. In particular, if the telefcope glafs be flat on one fide, and convex on the other; when the flat fide is turned towards the object, by a theorem, which he has laid down, the error from the figure comes out above 5000 times lefs than the other. This other inequality is fo great, that telefcopes could not perform fo well as they do, were it not that the light does not equally fill all the fpace O P, over which it is fcattered, but is much more denfe toward the middle of that fpace than at the extremities. And befides, all the kinds of rays affect not the fenfe equally ftrong, the yellow and orange being the ftrongeft,

the

Fig. 149.

Fig. 150.

Fig. 151.

Fig. 152.

Fig. 153.

Fig. 154.

Fig. 155.

Fig. 156.

Fig. 157.

Fig. 158.

Fig. 159.
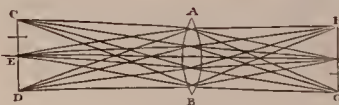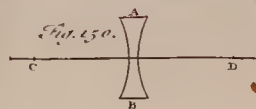
the red and green next to them, the blue indigo and vio-
let being much darker and fainter colours ; and it is ſhewn
that all the yellow and orange, and three fifths of the
brighter half of the red next the orange, and as great a
ſhare of the brighter half of the green next the yellow,
will be collected into a ſpace whoſe breadth is not above
the 250th part of the breadth of the glaſs. And the re-
maining colours, which fall without this ſpace, as they are
much more dull and obſcure than theſe, ſo will they be
likewiſe much more diffuſed; and therefore can hardly af-
fect the ſenſe in compariſon of the other. And agree-
able to this is the obſervation of aſtronomers, that
teleſcopes between twenty and ſixty feet in length re-
preſent the fixed ſtars, as being about 5 or 6, at moſt
about 8 or 10 ſeconds in diameter. Whereas other argu-
ments ſhew us, that they do not really appear to us of any
ſenſible magnitude any otherwiſe than as their light is
dilated by refraction. One proof that the fixed ſtars do
not appear to us under any ſenſible angle is, that when
the moon paſſes over any of them, their light does not, like
the planets on the ſame occaſion, diſappear by degrees, but
vaniſhes at once.

21. O u r author being thus convinced, that teleſcopes
were not capable of being brought to much greater perfection
than at preſent by refractions, contrived one by reflection, in
which there is no ſeparation made of the different coloured
light; for in every kind of light the rays after reflection
have the ſame degree of inclination to the ſurface, from
whence they are reflected, as they have at their incidence, ſo

that thofe rays which come to the furface in one line, will go off alfo in one line without any parting from one another. Accordingly in the attempt he fucceeded fo well, that a fhort one, not much exceeding fix inches in length, equalled an ordinary telefcope whofe length was four feet. Inftruments of this kind to greater lengths, have of late been made, which fully anfwer expectation [a].

Chap. V.
Of the RAINBOW.

I SHALL now explain the rainbow. The manner of its production was underftood, in the general, before Sir Isaac Newton had difcovered his theory of colours; but what caufed the diverfity of colours in it could not then be known, which obliges him to explain this appearance particularly; whom we fhall imitate as follows. The firft perfon, who exprefsly fhewed the rainbow to be formed by the reflection of the fun-beams from drops of falling rain, was Antonio de Dominis. But this was afterwards more fully and diftinctly explained by Des Cartes.

2. There appears moft frequently two rainbows; both of which are caufed by the forefaid reflection of the funbeams from the drops of falling rain, but are not produced by all the light which falls upon and are reflected from the drops. The inner bow is produced by thofe rays only which enter the drop, and at their entrance are fo refracted as to unite into a point, as it were, upon the farther furface of the drop, as is reprefented in fig. 160; where the contiguous rays a b, c d, e f, coming from the

[a] Philof. Tranf. No. 378.

fun,

fun, and therefore to fenfe parallel, upon their entrance in-
to the drop in the points *b*, *d*, *f*, are fo refracted as to meet
together in the point *g*, upon the farther furface of the drop.
Now thefe rays being reflected nearly from the fame point
of the furface, the angle of incidence of each ray upon
the point *g* being equal to the angle of reflection, the
rays will return in the lines *g h*, *g k*, *g l*, in the fame man-
ner inclined to each other, as they were before their inci-
dence upon the point *g*, and will make the fame angles with
the furface of the drop at the points *h*, *k*, *l*, as at the points
b, *d*, *f*, after their entrance ; and therefore after their emer-
gence out of the drop each ray will be inclined to the fur-
face in the fame angle, as when it firft entered it ; whence
the lines *h m*, *k n*, *l o*, in which the rays emerge, muft be
parallel to each other, as well as the lines *a b*, *c d*, *e f*, in
which they were incident. But thefe emerging rays being
parallel will not fpread nor diverge from each other in
their paffage from the drop, and therefore will enter the
eye conveniently fituated in fufficient plenty to caufe a
fenfation. Whereas all the other rays, whether thofe nearer
the center of the drop, as *p q*, *r s*, or thofe farther off, as
t u, *w x*, will be reflected from other points in the hin-
der furface of the drop ; namely, the ray *p q* from the point
y, *r s* from *z*, *t v* from *α*, and *w x* from *β*. And for this
reafon by their reflection and fucceeding refraction they
will be fcattered after their emergence from the foremen-
tioned rays and from each other, and therefore cannot en-
ter the eye placed to receive them copious enough to excite
any diftinct fenfation.

<div align="center">E e e 2</div>

3. The external rainbow is formed by two reflections made between the incidence and emergence of the rays; for it is to be noted, that the rays $g\,h$, $g\,k$, $g\,l$, at the points h, k, l, do not wholly pafs out of the drop, but are in part reflected back; though the fecond reflection of thefe particular rays does not form the outer bow. For this bow is made by thofe rays, which after their entrance into the drop are by the refraction of it united, before they arrive at the farther furface, at fuch a diftance from it, that when they fall upon that furface, they may be reflected in parallel lines, as is reprefented in fig. 161; where the rays $a\,b$, $c\,d$, $e\,f$, are collected by the refraction of the drop into the point g, and paffing on from thence ftrike upon the furface of the drop in the points h, k, l, and are thence reflected to m, n, o, paffing from h to m, from k to n, and from l to o in parallel lines. For thefe rays after reflection at m, n, o will meet again in the point p, at the fame diftance from thefe points of reflection m, n, o, as the point g is from the former points of reflection h, k, l. Therefore thefe rays in paffing from p to the furface of the drop will fall upon that furface in the points q, r, s in the fame angles, as thefe rays made with the furface in b, d, f, after refraction. Confequently, when thefe rays emerge out of the drop into the air, each ray will make with the furface of the drop the fame angle, as it made at its firft incidence; fo that the lines $q\,t$, $r\,v$, $s\,w$, in which they come from the drop, will be parallel to each other, as well as the lines $a\,b$, $c\,d$, $e\,f$, in which they came to the drop.

drop. By this means thefe rays to a fpectator commodioufly fituated will become vifible. But all the other rays, as well thofe nearer the center of the drop *x y*, *z α*, as thofe more remote from it *β γ*, *Δ ε*, will be reflected in lines not parallel to the lines *h m*, *k n*, *l o*; namely, the ray *x y*, in the line *ζ η*, the ray *z α* in the line *θ x*, the ray *β γ* in the line *λ μ*, and the ray *Δ ε* in the line *ι ξ*. Whence thefe rays after their next reflection and fubfequent refraction will be fcattered from the forementioned rays, and from one another, and by that means become invifible.

4. It is farther to be remarked, that if in the firft cafe the incident rays *a b*, *c d*, *e f*, and their correfpondent emergent rays *h m*, *k n*, *l o*, are produced till they meet, they will make with each other a greater angle, than any other incident ray will make with its correfponding emergent ray. And in the latter cafe, on the contrary, the emergent rays *q t*, *r v*, *s w* make with the incident rays an acuter angle, than is made by any other of the emergent rays.

5. Our author delivers a method of finding each of thefe extream angles from the degree of refraction being given; by which method it appears, that the firft of thefe angles is the lefs, and the latter the greater, by how much the refractive power of the drop, or the refrangibility of the rays is greater. And this laft confideration fully compleats the doctrine of the rainbow, and fhews, why the colours of each bow are ranged in the order wherein they are feen.

6. Suppose A (in fig. 162.) to be the eye, B,C,D,E,F, drops of rain, M n, O p, Q r, S t, V w parcels of rays of the fun, which entring the drops B, C, D, E, F after one reflection pass out to the eye in A. Now let M n be produced to n till it meets with the emergent ray likewife produced, let O p produced meet its emergent ray produced in x, let Q r meet its emergent ray in λ, let S t meet its emergent ray in μ, and let V w meet its emergent ray produced in ν. If the angle under M n A be that, which is derived from the refraction of the violet-making rays by the method we have here fpoken of, it follows that the violet light will only enter the eye from the drop B, all the other coloured rays paffing below it, that is, all thofe rays which are not fcattered, but go out parallel fo as to caufe a fenfation. For the angle, which thefe parallel emergent rays makes with the incident in the moft refrangible or violet-making rays, being lefs than this angle in any other fort of rays, none of the rays which emerge parallel, except the violet-making, will enter the eye under the angle M n A, but the reft making with the incident ray M n a greater angle than this will pafs below the eye. In like manner if the angle under O x A agrees to the blue-making rays, the blue rays only fhall enter the eye from the drop C, and all the other coloured rays will pafs by the eye, the violet-coloured rays paffing above, the other colours below. Farther, the angle Q λ A corre-fponding to the green-making rays, thofe only fhall enter the eye from the drop D, the violet and blue-making rays paffing above, and the other colours, that is the yellow and red,

red, below. And if the angle S μ A anfwers to the refraction of the yellow-making rays, they only fhall come to the eye from the drop E. And in the laft place, if the angle V ν A belongs to the red-making and leaft refrangible rays, they only fhall enter the eye from the drop F, all the other coloured rays paffing above.

7. But now it is evident, that all the drops of water found in any of the lines A κ, A λ, A μ, A ν, whether farther from the eye, or nearer than the drops B, C, D, E, F, will give the fame colours as thefe do, all the drops upon each line giving the fame colour; fo that the light reflected from a number of thefe drops will become copious enough to be vifible; whereas the reflection from one minute drop alone could not be perceived. But befides, it is farther manifeft, that if the line A $\approx$ be drawn from the fun through the eye, that is, parallel to the lines M n, O p, Q r, S t, V w, and if drops of water are placed all round this line, the fame colour will be exhibited by all the drops at the fame diftance from this line. Hence it follows, that when the fun is moderately elevated above the horizon, if it rains oppofite to it, and the fun fhines upon the drops as they fall, a fpectator with his back turned to the fun muft obferve a coloured circular arch reaching to the horizon, being red without, next to that yellow, then green, blue, and on the inner edge violet; only this laft colour appears faint by being diluted with the white light of the clouds, and from another caufe to be mentioned hereafter [a].

[a] § 11.

8. Thus

8. Thus is caufed the interior or primary bow. The drops of rain at fome diftance without this bow will caufe the exterior or fecondary bow by two reflections of the fun's light. Let thefe drops be G, H, I, K, L; X y, Z α, $\Gamma \beta$, $\Delta \epsilon$, $\odot \zeta$ denoting parcels of rays which enter each drop. Now it has been remarked, that thefe rays make with the vifible refracted rays the greateft angle in thofe rays, which are moft refrangible. Suppofe therefore the vifible refracted rays, which pafs out from each drop after two reflections, and enter the eye in A, to interfect the incident rays in π, ρ, σ, τ, φ refpectively. It is manifeft, that the angle under $\odot \varphi$ A is the greateft of all, next to that the angle under $\Delta \tau$ A, the next in bignefs will be the angle under $\Gamma \sigma$ A, the next to this the angle under Z ρ A, and the leaft of all the angle under X π A. From the drop L therefore will come to the eye the violet-making, or moft refrangible rays, from K the blue, from I the green, from H the yellow, and from G the red-making rays; and the like will happen to all the drops in the lines A π, A ρ, A τ, A φ, and alfo to all the drops at the fame diftances from the line A $\approx$ all round that line. Whence appears the reafon of the fecondary bow, which is feen without the other, having its colours in a contrary order, violet without and red within; though the colours are fainter than in the other bow, as being made by two reflections, and two refractions; whereas the other bow is made by two refractions, and one reflection only.

9. There

9. There is a farther appearance in the rainbow particularly defcribed about five years ago [a], which is, that under the upper part of the inner bow there appears often two or three orders of very faint colours, making alternate arches of green, and a reddifh purple. At the time this appearance was taken notice of, I gave my thoughts concerning the caufe of it [b], which I fhall here repeat. Sir Isaac Newton has obferved, that in glafs, which is polifhed and quick-filvered, there is an irregular refraction made, whereby fome fmall quantity of light is fcattered from the principal reflected beam [c]. If we allow the fame thing to happen in the reflection whereby the rainbow is caufed, it feems fufficient to produce the appearance now mentioned.

10. Let AB (in fig. 162.) reprefent a globule of water, B the point from whence the rays of any determinate fpecies being reflected to C, and afterwards emerging in the line CD, would proceed to the eye, and caufe the appearance of that colour in the rainbow, which appertains to this fpecies. Here fuppofe, that befides what is reflected regularly, fome fmall part of the light is irregularly fcattered every way; fo that from the point B, befides the rays that are regularly reflected from B to C, fome fcattered rays will return in other lines, as in BE, BF, BG, BH, on each fide the line BC. Now it has been obferved above [d], that the rays of light in their paffage from one fuperficies of a refracting body to the other undergo alternate fits of
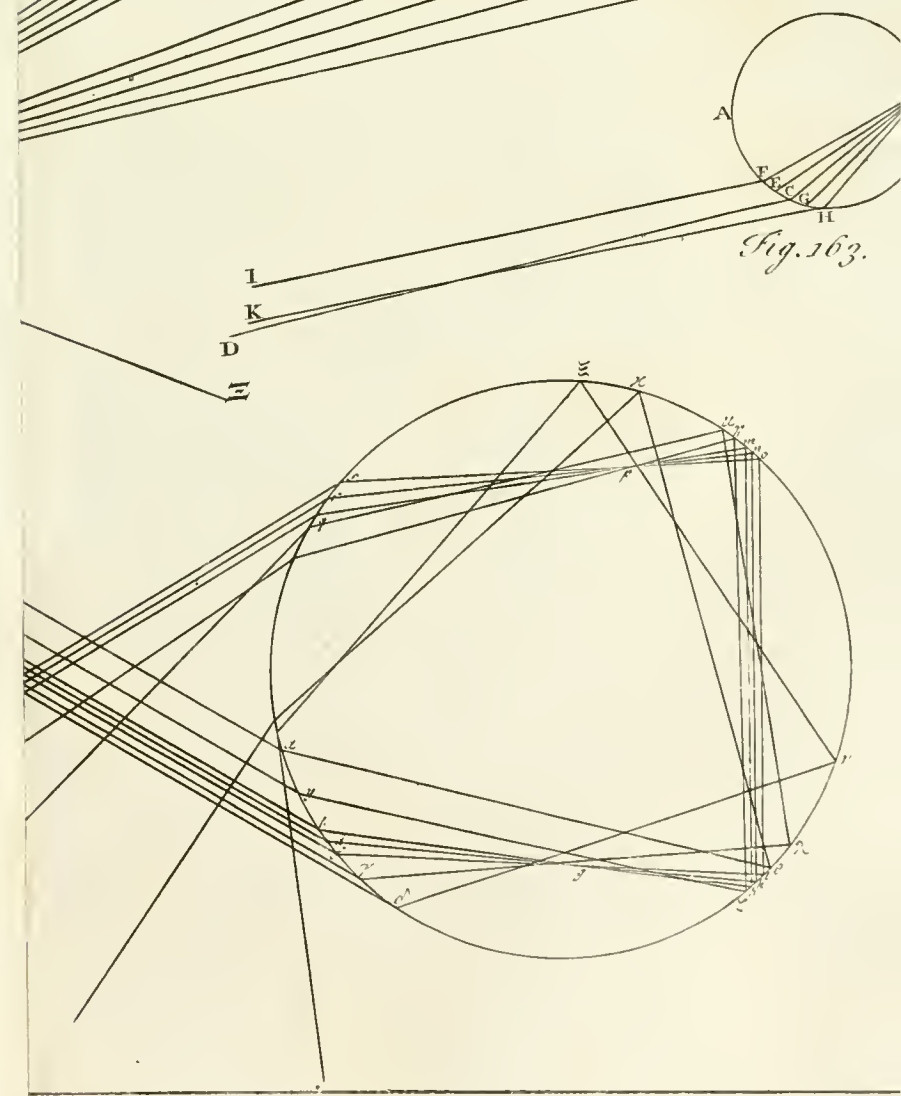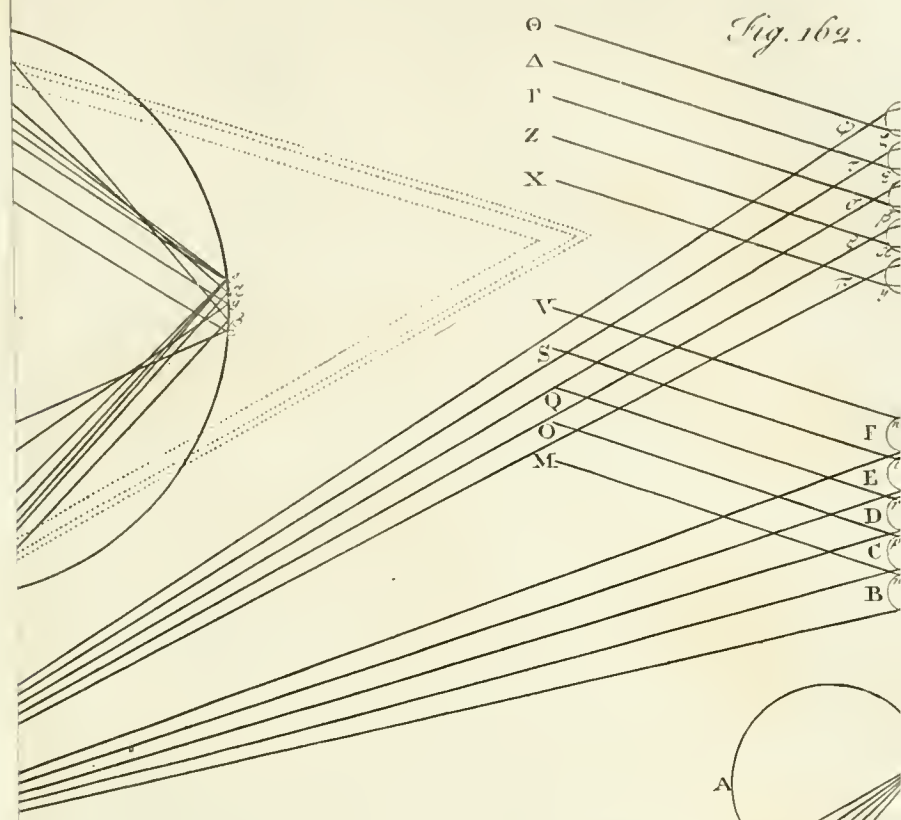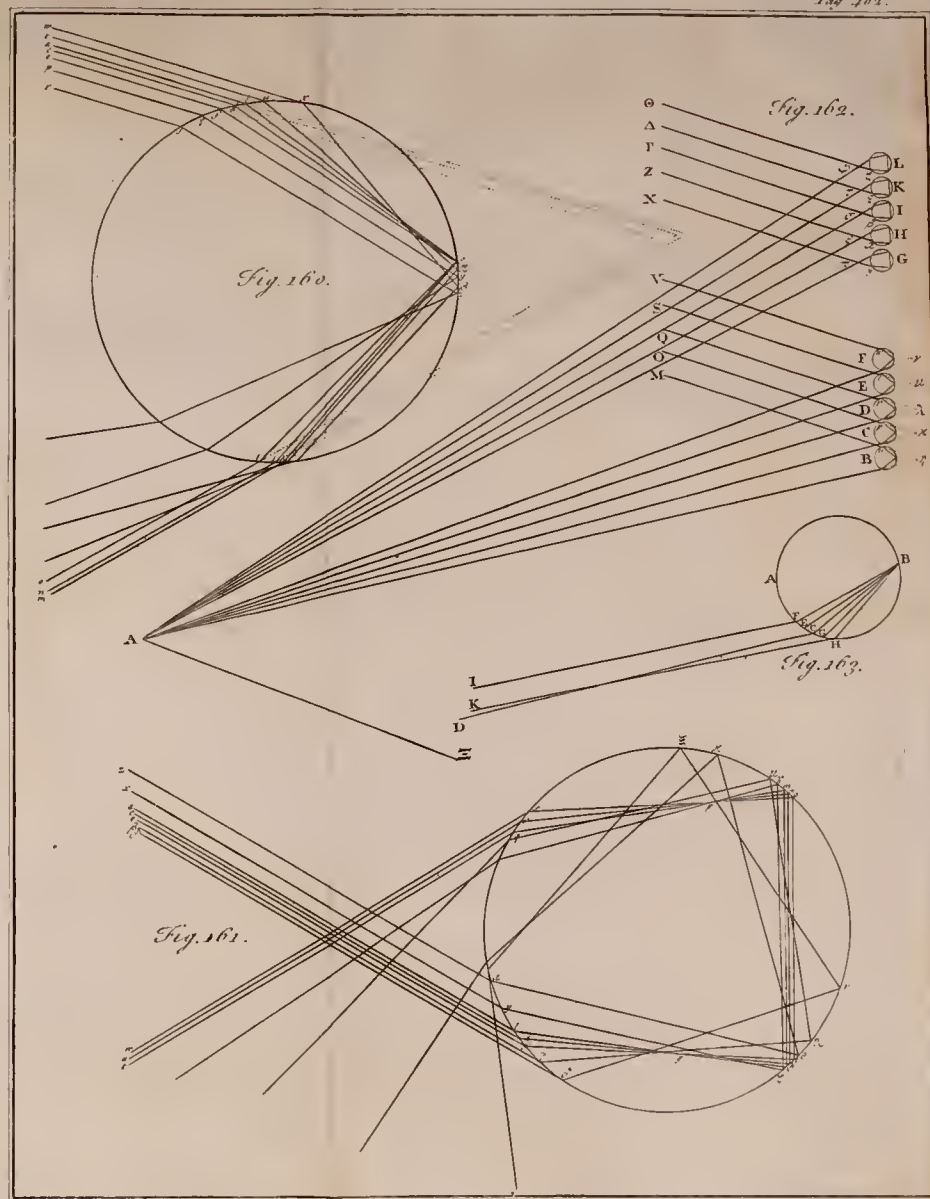
F f f   eafy

eafy tranfmiffion and reflection, fucceeding each other at
equal intervals; infomuch that if they reach the farther fu-
perficies in one fort of thofe fits, they fhall be tranfmitted;
if in the other kind of them, they fhall rather be reflected
back.   Whence the rays that proceed from B to C, and
emerge in the line CD, being in a fit of eafy tranfmiffion,
the fcattered rays, that fall at a fmall diftance without thefe
on either fide (fuppofe the rays that pafs in the lines B E,
B G) fhall fall on the furface in a fit of eafy reflection, and
fhall not emerge; but the fcattered rays, that pafs at fome
diftance without thefe laft, fhall arrive at the furface of the
globule in a fit of eafy tranfmiffion, and break through that
furface.   Suppofe thefe rays to pafs in the lines B F, BH;
the former of which rays fhall have had one fit more of eafy
tranfmiffion, and the latter one fit lefs, than the rays that
pafs from B to C.   Now both thefe rays, when they go out
of the globule, will proceed by the refraction of the water
in the lines F I, H K, that will be inclined almoft equally to
the rays incident on the globule, which come from the fun; but
the angles of their inclination will be lefs than the angle, in
which the rays emerging in the line CD are inclined to
thofe incident rays.   And after the fame manner rays fcatter-
ed from the point B at a certain diftance without thefe
will emerge out of the globule, while the intermediate rays
are intercepted; and thefe emergent rays will be inclined
to the rays incident on the globule in angles ftill lefs than
the angles, in which the rays F I and H K are inclined to
them; and without thefe rays will emerge other rays, that
fhall be inclined to the incident rays in angles yet lefs.   Now
by

Θ
Δ
Γ
Z
X

Fig. 162.

V
S
O
O
M

Γ
E
D
C
B

A

F E C G
H

Fig. 163.

I
K
D
Ξ

Fig. 162.

Fig. 160.

Fig. 163.

Fig. 161.

by this means may be formed of every kind of rays, besides the principal arch, which goes to the formation of the rainbow, other arches within every one of the principal of the same colour, though much more faint; and this for divers successions, as long as these weak lights, which in every arch grow more and more obscure, shall continue visible. Now as the arches produced by each colour will be variously mixed together, the diversity of colours observ'd in these secondary arches may very possibly arise from them.

11. In the darker colours these arches may reach below the bow, and be seen distinct. In the brighter colours these arches are lost in the inferior part of the principal light of the rainbow; but in all probability they contribute to the red tincture, which the purple of the rainbow usually has, and is most remarkable when these secondary colours appear strongest. However these secondary arches in the brightest colours may possibly extend with a very faint light below the bow, and tinge the purple of the secondary arches with a reddish hue.

12. The precise distances between the principal arch and these fainter arches depend on the magnitude of the drops, wherein they are formed. To make them any degree separate it is necessary the drop be exceeding small. It is most likely, that they are formed in the vapour of the cloud, which the air being put in motion by the fall of the rain may carry down along with the larger drops; and this may be the reason, why these colours appear under the upper

part

part of the bow only, this vapour not defcending very low. As a farther confirmation of this, thefe colours are feen ftrongeft, when the rain falls from very black clouds, which caufe the fierceft rains, by the fall whereof the air will be moft agitated.

13. To the like alternate return of the fits of eafy tranf-miffion and reflection in the paffage of light through the globules of water, which compofe the clouds, Sir Isaac Newton afcribes fome of thofe coloured circles, which at times appear about the fun and moon [a].

[a] Opt. B. II. part 4. obf. 13.

CONCLUSION.

SIR Isaac Newton having concluded each of his philofophical treatifes with fome general reflections, I fhall now take leave of my readers with a fhort account of what he has there delivered. At the end of his mathematical principles of natural philofophy he has given us his thoughts concerning the Deity. Wherein he firft obferves, that the fimilitude found in all parts of the univerfe makes it undoubted, that the whole is governed by one fupreme being, to whom the original is owing of the frame of nature, which evidently is the effect of choice and defign. He then proceeds briefly to ftate the beft metaphyfical notions concerning God. In fhort, we cannot conceive either of fpace or time otherwife than as neceffarily

farily exifting; this Being therefore, on whom all others depend, muft certainly exift by the fame neceffity of nature. Confequently wherever fpace and time is found, there God muft alfo be.   And as it appears impoffible to us, that fpace fhould be limited, or that time fhould have had a beginning, the Deity muft be both immenfe and eternal.

2. At the end of his treatife of optics he has propofed fome thoughts concerning other parts of nature, which he had not diftinctly fearched into.   He begins with fome farther reflections concerning light, which he had not fully examined.   In particular he declares his fentiments at large concerning the power, whereby bodies and light act on each other.   In fome parts of his book he had given fhort hints at his opinion concerning this[a], but here he exprefly declares his conjecture, which we have already mentioned[b], that this power is lodged in a very fubtle fpirit of a great elaftic force diffufed thro' the univerfe, producing not only this, but many other natural operations.   He thinks it not impoffible, that the power of gravity it felf fhould be owing to it.   On this occafion he enumerates many natural appearances, the chief of which are produced by chymical experiments. From numerous obfervations of this kind he makes no doubt, that the fmalleft parts of matter, when near contact, act ftrongly on each other, fometimes being mutually attracted, at other times repelled.

3. The attractive power is more manifeft than the other, for the parts of all bodies adhere by this principle.   And the
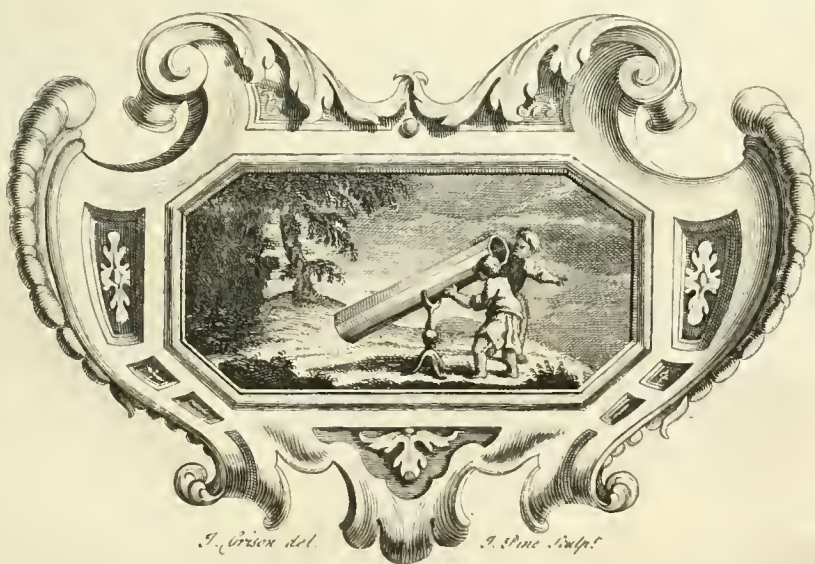
name

[a] Opt. pag. 255.   [b] Ch. 3. § 18.

name of attraction, which our author has given to it, has been very freely made use of by many writers, and as much objected to by others. He has often complained to me of having been misunderstood in this matter. What he says upon this head was not intended by him as a philosophical explanation of any appearances, but only to point out a power in nature not hitherto distinctly observed, the cause of which, and the manner of its acting, he thought was worthy of a diligent enquiry. To acquiesce in the explanation of any appearance by asserting it to be a general power of attraction, is not to improve our knowledge in philosophy, but rather to put a stop to our farther search.

F I N I S.

J. Grison del.   J. Pine Sculp.